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ENTHYMEMES

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Abstract

This paper presents two possible interpretations of the notion of enthymematic entailment. The first explains enthymematic entailment in terms of logical derivability; the second gives an explanation in terms of logical necessity and material implication. Two explanations, previously proposed by Anderson and Belnap, can be formulated as special cases of the second treatment. But the latter does not suffer from the drawbacks that these special cases can easily be seen to have.

ENTHYMEMES *

Many an argument accepted as correct in everyday conversation or in scientific discussion is invalid when considered in the framework of symbolic logic: every plausible symbolization of such an argument into a formal logical system will be invalid. One can however transform such an argument into a valid argument (i.e., one that has a valid natural symbolization) by adding to its premises certain hypotheses that are considered to be evident in the context in which the argument is given. Such elliptical arguments are called enthymemes.

There may be some dispute about what sort of things arguments really are. Here we will identify them with ordered pairs consisting of a set of sentences--the premises of the argument--and a sentence--the conclusion of the argument. This identification does not, we maintain, distort the issues with which this paper deals.

In a general discussion of enthymemes it is convenient not to require that every enthymeme be formally an invalid argument. An enthymeme is to be simply an argument that can be converted into a valid argument by adding certain sentences to its premises. Thus, some, though not necessarily all, arguments are enthymemes; and all valid arguments are enthymemes, though not all enthymemes are necessarily valid arguments.

Whether a given argument is an enthymeme depends on what sentences may be added as premises to convert it into a valid argument. If, for example,

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one admits all true sentences for this purpose, then for every true sentence ϕ, $<\wedge,\phi>$ (where $\wedge$ is the empty set) will be an enthymeme because we may add ϕ to the premises and obtain $<\{\phi\},\phi>$, which is a valid argument since ϕ follows from itself (in any plausible sense of "follows").

On the other hand, we may consider the limiting case where the set of sentences that we admit as additional premises is empty. Here an argument will be an enthymeme only if it is valid. In general, the question as to whether a given argument is an enthymeme depends on the set of sentences that may be added as premises. We will refer to this set as <u>the background knowledge</u>.

The concept of which we want to give a formal account is "The argument $<S,s>$ is an enthymeme relative to the background knowledge Δ," or as we will say henceforth:

(1) "s follows enthymematically from S, modulo Δ."

An explication of (1) within a formal system depends on the way in which we treat the word "follows." If we interpret "s follows from S" as "s is a logical consequence of S," then we may proceed thus:

Let $\mathfrak{L}$ be a formal language; let the relation $\Gamma \models \phi$ (read: "ϕ is a consequence of Γ") between sets Γ of formulae of $\mathfrak{L}$ and formulae ϕ of $\mathfrak{L}$ be given, and be such that if $\Gamma \subseteq \Gamma'$ and $\Gamma \models \phi$, then $\Gamma' \models \phi$.

We define:

<u>Definition 1</u>. $\Gamma \models_{(1)} \phi$ (Mod Δ) if $\Gamma \cup \Delta \models \phi$.

<u>Definition 2</u>. $\Gamma \models_{(2)} \phi$ (Mod Δ) if there is a finite subset Δ' of Δ such that $\Gamma \cup \Delta' \models \phi$.

<u>Definition 3</u>. $\Gamma \models_{(3)} \phi$ (Mod Δ) if there is a $X \in \Delta$ such that $\Gamma \cup \{X\} \models \phi$.

Clearly:

(2) for all Γ and ϕ, if $\Gamma \models_{(3)} \phi$ (Mod Δ) then $\Gamma \models_{(2)} \phi$ (Mod Δ),

and

(3) if $\Gamma \models_{(2)} \phi$ (Mod Δ) then $\Gamma \models_{(1)} \phi$ (Mod Δ).

But the converses of (2) and (3) are not true in general. The converse of (3) holds if and only if $\mathcal{L}$ is <u>compact</u> (indeed, the converse of (3) is a natural definition of compactness). The converse of (2) holds in particular if Δ is closed under conjunctions.

We also consider the phrase:

(4) "ϕ follows enthymematically from ψ"

where ψ is a sentence rather than a set of sentences. It seems natural to explicate (4) as "ϕ follows enthymematically from $\{\psi\}$" where the latter phrase can be defined by definition 1, 2, or 3. However, at this point our approach seems less convincing. On the one hand enthymeme (modulo Δ) seems to be a logical ternary relation, the arguments of which are a set of sentences, a sentence, and a set of sentences, respectively; and as such should be explained in terms of metamathematical notions. On the other hand (4) seems to be nothing but a paraphase for:

(5) "ϕ follows from ψ"

in its usual contextually dependent sense. And even though the word "follows" in (4) or (5) does not have the grammatical character of a sentential connective, other English expressions that are presumably paraphases of (4), like

(6) "ϕ, since ψ," "ϕ, for ψ," "ψ; therefore ϕ"

clearly are built up from ϕ and ψ by application of a binary sentential connective. Thus, if we want to give a formal explication of (6) we certainly ought to treat "since" (in the sense intended here) as a binary sentential connective.

In order to give such an explication, let us turn to general pragmatics and let us represent "ϕ follows from ψ" (in the absolute, not contextually dependent, sense of "follows") as $\Box(\phi \rightarrow \psi)$. Let $\mathfrak{L}$ be a language for general pragmatics that contains the 1-place sentential operator '$\Box$' and the 2-place sentential operator '$\ominus\!\!\rightarrow$'. Let $\mathfrak{L}'$ be the language that we obtain by omitting '$\ominus\!\!\rightarrow$' from $\mathfrak{L}$. We define the semantics for $\mathfrak{L}$ in terms of a given semantics for $\mathfrak{L}'$ as follows:

A <u>standard interpretation</u> for $\mathfrak{L}$ is a pair $\langle\mathfrak{A},\Delta\rangle$ where $\mathfrak{A}$ is a possible interpretation* for $\mathfrak{L}'$ and Δ is a set of sentences of $\mathfrak{L}'$. Let $\langle\mathfrak{A},\Delta\rangle$ be a standard interpretation for $\mathfrak{L}$. Let $\mathfrak{A} = \langle A,F,R\rangle$. Satisfaction in $\langle\mathfrak{A},\Delta\rangle$ is defined as follows: Let D be the set of all those $j \in \text{Dom } A$ such that for all $X\in\Delta$, X is true$_{\mathfrak{A},j}$. Let $\mathfrak{A}'$ be the triple $\langle A,F,R'\rangle$, where

(i) $\text{Dom } R' = \text{Dom } R \cup \{\ominus\!\!\rightarrow\}$,

(ii) $R' \upharpoonright \text{Dom } R = R$,

(iii) $R'_{\ominus\!\rightarrow} = \{\langle i,I,K\rangle : i \in \text{Dom } A,\ I \subseteq \text{Dom } A,$
$K \subseteq \text{Dom } A \text{ and } I\cap D \subseteq K.\}$

* See [3]

Then for any formula $\emptyset$ of $\mathfrak{L}$ and assignment $\underline{a}$ (to the variables, of appropriate objects connected with $\mathfrak{A}$) and $i \in$ Dom A, we put:

(7) $\underline{a}$ $sat_{\langle \mathfrak{A}, \Delta \rangle, i}$ $\emptyset$ iff $\underline{a}$ $sat_{\mathfrak{A}', i}$ $\emptyset$.*

One easily verifies that if $\emptyset$ is a formula of $\mathfrak{L}'$, then

(8) $\underline{a}$ $sat_{\langle \mathfrak{A}, \Delta \rangle, i}$ $\emptyset$ iff $\underline{a}$ $sat_{\mathfrak{A}, i}$ $\emptyset$.

Further we see that for sentences $\emptyset$, ψ of $\mathfrak{L}'$

(9) $\emptyset \ e\!\!\rightarrow \psi$ is $true_{\langle \mathfrak{A}, \Delta, i}$ iff for all $j \in$ Dom A which are such that for all $X \in \Delta$, X is $true_{\langle \mathfrak{A}, \Delta \rangle, j}$ it is the case that if $\emptyset$ is $true_{\langle \mathfrak{A}, \Delta \rangle, j}$ then ψ is $true_{\langle \mathfrak{A}, \Delta \rangle, j}$.

Thus, in any standard interpretation $(\mathfrak{A}, \Delta)$ for $\mathfrak{L}$, "$\emptyset \ e\!\!\rightarrow \psi$" can be regarded as meaning: "ψ follows enthymematically from $\emptyset$ relative to the background knowledge Δ."

The representation of enthymematic implication developed here shows a strong resemblance to Definition 1 above. However, by modifying (iii) above we obtain representations which show resemblance to Definitions 2 and 3 respectively, rather than to Definition 1. For replacement of (iii) by

(iii') $R'_{e\rightarrow}$ $\{\langle i, I, K \rangle$: $i \in$ Dom A and $I \subseteq$ Dom A and there is a finite subset Δ_o of Δ such that $K \supseteq I \cap \{j \in$ Dom A: for all $X \in \Delta_o$ X is $true_{\mathfrak{A}, j}\}\}$

instead of (9) gives

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* For the definition of '$\underline{a}$ $sat_{\mathfrak{A}', i}$ $\emptyset$' see [3].

(9') $\phi \rightarrowtail \psi$ is $\text{true}_{<\mathfrak{A},\Delta>,i}$ if there is a finite subset Δ_o of Δ such that for all $j \in$ Dom A it is the case that if for all $X \in \Delta_o$, X is $\text{true}_{<\mathfrak{A},\Delta>,j}$ then if ϕ is $\text{true}_{<\mathfrak{A},\Delta>,j}$, ψ is $\text{true}_{<\mathfrak{A},\Delta>,j}$;

and replacement of (iii) by

(iii") $R'_{\rightarrowtail} = \{<i,I,K>: i \in \text{Dom } A \text{ and } I \subseteq \text{Dom } A$ and $K \subseteq \text{Dom } A$ and there is a $X \in \Delta$ such that $K \supseteq I \cap \{j \in \text{Dom } A$: X is $\text{true}_{\mathfrak{A},j}\}\}$

gives (9") $\phi \rightarrowtail \psi$ is $\text{true}_{<\mathfrak{A},\Delta>,i}$ if there is $X \in \Delta$ such that for all $j \in$ Dom A if X is $\text{true}_{<\mathfrak{A},\Delta>,j}$ and ϕ is $\text{true}_{<\mathfrak{A},\Delta>,j}$ then ψ is $\text{true}_{<\mathfrak{A},\Delta>,j}$.

Clause (iii") leads to a formal explication of enthymematic implication that comes close to the proposal given by Anderson and Belnap [1], p. 719: they define "p enthymematically implies q" (where 'p' and 'q' are propositional variables) as

(10) $(\exists r)\,(r \wedge (p \wedge r \prec q))$, where $\prec$ is the entailment relation previously formalized by them in their system E [2]. Indeed, Anderson and Belnap reject the explication of "p follows from q" by "$\Box\,(q \rightarrow p)$," for the reasons hinted at in [2].

If, however, we disregard this difference in the representation of "p follows from q," their proposal is really a special case of our last explication (the one based upon clause (iii"). Indeed we may restrict the notion of a standard interpretation for $\mathcal{L}$ to those pairs $<<A,F,R>,\Delta>$ where $0 \in$ Dom A (we think of 0 as the point of reference to the actual world)

and Δ is the set of sentences of $\mathcal{L}$ which are true$_{<<A,F,R>,\Delta,0}$.
In such interpretations $<\mathfrak{A},\Delta>$ we have according to (iii"),

(11) $\phi \to \psi$ is true$_{<\mathfrak{A},\Delta>,0}$ if there is a $X \epsilon \Delta$ such that for all $j \in$ Dom A if X is true$_{<\mathfrak{A},\Delta>,0}$ and ϕ is true$_{<\mathfrak{A},\Delta>,0}$ then $\phi e\!\!\to \psi$ is true$_{<\mathfrak{A},\Delta>,0}$.

Then, if ψ is true$_{<\mathfrak{A},\Delta>,0}$, ϕ e→ψ is true$_{<\mathfrak{A},\Delta>,0}$. (Take ψ for X in (11); if ϕ is not true$_{<\mathfrak{A},\Delta>,0}$ then again, taking ϕ for X, we see that ϕ e→ψ is true$_{<\mathfrak{A},\Delta>,0}$. So if $\phi \to \psi$ is true$_{<\mathfrak{A},\Delta>,0}$ then ϕe→ψ is true$_{<\mathfrak{A},\Delta>,0}$. On the other hand, if $\phi \to \psi$ is not true$_{<\mathfrak{A},\Delta>,0}$, then clearly ϕ e→ψ is not true$_{<\mathfrak{A},\Delta>,0}$.)

Enthymematic entailment thus reduces to material implication. That in such interpretations enthymematic entailment collapses with material implication is in part due to the way in which we have represented "follows from." But the fact that "ϕe→ψ" holds whenever ψ is true is independent of this representation, and this fact alone makes this notion of entailment unsatisfactory.

In another proposal, Anderson and Belnap render "p enthymematically implies q" as '$(\exists r)\ (\Box r \wedge (p \wedge r \to q))$' ([1], p. 721). This proposal corresponds to the following restriction on standard interpretations:
We consider only those interpretations $<<A,F,R>,\Delta>$ where $0 \in$ Dom A and Δ_N is the set of sentences of $\mathcal{L}$ that are true$_{\mathfrak{A},i}$ for all $i \in$ Dom A.
Then according to (9"):

ϕe→ψ is true$_{<\mathfrak{A},\Delta_N>0}$ iff for all $j \in$ Dom A if ϕ is true$_{<\mathfrak{A},\Delta_N>,j}$ then ψ is true$_{<\mathfrak{A},\Delta_N>j}$, and so $\phi \,e\!\rightarrow \psi$ reduces to $\Box\,(\phi \rightarrow \psi)$; so this solution is also unsatisfactory. Indeed, the interesting cases are those where $\Delta_O \subset \Delta \subset \Delta_N$. Only in such a case may we expect some sentences $\phi \rightarrow \psi$ to be true (at 0), whereas $\phi \,e\!\rightarrow \psi$ is false (at 0) and at the same time some sentences $\phi \,e\!\rightarrow \psi$ to be true, whereas $\Box\,(\phi \rightarrow \psi)$ fails.

References

[1] Anderson, A.R., and Belnap, N.D.,"Enthymemes," Journal of Philosophy LVIII; No. 23, 1961, pp. 713-723.

[2] _______ "The Pure Calculus of Entailment," Journal of Symbolic Logic, Vol. 27, No. 1, March 1962.

[3] Montague, Richard, Pragmatics and Extensional Logic (forthcoming).