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INHALT

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QUANTIFICATION AND REFERENCE IN MODAL AND TENSE LOGIC

This paper is based on ideas which occurred to me between the summer of 1970 and the fall of 1972. In its present form it is little more than a compilation of these ideas. A more polished version should wait, I feel, until they have been subjected to a good deal of criticism from others. I hope very much therefore, that circulation of these pages among the members of the present conference may provoke some critical comments.

Throughout my concern will be with the proper semantical analysis of sentences in which singular terms occur inside the scope of tense or modal operators. These singular terms may be complex descriptions, proper names, or variables bound by quantifiers which themselves occur either inside or outside the scope of the operator in question. We will see that each of these different types of singular terms gives rise to its own problems, though most of these are of course related.

The central idea of this paper concerns the semantics of modal quantification theory. It is explained in section 3. I first conceived of this particular way of looking at the semantics of quantified modal logic in the summer of 1970. Public reports on the issue were given in two talks during the spring of 1972, one to the London Philosophy Group and one to the Department of Philosophy of the University of Uppsala. The following sections (4-6) contain elaborations of the central idea in various directions, and in particular an investigation whether, and if so to what extent, the central idea applies to tense as well as to modal quantification theory. Many of the remarks contained in these section are suggestions rather than theses, and I will be particularly grateful for comments on them. The material contained in sections 1 and 2 is not in any way original; however, it seemed necessary to include it in order to render the subsequent parts accessible to those who have no close acquaintance with this particular formal way of developing intensional and pragmatic logic.

It is only after I had formulated the semantics contained in section 3, — though before I wrote the sections 4-6 — that I read Kripke's 'Naming and Necessity'. It seems to me now that many features of the semantical theory I here sketch might be regarded as formal counterparts of points which Kripke makes. So I am left with the unhappy feeling that what I am submitting is likely to appear as rather derivative. Yet, some of the observations in the last three sections I have not encountered so far in other published work; so for all I know they are new; and the material in section 3 is indispensible for their proper understanding, — which might justify its inclusion.

SECTION 1

The problems with which this paper is concerned will all be discussed with reference to quite simple formal languages of tense and modal logic (or extentions thereof which will be explicitly described later on as the need for them arises), and within the framework provided by the kind of model theory for these languages which has by now won wide—if not universal — acceptance by those concerned with such languages. This theory is least controversial when applied to languages of sentential (tense or modal) logic — although the application to modal logic already raises important and difficult philosophical questions, viz. those pertaining to the nature of possible worlds. Thus it seems best to first explain the theory in connection with these sentential languages. This has the additional advantage that the essential new features which distinguish the theory from the standard model theory for extensional logic appear clearly, unencumbered by the technical complications connected with satisfaction which necessarily arise in any semantical treatment of languages that contain quantifiers.

The languages of sentential tense and modal logic which I will here consider are both based on a simple language L_0 of ordinary sentential logic. Its symbols are:

1) the sentential letters $q_1, q_2, q_3, \ldots$;

2) the 1-place sentential connective $\neg$ (‚not'), and the 2-place sentential connectives $\rightarrow$, (‚if ..., then'), $\wedge$ (‚and'), $\vee$ (‚or'), and $\longleftrightarrow$ (‚if and only if').

According to the standard model theory for extensional logic a model for L is simply a function which assigns to each letter q_i a truthvalue, 1 (‚true') or 0 (‚false'). The truth values of arbitrary complex formulae of L in the model M are then determined by the general recursive clauses (where $[\varphi]_M$ stands for the truth value of φ in M):

T1.	$[\neg\varphi]_M = 1$	iff	$[\varphi]_M = 0$
T2.	$[\varphi \rightarrow \psi]_M = 1$	iff	either $[\varphi]_M = 0$ or $[\psi]_M = 1$
T3.	$[\varphi \wedge \psi]_M = 1$	iff	$[\varphi]_M = 1$ and $[\psi]_M = 1$
T4.	$[\varphi \vee \psi]_M = 1$	iff	$[\varphi]_M = 1$ or $[\psi]_M = 1$
T5.	$[\varphi \longleftrightarrow \psi]_M = 1$	iff	either $[\varphi]_M = 1$ and $[\psi]_M = 1$, or $[\varphi]_M = 0$ and $[\psi]_M = 0$

We may further characterize the *logical truths*, or *valid formulae* of L_0 as those formulae which are true in all models, and stipulate that a formula φ is a *logical consequence of* a set Γ of formulae of L_0 if φ is true in each model in which all members of Γ are true.

In any model of the kind discussed each formula of L_0 has got exactly one truth value: The formula is either simply true in the model or else simply false in the model.

One of the uses to which symbolic languages such as L are often put is the formal representation by formulae of these languages of sentences that belong to natural languages. It is tempting to interpret the correspondence between, say, L_0 and English which must exist if there is to be any sense at all to this practice of symbolic representation, in such a way that the models for L_0 are the formal counterparts of what might intuitively be regarded as the possible models for English. Now it is by no means clear what a possible model for English should be in general. But it seems plausible at least that the actual world would provide one of these models. However, the relation between English and the actual world is importantly different from that between L_0 and its models as defined above. For, as I remarked already, in any model of L_0 each formula of L_0 is either true (once and for all) or false (once and for all); of most English sentences it is impossible to maintain that they are true or false simpliciter. They will be true when used on one occasion, and false when used on another. Indeed it is in part for this reason that it is often denied that that sentences can have truth values at all; it is rather, so the contention goes, the statements we make by using sentences on particular occasions which can be said to be true. I will not adress myself to this issue here, but limit myself to the observation that if sentences can be said to have truth values at all they can in the majority of cases only be said to have them relative to particular contexts of use.

There are various ways in which we could try to explain away this model theoretic discrepancy between English and L_0. We may conclude that it is not the actual world as such which we should regard as one of the possible models for English, but that what functions as the actual model is really the world when perceived from the perspective of a particular context of use. Or we may conclude that most so-called ‚sentences' of English are not really sentences in the logical sense of the word, but rather formulae containing, usually covertly, certain free variables whose values are determined by the particular speech situations in which these 'sentences' are used.

I will not discuss the merits of these proposals now; instead I will state an alternative approach which consists in removing the discrepancy by so modifying the model theory for L that it squares with the relation between English and the world as it appears to the naïve observer:

Let C be a collection of (possible) contexts (of speech use). A *model relative to* C is, by definition, a function which assigns to each pair consisting of a letter q_i and a member c of C a truthvalue, the 'truthvalue of

q_i in c' in the model. The truthvalue in such a model M of a complex formula of L_0 in a context c is again determined by general recursive clauses T^C1-T^C5, where T^C1 is:

$$T^C1 \quad [\neg\varphi]^C_{M,c} = 1 \text{ iff } \quad [\varphi]^C_{M,c} = 0,$$

and the remaining clauses are obtained in a similar way from the clauses T2-T5 viz. by adding the superscript C and the subscript c to all occurring expressions of the form $[\ ]_M$.

We may now define a formula as *valid relative to* C if it is true in all members of C in all models relative to C. We may also characterize a formula as *valid absolutely* if it is valid relative to all non-empty sets (of contexts) C. (We could of course similarly adapt our earlier definition of logical consequence.)

It is clear from the recursive clauses T^C1-T^C5 that the truth value of a complex formula of L_0 in a given context is completely determined by the truthvalues of its components in that same context c. Indeed, it is precisely in this way that the so-called truthfunctional character of the connectives of L_0 manifests itself within the framework of context dependent model theory. The new framework allows, however, also for the semantical analysis of connectives which are not truthfunctional in this sense. For example, we may extend L with the 1-place connective P, reading ‚it was the case that'. The truth value of a formula P φ in a context c will then not be determined by the truth value of φ in c itself; rather it will be true in c depending on whether φ is true in some (appropriate) context which temporally precedes c: if and only φ is true in such a context will P φ be true in c. Similarly we might introduce a 1-place connective F, reading ‚it will be the case that'; the truthvalue of F φ in c will then depend on whether there are propriate contexts later than c in which φ is true.

We can give a simple coherent formulation of the model theory for the language L_0 (P, F) (i.e. L_0 extended with P and F) if we assume that the only contextual feature on which truthvalues explicitly depend is the time of utterance. (This of course fails to be true of many English sentences; it holds for ‚There is more gold in the universe than silver' and, approximately at least, for ‚England's Head of State is a woman', but not e.g. for ‚It is raining', which may be true in one context of use and yet false in another which, though simultaneous with the first, differs from it in location. On this assumption we may identify contexts with the times of their occurrences and replace the last theory by the following:

Let $\mathcal{T}$ be a linear structure, i.e. a pair $\langle T, < \rangle$, where T is a set (to be thought of as the set of moments of time) and $<$ a linear ordering of T (to be thought of as the earlier-later relation between moments. A *model for* L_0 (P, F) *relative to* $\mathcal{T}$ is a function which assigns to each q_i and each t in T a truth value, the truth value of q_i at t. The truth values in such models of the complex formulae of L_0 (P, F) are again be determined by general recursive clauses:

$T^\tau 1 \quad [\neg\varphi]^\tau_{M,t} = 1$ iff $[\varphi]^\tau_{M,t} = 0$

Similarly for $T^\tau 2$–$T^\tau 5$

$T^\tau 6 \quad [P\,\varphi]^\tau_{M,t} = 1$ iff $[\varphi]^\tau_{M,t'} = 1$ for some t' in T such that t' < t.

$T^\tau 7 \quad [F\,\varphi]^\tau_{M,t} = 1$ iff $[\varphi]^\tau_{M,t'} = 1$ for some t' in T such that t < t'.

A formula of L_0 (P,F) is *valid relative to* $\mathcal{T}$ if it is true in every model relative to $\mathcal{T}$ at each member of T. It is an interesting fact – though of little consequence for the particular problems which are discussed in the later sections of this paper – that the set of formulae valid relative to τ depends essentially on the structure of τ. For this reason it is of interest to introduce besides the notions of validity relative to particular time structures and absolute validity also the notion of validity relative to a class of linear structures: φ is *valid relative to* the class $\mathfrak{K}$ of linear orderings if φ is valid relative to every member of $\mathfrak{K}$. Interesting cases are e.g. the one where $\mathfrak{K}$ is the class of all dense linear orderings without endpoints, or that where $\mathfrak{K}$ is the class of all linear orderings which are discrete – in the sense that for any element in the ordering for which there is a later (earlier) element there is a next (immediately preceding) element.

This very same type of model theory can also be employed for the analysis of modal notions, in particular for modal connectives such as ‚it is necessarily the case that', or ‚it is possibly the case that'. As the latter of these expressions is equivalent to ‚it is not necessarily the case that not' I will pay explicit attention only to the former.

So let's extend L_0 to $L_0(\Box)$ by adding the 1-place connective reading ‚it is necessarily the case that'. The idea on which the semantics for $L_0(\Box)$ rests goes back to Leibnitz, according to whom to be the case necessarily is to be the case in all possible worlds. In Kripke's refinement of this idea a formula of the form $\Box\,\varphi$ is true in a given world w iff φ is true in all worlds which are possible from the point of view of w; where the eventuality that what is possible from the perspective of one world is not so from the perspective of some other is not excluded.

This leads to the following model theory for $L_0(\Box)$:

Let $\mathfrak{W}$ be a pair ⟨W, R⟩ where W is a set (of possible worlds) and R a binary relation on W (to be thought of as the relation which holds between w and w' if w' is a possible alternative from the perspective of w). A *model for* $L_0(\Box)$ *relative to* $\mathfrak{W}$ is a function which assigns to each pair $\langle q_i, w\rangle$ a truth value, the truth value of q_i in w. The recursive clauses that determine the truth values of the complex formulae now take the following form:

$T^\omega 1 \quad [\neg\varphi]^\omega_{M,w} = 1$ iff $[\varphi]^\omega_{M,w} = 0$

Similary for $T^\omega 2$–$T^\omega 5$

$T^\omega 6 \quad [\Box\varphi]^\omega_{M,w} = 1$ iff $[\varphi]^\omega_{M,w'} = 1$ for all w' ε W such that w Rw'

Definitions for $L_0(\Box)$ of validity, whether absolute, relative to particular pairs $\langle W, R\rangle$ or relative to classes of such pairs can be given in exact analogy with those for $L_0(P, F)$.

The problem of what possible worlds are (if they are anything at all!) goes beyond the scope of this paper. I do myself believe that reasonable sense can be made of the notion; here I will simply assume this. We will see shortly, however, that even if we accept the notion of a possible world as such there are still serious questions to be met when the same basic idea is applied to the model theory of quantified modal logic.

SECTION 2

The languages of sentential tense and modal logic we considered were obtained through the addition of tense or modal operators to the language of ordinary sentential logic L_0. In the same way we may obtain languages for tense or modal predicate logic by means of such additions to a language of predicate logic. The predicate language from which we will take our departure here is the language L, which contains apart from the usual logical vocabulary (the connectives $\neg$, $\rightarrow$, $\wedge$, $\vee$, $\longleftrightarrow$, the quantifiers $\exists$, and $\forall$, the identity sign $=$, and the individual variables v_1, v_2, v_3, . . .) a only non-logical constants the 1-place predicate letter Q, the 2-place predicate letter R and the two individual constants c and d. We will consider the language L(P,F) of tense predicate logic, and the language $L(\Box)$ of modal predicate logic, which result if we add to L either P and F or $\Box$. The model theory for these languages can be obtained by modifying the standard model theory for L in the same way in which we had to alter the semantics for L_0 in order to arrive at the model theory for L_0 (P,F) and L_0 ($\Box$). However, as we will soon be forced to notice, the new modifications raise problems far more serious than those encountered in connection with the sentential languages, especially in the case of modal logic. Let us therefore first consider the semantics for L (P,F). (We will see later that the semantics for languages of tense predicate logic is indeed much more problematic than it may appear at first. Yet as a first approximation the model theory for L(P,F) which I will give in the course of this section will do. In any case it seems the best way to get started on the problems which will receive more careful attention subsequently.

I will base this model theoretic account of L(P,F) – and subsequently that of $L(\Box)$ – on a formulation of the standard model theory for L which is slightly different from the usual presentations of that theory. It seems therefore advisable to begin by giving this formulation of the theory as it applies to L itself. A *model* M *for* L provides:

i) a universe (of discourse) U^M.

ii) an appropriate, extensional, interpretation for the predicate Q, i.e. a subset Q^M of U^M, the ‚set of things satisfying Q'.

iii) an appropriate extensional interpretation of R, i.e. a set R^M of ordered pairs of elements of U^M, the set of those pairs of things a and b such that ‚a stands in the relation R to b'.

iv) appropriate extensional interpretations for c and d, i.e. particular elements c^M and d^M of U^M, the ‚referents', or ‚denotata', of c and d, respectively.

(It is not important here in exactly what form these various items of information are cast — we might define a model as an ordered sequence of the objects mentioned under i)-iv), but this immaterial for our purposes.)

It is obvious how i)-iv) determine the truth values of atomic sentences; thus Q(c) is true in M iff the object c^M associated with c in iv) belongs to the extension Q^M of Q in M associated with it in ii); and R(c,d) is true in M iff $\langle c^M, d^M \rangle$ belongs to R^M. It would be desirable to determine the truth values of all complex sentences of L by general recursive clauses, just as we did in the case of L_0. This requires in particular new clauses which express the truth values of sentences of the forms $(\exists v_i)\varphi$ and $(\forall v_i)\varphi$ in terms of simpler sentences. The clauses which suggest themselves naturally are:

T8 $[(\exists v_i)\varphi]_M = 1$ iff $[[\varphi]^{\alpha}/v_i]_M = 1$ for some individual constant α

T9 $[(\forall v_i)\psi]_M = 1$ iff $[[\varphi]^{\alpha}/v_i]_M = 1$ for all individual constants α

($[\varphi]^{\alpha}/v_i$ is the result of replacing all free occurrences of v_i by α)

But these clauses will only be acceptable in case every object in U^M is the referent of some individual constant; for if this is not so it might be that the only objects which satisfy Q are not referents of any constants; and consequently the sentence $(\exists v_1)Q(v_1)$, though intuitively true, would be marked as false by T8; and $(\forall v_1)\neg Q(v_1)$, though false intuitively, would come out true under T9.

The traditional method for getting around this difficulty consists in defining by recursion the relation which holds between an arbitrary formula — whether with or without free variables — and an infinite sequence of individuals in the universe of the model iff the sequence satisfies the formula in the model: It then turns out that any sentence (i.e. formula without free variables) is satisfied by one sequence iff it is satisfied by all; we may therefore say that it is *true* in M if some (or equivalently every) sequence satisfies it.

It is possible however to retain the clauses T8 and T9 if we allow for the passage from the given language with the truth definition for which we are concerned to extentions of that language obtainable through the addition of new individual constants. Given a model M for the language of our concern, say L, and such an extention L′ of L by a set S of new constants, we may consider models for L′ which are extentions of M in the sense that they provide the same information that is contained in the clauses i)-iv) for M. In addition the extention M′ will have to provide v) a referent $s^{M'}$ in U^M for each of the new constants $s \varepsilon S$. Let us call such a model M′ for an extention L′ of L a *referentially perfect* model for L′ if it coordinates each element of U^M with exactly one constant in S. In such a referentially perfect model we can define the truthvalues of all sentences of L′ recursively without any reference to formulae that contain free variables. For this recursive definition we may retain T1 – T5 and modify T8 and T9 into

T8′ $[(\exists v_i)\varphi]_M = 1$ iff $[[\varphi]^{s}/v_i]_M = 1$ for some constant $s \varepsilon S$,
T9′ $[(\forall v_i)\varphi]_M = 1$ iff $[[\varphi]^{s}/v_i]_M = 1$ for each constant $s \varepsilon S$.

The truth value of a sentence of L may then be defined as the truth value of that sentence in any referentially perfect extention M′ of M for a language L′ obtained by adding new constants to L. (This definition is of course acceptable only on the provision that one will obtain the same truth value whichever extensions L′ and M′ one may consider; but this is quite easily proved.)

To obtain a suitable modeltheory for L(P,F) we should extend our earlier observations on the relation between English and the world. Just as a sentence may change its truth value with time, so may a predicate change its extension or a singular term its reference. A model for L(P,F), relative to the structure τ, should therefore provide the same items as do the models for L, *but one such item for every moment in T*. There is some special difficulty in connection with the specification of the universe(s) of the model: should the model specify just one universe, or rather for each $t \varepsilon T$ a universe (of things existing) at t. The solution for which I will opt in first instance is to specify separately for each $t \varepsilon T$ a universe $U^M{}_t$ of things which exist at t in M, but to let the quantifiers range over the union $U^M = \bigcup_{t \varepsilon T} U^M{}_t$ of all these. What exactly is implied by this will be discussed afterwards.

The previous observations naturally lead to the characterization of a *model for* L(P,F), *relative* to $\mathcal{T}$, as something which provides us with:

(1) for each $t \varepsilon T$ a set $U^M{}_t$ – the set of ‚things existing at t‘. We put $U^M = \bigcup_{t \varepsilon T} U^M{}_t$.

(2) for each $t \varepsilon T$ an appropriate interpretation of $Q^M{}_t$ of Q at t, i.e. a subset of U^M.

(3) for each $t \varepsilon T$ an appropriate interpretation R^{M}_{t} of R at t, i.e. a set of pairs of elements of U^{M}.

(4) for each $t \varepsilon T$ appropriate interpretations c^{M}_{t} and d^{M}_{t} of c and d at t, i.e. elements of U^{M}.

The *truth value* of a sentence φ of L(P,F) in a model M at a time t is, as before, defined in terms of the truth value of φ at t in a referentially perfect model for an extension L′ of L(P,F) with new individual constants; where by *a referentially perfect model relative to $\mathcal{T}$ for* a language $L' = L(P,F) \cup S$ (where S is a set of new individual constants) we now understand a pair $\langle M,H \rangle$ where M is a model for L(P,F) relative to $\mathcal{T}$ and H is a 1-1 correspondence between S and U^{M}. (Thus the members of S do *not* change their reference with time.)

The truth value of φ in such a referentially perfect model is again defined by recursion. We give only some of the clauses of the definition:

$$[Q(c)]^{\tau}_{M,t} = \begin{cases} 1 \text{ if } c^{M}_{t} \varepsilon Q^{M}_{t} \\ 0 \text{ otherwise} \end{cases}$$

$$[R(s,d)]^{\tau}_{M,t} = \begin{cases} 1 \text{ if } \langle H(s), d^{M}_{t} \rangle \varepsilon R^{M}_{t} \\ 0 \text{ otherwise} \end{cases}$$

$$[c = d]^{\tau}_{M,t} = \begin{cases} 1 \text{ if } c^{M}_{t} \text{ equals } d^{M}_{t} \\ 0 \text{ otherwise} \end{cases}$$

$$[\neg\varphi]^{\tau}_{M,t} = \begin{cases} 1 \text{ if } [\varphi]^{\tau}_{M,t} = 0 \\ 0 \text{ otherwise} \end{cases}$$

$$[(\exists v_i)\varphi]^{\tau}_{M,t} = \begin{matrix} 1 \text{ if } [[\varphi]^{s}/v_i]^{\tau}_{M,t} = 1 \text{ for some } s \varepsilon S \\ 0 \text{ otherwise} \end{matrix}$$

$$[P\varphi]^{\tau}_{M,t} = \begin{cases} 1 \text{ if } [\varphi]^{\tau}_{M,t'} = 1 \text{ for some } t' < t \\ 0 \text{ otherwise} \end{cases}$$

Logical truth and logical consequence, either with respect to particular time structures or with respect to classes of such structures, are defined as before.

There are some aspects of this model theory which we ought to discuss before accepting it as a sound formalisation of our intuitive understanding of the semantics of tensed discourse. — I mentioned already the question concerning the sets U^{M}_{t}, and U^{M}, but there are other points as well.

(1) Is it plausible that Q^{M}_{t} should in general be allowed to be a subset of the set U^{M}; or should we rather demand that Q^{M}_{t} always be a subset of the set U^{M}_{t} of things existing at t?

No, we should not insist that the extension of Q at t be always a set consisting only of things that exist at t. Think for example of the English predicates ‚is remembered by someone' and ‚is famous'. The present extensions of these predicates *do* in fact contain individuals which do not exist now. (One might feel that these examples are not particularly convincing, for it may seem that the predicates are essentially complex and that they could perhaps be analysed in terms of tense operators, quantification and more fundamental predicates whose extensions are always subsets of what exists at the time in question. However, I do not see how to carry out such an analysis.)

A similar question can be raised — and a similar answer given — with regard to the extension of t of R and the denotations of c and d.

(2) In connection with c and d, however, we may ask the additional question in how far it is acceptable that they may change their denotations with time. That predicates change their extensions with time is plausible enough; if that were not the case there could be no change of things. For the change of a thing consists in its acquiring properties that it didn't have before and its losing some of those which it did have. But is it plausible that a *name* should change its denotation?

The answer to this last question is almost certainly: 'No'. However, this is not necessarily an indictment of our semantics. For we may regard the individual constants of our system as the formal counterparts not just of proper names, but rather as the counterparts of arbitrary singular terms. And some singular terms, such as ,the president of the U.S.', ,the youngest child of J. S. Bach' or ,the lover of Cleopatra', do of course vary their denotation with time. If, however, we insist that all our constants are proper names, then we should add to our semantics the extra condition $c^M{}_t = c^M{}_{t}$, for any t, t' ε T — and similarly for the other constants.

(3) Doubts similar to those which provoked the questions under (1) and (2) may be felt in connection with our treatment of the quantifiers. According to the clause given above for $\exists$, $(\exists v_i)\ \varphi(v_i)$ is true at t as long as there is something — whether it exists at t or not — which satisfies $\varphi(v_i)$ at t. But should we not insist that $(\exists v_i)\ \varphi(v_i)$ be true at t *only* if something *existing* at t satisfies $\varphi(v_i)$ at t?

No; if we do not want to preclude adequate treatment of sentences such as ,There is someone more famous than anyone in existence now', we should not.

In ordinary discourse, however, quantifiers are often used in a narrower sense than the one symbolised here. Such stricter sense is e.g. intended in any ordinary use of a sentence such as: ,There are no real statesmen any more these days'. (Clearly Cicero or Disraeli can't be adduced to refute this assertion!) We could account for such quantifiers by adding corresponding symbols as primitives to our formal language with the appropriate additional clauses in the truth definition.) But we can also express them in terms of the quantifiers which are already part of our formalism if we add a special one-place predicate. E, reading ,exists', to our symbols — and provide for it the truth condition:

$$[E(s)]_{M,t} = \begin{cases} 1 \text{ if } H(s)\ \varepsilon\ U^M{}_t \\ 0 \text{ otherwise} \end{cases}$$

(and similarly for the constants of L)

We can then symbolize 'there is an actually existing v_i such that $\varphi(v_i)$ as $(\exists v_i)(E(v_i) \wedge \varphi(v_i))$ and 'all actually existing v_i are such that $\varphi(v_i)$' as $(\forall v_i)(E(v_i) \rightarrow \varphi(v_i))$.

There is one serious flaw in the semantics presented here. All examples considered in connection with questions whether we should quantify only over actual existents or over what either exists, has existed, or will exist, or whether the extension of a predicate at t might contain objects other than those existing at t, depended on the possibility of referring to things *after* they have perished. But none of them indicates that there is any sense in referring to an object *before* it has come into existence. And it is indeed very doubtful that there *is* any sense to this. Thus while we should allow into the domain quantification at t not just the things which exist at t but also those which existed at some earlier time, we should perhaps refuse objects which come into existence only later. We will return to this question later on.

It should be noted that some rules of inference which are valid for extensional logic fail within the realm of tense predicate logic. First, we cannot infer from $c = d$ and $\varphi(c)$ that $\varphi(d)$. For example $PQ(d)$ does not follow from $PQ(c)$ and $c = d$, for even if c and d denote the same object now, they could well have denoted different objects at previous times. So there might be a past time at which the object thus denoted by c has Q while at no time in the past does the object denoted by d have Q. Example: Nixon = The President of the U.S.; Nixon has been less than 10 years old. So the president of the U.S. has been less than 10 years old. – Remark, however, that there is only reading on which the last sentence is false. If we read it as ‚Whoever is now president of the U.S. has been less than 10 years old' it is true. This latter reading can be represented in our formalism by the sentence $(\exists v_i)(v_i = d \wedge PQ(v_i))$ and this sentence does indeed follow from $c = d$ and $PQ(c)$ (provided c denotes the same object at all times – i.e. behaves as a proper name).

In a similar fashion we can show that it is in general not correct to infer $(\exists v_i)\ \varphi(v_i)$ from $\varphi(d)$ or $\varphi(d)$ from $(\forall v_i)\ \varphi(v_i)$: there is a reading of ‚The president has always been over 20 years old' in which it does not imply ‚Someone has always been over 20 years old'. In a similar reading ‚The president has been less than 10 years old' cannot be inferred from ‚Everyone has been less than 10 years old'.

It is important to realize the reasons for these failures, as they give an important insight into the (similar) reasons for failure of the same principles within modal quantification theory.

Modal Predicate Logic

As I remarked earlier, the semantics for $L(\Box)$ raises problems more serious than those encountered either in the development of sentential modal logic or that of tense predicate logic. Of the various proposals for such a semantics now available in the literature I shall consider here two. I will then, in Section 3, make a third proposal which to my knowledge has thus far not been available in print. The first is a direct formal analogue of our semantics for tense predicate logic. According to this proposal a *model* M for L(□) provides:

(1) A family W^M of worlds and an alternative relation R^M on this family.

(2) For each $w \in W^M$ a set U^M of individuals existing in w. We put $U^M = \bigcup_{w \in W^M} U^M{}_w$

(3) For each $w \in W^M$ appropriate extensions $Q^M{}_w$, $R^M{}_w$ in w of Q and R respectively, i.e. a subset of U^M and a subset of $U^M \times U^M$, for any sets A,B A x B is the set of all ordered pairs $\langle a,b \rangle$ where $a \in A$ and $b \in B$, and appropriate denotations $c^M{}_w$, $d^M{}_w$ of c and d, i.e. elements of U^M.

From now on I will regard the possible worlds and the alternative relations on them as part of the model, rather than consider $\mathfrak{W}$ separately and then define models relative to it. This strikes me as intuitively more plausible, especially in view of later developments. From a formal point of view the new approach is of course equivalent to the old one.

Truth in these models is again defined via truth in referentially perfect extensions of them. The truth value of a sentence in a referentially perfect extension is defined in exactly the same fashion as in the case of tense predicate logic, except that here the clauses for P and F are replaced by

$$[\Box\varphi]_{M,w} = \begin{cases} 1 \text{ if } [\varphi]_{M,w'} = 1 \text{ for all } w' \in W^M \text{ such that } wR^Mw' \\ 0 \text{ otherwise} \end{cases}$$

Among the virtues of this semantics (which, by the way, it shares with the other proposals presented below), we should notice in particular the explanation it provides for what have long been regarded as anomalies of modal quantification theory, e.g. the failure of the inference from $(\forall v_i)$ $\varphi(v_i)$ to $\varphi(d)$. (Thus while presumably ‚Every number either is necessarily greater than nine or else necessarily less than or equal to nine' is true, ‚the number of planets either is necessarily greater than nine or else necessarily less than nine' is — at least on one reading — false.) Clearly the explanation of this phenomenon is the same as in the case of tense logic: the term, ‚the number of planets' denotes different numbers in different worlds.

However the semantics here discussed raises serious philosophical questions which do not appear as equally urgent in connection with the analogous semantics for tense predicate logic. The main problem is how to make sense of the question whether a certain individual from this world does or does not exist in a given other world. The answers to such questions are settled by fiat in our models, viz. through the sets U^M. But what justification is there for that procedure? What justification could there be for saying that object b in w' *is* indeed the object a from w, rather than, say, some other object c in w (which perhaps resembles b so much that the latter definitely appears to be c, and more so than it appears to be a)? This problem does not arise in the same way with regard to our model theory for L(P,F). For even if there may in particular cases be practical obstacles to our coming to *know* if what is encountered at t is indeed the same as what was encountered at t', and even if on occasion these obstacles may indeed be insuperable, the very idea of what are undoubtedly the central instances of the concept of an individual, — viz. physical objects, — implies that they exist at more than one time. In all (or at any rate nearly all) cases there exists at least the conceptual possibility of ‚following' what was encountered at t' through time and space and thus verifying whether it is indeed what is encountered at t.

But what would it mean to follow an individual from one possible world to another? The discomfort produced by this question has led to a rather different conception of the semantics of modal quantification theory. According to this conception no individual belongs to more than one world. Nonetheless it is possible for a sentence such as $\neg Q(a) \wedge \Diamond Q(a)$, about an individual a in a world w, to be true in w. It will be true, as long as there is an alternative world w' which contains a *counterpart* a' of a that does have Q in w', — while on the other hand a fails to have Q in its own world w. ($\Diamond$ is short for $\neg\Box\neg$).

Formally this leads to the following notion of a *model* for L($\Box$). A *model* M *for* L($\Box$) provides:

(1) A family W^M of possible worlds and an alternative relation R^M on this family.
(2) For each $w \in W^M$ a set U^M of individuals belonging to w. For $w \neq w' \in W^M$, $U^M{}_w \cap U^M{}_{w'} = \emptyset$. Again we put $U^M = \bigcup_{w \in W^M} U^M{}_w$.
(3) An appropriate extension $Q^M{}_w$ for Q in w which, according to the present conception, should be a subset of $U^M{}_w$, rather than of U^M. Similarly a subset $R^M{}_w$ of $U^M{}_w \times U^M{}_w$ and elements $c^M{}_w$, $d^M{}_w$ of $U^M{}_w$ as extensions of w of R, c and d, respectively.
(4) A counterpart relation C^M between the members of $U^M{}_M$. For any $w \in W^M$ and $a, b \in U^M{}_w$ we have $C^M(a,b)$ iff $a=b$.

As before *truth* in arbitrary models is defined in terms of truth in their referentially perfect extensions. The recursive definition of truth in referentially perfect models proceeds just as before exept that we replace the clauses for $\exists$ and $\Box$ by:

$$[(\exists v_i)\varphi]_{M,w} = \begin{cases} 1 \text{ if } [[\varphi]^{s}{}_{vi}]_{M,w} = 1 \text{ for some } s \in S \text{ with } H(s) \in U^M{}_w \\ 0 \text{ otherwise} \end{cases}$$

$[\Box\varphi(s_1 \ldots s_n)]_{M,w} =$ 1 if for every $w' \in W^M$ with wR^Mw' and $s_i' \ldots s_n'$ s.th. $H(s_i') \in U^M{}_{w'}$, and $C^M(H(s_i), H(s_i'))$ for $i = 1 \ldots n$, $[\varphi(s_i' \ldots s_n')]_{M,w'} = 1$
0 otherwise

(in the second clause $s_1 \ldots s_n$ are *all* the new constants occurring in φ.)

It is not clear whether we should in general impose further restrictions on the counterpart relation. We might for example consider the condition that whenever $C^M(b,a)$ also $C^M(a,b)$; or the condition that for any individual a in w and alternative w' for w there is at most one counterpart of a in w'. However, it is by no means evident that we should adopt these conditions. In particular, if we think of the counterpart relation as characterizable in terms of similarity it rather appears as if these conditions do not hold in general. For example there could be a b in w' which is more similar to a in w than is anything else in w', and which moreover is similar enough to a to be its counterpart. But at the same time w might contain an object a' which is more similar to b than a is. So while b is the counterpart of a, a' rather than a is the counterpart of b. Also there might be in w' two objects b and b' which, though different, are equally similar to the object a in w, and more similar to a than the other things in w' are. It would then seem reasonable to regard both b and b' as counterparts of a.

It is interesting to observe, however, that unlike the conditions on the alternative relation R^M, those on C^M generally have no influence on the set of logical truths. This is so in particular for the conditions just considered. The only condition of a relatively simple form which I know to have such an influence is the assumption that every individual a in wo has at least one counterpart in every possible alternative for w. But it seems unacceptable to impose this condition in general.

It is worth noticing — also in connection with what is to follow — that if we do not accept this condition the formula

$$\Box(Q(s_1) \rightarrow Q(s_2)) \rightarrow (\Box Q(s_1) \rightarrow \Box Q(s_2))$$

will not be a logical truth. For the truth in w of $\Box(Q(s_1) \rightarrow Q(s_2))$ and $\Box Q(s_1)$ depends on the truth of $Q(s_1) \rightarrow Q(s_2)$ and $Q(s_1)$, respectively, only in those possible alternatives of w which contain counterparts of the object denoted by s_1; while $Q(s_2)$ will be true in w only if $Q(s_2)$ is true in all those alternatives of w in which there are counterparts of the object denoted by s_2 — but not necessarily of that denoted by s_1.

SECTION 3

The difficulty with counterpart theory becomes apparent when we try to give an account of the *nature* of the counterpart relation. Without such an account C^M would raise, and fail to answer, exactly the same questions as the sets $U^M{}_w$ of our earlier models, and thus the present approach would hardly be an improvement over the previous one.

But what could such an account of C^M be? The only one that seems to be at all plausible (and which was in fact suggested by David Lewis) is in terms of similarity. But it appears unlikely that such an account could ever be adequate.

To see this, consider the statement: ‚It could have been the case that though John were exactly what he is in fact, I would have him a great deal more different from what I am than I am in fact different from him.' Clearly there ought to be interpretations in which this statement is true. (A defender of counterpart theory might contend that it is by no means *obvious* that the statement is contingent rather than logically false; but I am inclined to take (my) intuition on this point as conclusive). However, within the framework of counterpart theory, no such interpretation seems possible, if the counterpart relation is understood in terms of similarity. For such an interpretation should comprise, besides the actual world w in which the statement itself is true, another world w' containing two individuals a and b such that:

(1) a coincides in all properties with the actual individual John of w
(2) b shows greater difference from what I am in w than does John (and therefore a).
(3) b is my counterpart in w'.

Clearly these three conditions are incompatible if the counterpart of an individual in some other possible world is never less similar to that individual than are all the other individuals in that world.

(Remark: It should be noted that the above statement cannot be symbolized in the languages of quantified modal logic we have considered so far. A language adequate to such a symbolization would need to go beyond the present framework in two respects:

(a) it would need a modal operator Ac which always refers back to the actual world — so that for any world w Ac φ is true in w iff φ is true in the actual world. (An exact account of this operator is given later on).
(b) it ought to contain quantification over properties or attributes as well as over particulars. I will not work out a semantics for such higher order modal quantification theory. However, it appears that any such semantics based on the counterpart relation ought to render the sentence:

$$(\forall v_1)\Box(\forall v_2)(\forall v_3)[[(\forall P)(Ac\,P(v_i) \wedge P(v_2) \rightarrow P(v_3)) \wedge (\exists P)(P(v_3) \wedge AcP(v_1) \wedge \neg P(v_2))] \rightarrow v_1 \neq v_2]$$

valid. Consequently the natural symbolization of our original statement in this extended formalism — viz.

$$\Diamond\ [(\forall P)\ (P(c_j) \longleftrightarrow Ac\ P(c_j))\ \wedge\ (\forall P)\ (P(c_i)\ \wedge\ Ac\ P(c_j) \rightarrow Ac\ P(c_j))\ \wedge\ (\exists P)\ (P(c_j)\ \wedge\ \neg\ P(c_i)\ \wedge\ Ac\ P(c_i)]$$

(where c_j denotes John and c_i denotes me) would be logically false).

Thus the counterpart theorist rightly objects to the first semantics for L (□) that it forces us into asking questions to which it provides no real answers but which it settles by fiat. He himself does try to give real answers to these questions; but unfortunately the particular answer which we discussed here is not adequate and it is not clear what other kind of answer could plausibly be tried.

But the real problem is, I believe, not so much that this — or any other — *particular* answer is wrong. It is rather that the questions shouldn't be asked to begin with. To see how they can be avoided let us consider what appear to me to be the simplest and most natural statements whose analyses would involve us, according to the views discussed so far, in cross-world identification, — viz. counterfactual statements about actual individuals mentioned by name; take e.g. the statement:

‚John could have had five children by now.'

The tradition has it that this statement is true in so far as there is some possible world in which (a) John exists and, moreover, in which (b) he has five children by now. This formulation involves us once again in the difficulty that has been confronting us all along: that of having to consider arbitrary possible worlds and deciding, or at least making sense of, the question whether a particular actual individual is contained in them or not. For those that do contain the individual we may then proceed to ask if in them he has five children at the appropriate point of his lifetime; only if there is a world which provides a positive answer to both these questions is the counterfactual statement true.

But if we state these same truth conditions slightly differently we come to see the whole issue in a different — and, I believe, more appropriate — light: The statement is true, we might say, as long as there is an alternativ-world-in-which-John-exists, in which he has (by now) five children — and here the dashes are meant to suggest that what they connect is to be understood as an indivisible whole. In other words, the statement is true as long as there is a *John-alternative* to the actual world in which he has got five children. In the evaluation of this particular modal statement only such worlds come into play in which John exists; *the very notion of an alternative possible world relevant to the evaluation of this particular statement* implies John's existence as a matter of course.

The moral of this is that, generally, there is not just one, fixed, alternative relation which does equal duty in the analysis of all modal statements, but rather that each statement requires its own alternative relation. *How* the statement *determines* this relation is a difficult matter demanding careful and detailed analysis. This problem falls under the more general question of how the alternative relation depends on context. Apart from proper names there are various other grammatical types of expressions (such as e.g. natural kind words and other predicates that are meaningful only if certain contingent features of the actual world obtain) which exert upon the alternative relation relevant to the analysis of a particular modal operator in the scope, or immediate environment, of which they occur an effect which can be isolated and subjected to systematic analysis. Here, however, I will only be concerned with the effect produced by the explicit mentioning of individuals. The analysis of this aspect of the problem seems relatively straightforward, but will nonetheless exhibit the most important new logical features to which the present conception gives rise.

Suppose that V is the set of individuals mentioned by the sentence $\Box\, \varphi$ in w. Then the set of possible worlds in which φ should be true in order that $\Box\, \varphi$ be true in w is determined not just by w, but rather by w and V together. This leads to the following characterization of models.

A *model* for $L(\Box)$ provides:

(1) A set W^M of possible worlds

(2) For each $w \in W^{M}$, a set U^M of individuals existing in w.

(3) A function F^M which assigns to each pair $\langle w, V\rangle$ where $w \in W^M$ and $V \subseteq U^M{}_w$, a subset of W^M, the V-alternatives for w. If $W' = F^M(\langle w, V\rangle)$ then for all $w' \in W'$ and $u \in V$, $u \in U^M{}_{w'}$. Further for any w and $V \subseteq V' \subseteq U^M{}_w$, $F^M(\langle w, V'\rangle) \subseteq F^M(\langle w, V\rangle)$.

(4) Appropriate extensions of w for Q, R, c and d. According to the present conception we never have to ask of an individual whether it has Q in a world in which it does not exist, thus it is appropriate to insist that $Q^M{}_w$ be a subset of $U^M{}_w$. Similarly for $R^M{}_w$, $c^M{}_w$ and $d^M{}_w$.

In defining truth in such a model we are faced with a choice between two alternatives. (Actually there are more alternatives; but the ones discussed here are the most striking and natural cases.) Consider the sentence

$$\Box\, Q(s_1) \rightarrow \Box\, Q(s_2).$$

The truth value of this sentence in w will depend on the truth values in w of $\Box\, Q(s_1)$ and $\Box\, Q(s_2)$. Now according to what I said before the truth value of $\Box\, Q(s_1)$ in w will depend on the truth values of $Q(s_1)$ in all those worlds in which the object denoted by s_1 exists — i.e. all the $\{s_1\}$ — alternatives of w. But perhaps we should take into account as well the other individual mentioned in our assertion, viz. the object denoted by s_2, when we evaluate $\Box\, Q(s_1)$ in w? I can find no conclusive argument why one approach should be better than the other and therefore will give the truth definitions ensuing from either conception.

I. Consider first the approach according to which only the individuals mentioned in φ are relevant to the evaluation of □ φ. In this case the truth definition is quite straightforward. To give the two most important clauses:

$$[(\exists v_i)\varphi]_{M,w} = \begin{cases} 1 \text{ if there is an s such that } [[\varphi]^{s}/v_i{}_{M,w} = 1 \text{ and } H(s) \varepsilon U^M_w \\ 0 \text{ otherwise} \end{cases};$$

$$[\Box\varphi]_{M,w} = \begin{cases} 1 \text{ if } [\varphi]_{M,w'} = 1 \text{ for all } w' \varepsilon F^M(\langle w, V\rangle). \\ 0 \text{ otherwise} \end{cases},$$

Where V is the set of all individuals mentioned in φ, i. e. the set of objects H(s) where s occurs in φ together with $c^M{}_w$ and/or $d^M{}_w$ provided c and/or d occur in φ).

It should be noted that the last clause makes sense only on the assumption that all the individuals which are mentioned by constants in φ belong to w. In the case that φ contains no special constants this reduces to the assumption that $c^M{}_w$ and $d^M{}_w$ belong to $U^M{}_w$ which we assumed to be the case. However if this assumption is satisfied for a certain formula φ which we evaluate in w, it will be similarly satisfied for all the subformulae evaluations in w or other worlds to which the evaluation of φ in w could lead.

Here we may question the advisability of including $c^M{}_w$ and $d^M{}_w$ in the set V (provided the constants occur in ^{W}). The question whether this should be done depends on whether we regard c and d as proper names or not. If we do then $c^M{}_w$ and $d^M{}_w$ should be included. But then we should also insist that c and d denote in any relevant alternative the same object that they denote in w — formally for any w' ε $F^M(\langle w, \{c^M{}_w\}\rangle)$, $c^M{}_w$ = $c^M{}_{w'}$, (and similarly for d).

II. According to the second conception we have to take into account, when evaluating □ φ in w, not just the individuals mentioned in φ but also those which are mentioned elsewhere in the assertion of which □ φ is part. We have to carry the information what these individuals are along in the truth definition; i. e. what we should define by recursion is not just ,the truth value of φ in M, in w', but rather ,the truth value of φ in M, in w, when part of an utterance which mentions all the individuals in the set V (in symbols $[\varphi]^V{}_{M,w}$)'. The important clauses of the definition are:

$$[(\exists v_i)\varphi]^V{}_{M,w} = \begin{cases} 1 \text{ if there is a u } \varepsilon\, U^M{}_w \text{, such that for some s,} \\ H(s) = u \text{ and } [[\varphi]^{s}/v_i]^{V\cup\{u\}}{}_{M,w} = 1 \\ 0 \text{ otherwise} \end{cases}$$

$$[\Box\,\varphi]^V{}_{M,w} = \begin{cases} 1 \text{ if for all } w' \varepsilon F^M(\langle w, V\rangle), [\varphi]^V{}_{M,w'} = 1 \\ 0 \text{ otherwise} \end{cases}$$

Here it is primarily the first clause which deserves comment, and especially the fact that we require $[\varphi]^{\varsigma}/v_i$ to be true relative to $V \cup \{u\}$ rather than V. That this is necessary becomes evident when we consider a sentence like $(\exists v_i) \Box Q(v_i)$. This sentence is true in w if there is an object a in w which satisfies $\Box Q(v_i)$ in w; and it does so only if it satisfies $Q(v_i)$ in all relevant alternatives for w, and these are of course alternatives in which u exists. Thus even if we ask for the truth value of $(\exists v_i) \Box Q(v_i)$ in w relative to the empty set we are led to ask questions about $Q(v_i)$ relative to the set $\{u\}$.

This last remark adumbrates the appropriate definition of the *truth value of an assertion of* φ *in* w, *in* M. That truth value should be, by definition, $[\varphi]^{V}_{M,w}$, where V is the set of individuals mentioned in φ. In case φ contains no special constants this reduces either to the empty set or to the set consisting of those of c^{M}_{w}, d^{M}_{w} for which the corresponding constant (i.e. c or d) occurs in φ. Again the question which of these sets should be chosen depends on whether we treat c and d as proper names or not.

The notions of logical truth to which approaches I and II lead will of course depend (as usual) on what further general assumptions we make about the class of models – in particular about the structural properties of F^M. But in any case there is an important difference. According to approach I the same formulae of the form $\Box(\varphi \rightarrow \psi) \rightarrow (\Box \varphi \rightarrow \Box \psi)$ fail to be logically true that are not logically true according to counterpart theory. But according to the second approach all of them are logically true.

In the second approach there is however a peculiarity as regards the relationship between logical truth and logical consequence – at least if we define the latter in the usual fashion by saying that φ is a logical consequence of ψ iff for every M and $w \in W^M$ if ψ is true in M in w then so is φ. It is then no longer true that for any φ and ψ, φ is a logical consequence of ψ iff $\psi \rightarrow \varphi$ is a logical truth, i. e. the deduction theorem fails. To give an example that indicates what is going on here:

‚If Adam couldn't but have fallen in love with Eve, he couldn't but have fallen in love'

is a logical truth.

However it is plausible that
‚Adam couldn't but have fallen in love'

is not a logical consequence of:

‚Adam couldn't but have fallen in love with Eve'

For there may be a model in which Adam falls in love with Eve in every possible paradise in which she is created, while there are some other paradises where Adam doesn't fall in love (simply because there is no one to fall in love with). Thus someone could defend his assertion that Adam could have failed to fall in love by saying that God could after all very well have decided not to produce Eve at all. But this will not do as justification for the assertion that Adam could have failed to fall in love with Eve. For the assertion already implies the creation of Eve.

This explanation may also indicate, however, that there might be something wrong with our definition of logical consequence. The main point of the notion of logical consequence is after all its connection with the concept of valid inference. But it is plausible that we *can* validly infer ‚Adam couldn't but have fallen in love' from ‚Adam couldn't but have fallen in love with Eve', for in the context of ‚Adam couldn't but have fallen in love with Eve', ‚Adam could't but have fallen in love' is understood anyway as implicitly referring only to worlds in which Eve exists.

Thus we should rather define, perhaps, ‚φ is a logical consequence of ψ' as: ‚for any M, $w \in W^M$ and $V \subseteq V' \subseteq U^M{}_w$, if $[\psi]^V{}_{M,w} = 1$ then $[\varphi]^{V'}{}_{M,w} = 1$'. This definition embodies the intuition that in order to know whether φ can be safely inferred from ψ we only have to be sure that φ is true whenever ψ is true, where we take into account at least as many individuals when evaluating φ as we do when evaluating ψ.

SECTION 4

I will now return to tense predicate logic and indicate how the mentioned observations on the semantics of modal logic find application there. I will, at the same time, reformulate the semantics of predicate logic so as to eliminate the unplausible feature of my previous formulation, viz. that quantifiers range not only over past and present, but also over future individuals. Finally I will make a modification which has basically only technical significance, but which yields slightly more natural subsequent truth definitions: instead of defining the truth value of a sentence $(\exists v_i)$ $\varphi(v_i)$ in a referentially perfect model (at a time) in terms of the truth values of these sentences φ (s), and then defining the truth values of these sentences in arbitrary models as their truthvalues in referentially perfect extensions of these models, I will introduce the original Tarskian method of defining recursively the *satisfaction value of* a, possibly open, formula *in* a model (*at* a time) *by* an assignment, the truth value of a sentence can then be defined as the satisfaction value of the sentence by some, -or, equivalently, by every — assignment.

An *assignment for* a language L *in* a model M is a function which maps the variables of L into U^M. If *a* is an assignment for L in M and $u \in U^M$ then by $[a]^u/v_i$ we understand the assignment a' such that $a'(v_i) = u$ while for $j \neq i$ $a'(v_j) = a(v_j)$. Reformulated in terms of satisfaction by assignments our old truth definition for tense predicate logic takes the following form (again we give only some of the clauses in the recursive satisfaction definition; we abbreviate the *satisfaction value of in* M *at* t, *relative to* according to the assignment *a* as $[\varphi]^{\tau}{}_{M,c,a}$):

$[\vee_i = c]_{M,t,a} = 1$ iff $a(\vee_i) = c^{M}{}_t$
$[R(\vee_i, \vee_j)]_{M,t,a} = 1$ iff $< a(\vee_i), a(\vee_j) > \varepsilon\, R^{M}{}_t$
$[\neg\varphi]_{M,t,a} = 1iff[\varphi]_{M,t,a} = 0$
$[\exists \vee_i)\varphi]_{M,t,a} = 1$ iff there is a u in U^M such that $[\varphi]^{\tau}{}_{M,t,[a]}{}^{u}/v_i = 1$
$[P\varphi]^{\tau}{}_{M,t,a} = 1$ iff there is a $t' < t$ in T such that $[\varphi]_{M,t',a} = 1$
$[F\varphi]_{M,t,a} = 1$ iff there is a $t' > t$ in T such that $[\varphi]_{M,t',a} = 1$

The *truth value of* a sentence φ *in* M, *at* t *relative to* $\mathcal{T}$,$[\varphi]^{\tau}{}_{M,t}$, is by definition the satisfaction value of φ in M at t, relative to $\mathcal{T}$ by any assignment in M. (As a sentence is satisfied by every assignment as soon as it is satisfied by any, this definition is unambiguous.)

It is a difficult question whether a proper name – in the true logical sense of the term – should necessarily denote; whether, in other words, an expression which does not denote an object of the sort names are generally used to denote can be a proper name. Later on I will pay more attention to this question. Until then, however, I will assume that proper names must always denote (objects of the proper sort).

The new semantics should reflect the intuition that no reference can be made to a thing before it has come into being. This implies, in the first place, that at any time t the quantifiers should range not over U^M but rather over $U_{t'} \leqq_t U_{t'}{}^{M}$ – let us, in order to smooth notation, abbreviate this last expression, i.e. $U_{t'} \leqq_t U_{t'}{}^{M}$, as $\bar{U}_t{}^{M}$. Secondly, it implies that the range of a predicate constant at t should be restricted to $\bar{U}_t{}^{M}$; thus, in particular, $Q_t{}^{M} \subseteq \bar{U}_t{}^{M}$, and $R_t{}^{M} \subseteq \bar{U}_t{}^{M} \times \bar{U}_t{}^{M}$. (So this is an extra condition to be imposed upon the models we consider.) Thirdly, it has certain implications for those individual constants of the language which – as I will henceforth assume all our individual constants do – stand for proper names.

If we make this assumption then we can no longer maintain what was implied in all previous developments: that the syntax of a language – in particular the concept of a well-formed sentence of the language – is independent of time. For it follows from our assumption that a proper name which names an object that comes into being only at a certain moment, can be a true proper name only from that moment onwards. Thus if the language contains any name at all for an object whose life-span extends only partway into the past, it can have done so only from the time of that object's genesis. Thus if we want to admit such names at all, we must allow that the language's syntax (i.e. at least its vocabulary) change with time.

Another problem that proper names render manifest (even if it arises, as we will see, also in their absence) regards the interpretation of formulae of the form Pφ. Consider for example the sentence PQ(c). Let us suppose that c denotes an object which came into existence at t_0 and at no time in the interval $[t_0, t_1)$ had the property Q. What, in these circumstances, is the truth value of PQ(c) at t?

According to the standing truth definition, the sentence is true at t_1 iff there is a time t earlier than t_1 such that Q(c) is true at t. Now we assumed that there is no such time t in the interval (t_0, t_1); so the truth value of PQ(c) at t_1 seems to depend on the truth value of Q(c) at a time previous to t_0. But what sense does it make to ask such a question? For the object didn't exist then and, moreover, in view of my earlier assumption about proper names, the name c, and consequently the sentence Q(c), cannot even have been part of the language at such time.

In fact the first objection already suffices by itself to show the inadequacy of the truth definition clause for formulae of the form $P\varphi$, given our new conception of models. And the difficulty that this objection points at arises whether the language contains names or not. Indeed, consider the sentence $(\exists \vee_1) PQ(\vee_1)$. The truth value of this sentence at a time t leads to the same nonsensical questions about satisfaction of the formula $Q(\vee_1)$ by an object at times preceding the genesis of that object.

Intuitively we would all agree, I think, that the sentence PQ(c) is false at t, if the circumstances are as I supposed them to be, and that intuition I believe to rest on the feeling that the only times to be considered in the evaluation of PQ(c) at t_1 are those times previous to t_1 *during which the object denoted by* c has been in existence. Similarly $(\exists \vee_1) PQ(\vee_1)$ we would regard as false at t if every object existing at or previous to t had at no time of *its* past the property Q. Thus here we have a similar dependence of the semantic reach of the operator P of what things are mentioned inside its scope. This leads to the following modifications of our formal system.

Let us first consider a language L for tense predicate logic which contains no individual constants. (We may suppose for convenience that the only predicate constants of L are Q and R. The *models for* L, *relative to* $\mathcal{T}$, are to specify the same items that were mentioned earlier. As in the case of modal logic we must make the truth, or, rather satisfaction, definition dependent on a set of explicitly mentioned objects. And, as in the case of modal logic, we are again faced with the alternative of taking all mentioned objects into account, or only those which occur in the scope of the operator that begins the formula we are evaluating. I simply do not know which of the two alternatives is the better theory, as I can think of usages in agreement with either. I will give here only the formulation of the theory where all the objects mentioned in the context are taken into account (some of which may be mentioned only at places outside the scope of the tense operator under consideration). The other alternative can easily be reconstructed from its analogue in modal logic, together with the definition which now follows.

Apart from this general modification of the satisfaction definition there are three clauses which require specific changes, those for $(\exists \vee_i)\varphi$, $(\forall \vee_i)\varphi$ and $P\varphi$. We get $[(\exists \vee_i)\varphi]^{\tau, \vee}_{M,t,a} = 1$ iff there is a u ε $\bar{U}_t^M$ such that

$[\varphi]^{\tau, V \cup \{u\}}_{M,t,[a]^{u}/v_i} = 1$, and similarly for $(\forall v_i)\varphi$; and $[P\varphi]_{M,t,a} = 1$ iff there is a $t' < t$ such that $V \subseteq \bar{U}_{t'}^{M}$ and $[\varphi]^{\tau, V}_{M,t,a} = 1$. Again the *truth value of* the sentence φ *in* M *at* t *relative to* $\mathcal{T}$, $[\varphi]^{\tau}_{M,t}$, is defined as $[\varphi]^{\tau}_{M,t} = [\varphi]^{\tau, V}_{M,t}$, where V is the set of objects mentioned explicitly in Q. (In the languages now under consideration, which contain no individual constants, V is of course the empty set.)

Let us now consider languages which contain individual constants.

As we have seen, of such languages not even the syntax is separable from their interpretations. It will be useful, however, to single out that part of the syntax which we will here assume to be independent of the model, viz. that fragment in which the individual constants do not occur. Thus let L again be a language for tense predicate logic without individual constants and M a model for L, relative to $\mathcal{T}$. By a *possible extension of* L (*with individual constants*) *in* M we understand a sequence $\{< L_t, F_t >\}_{t \varepsilon \tau}$, where for each t, L_t is an extension of L with individual constants and F_t is a function from the individual constants of L_t into $\bar{U}_t^M$; moreover, for each t, $t' \varepsilon$ T such that $t < t'$, $L_t \subseteq L_{t'}$ and $F_t \subseteq F_{t'}$. (As the last condition indicates, I assume that although new names may be added to the language, no names ever disappear from it — this assumption can of course easily be dropped by whoever regards it as misleading. However, it does somewhat simplify technical matters). I will at first make the unnatural additional assumption that a name becomes part of the language as soon as the object it names comes into being. Thus we have for all $t \varepsilon$ T: $u \varepsilon \bar{U}_t^M$ iff $(F_t^M)^{+}(u) \varepsilon L_t$. The satisfaction definition for a possible extension $\{< L_t, F_t >\}_{t \varepsilon \tau}$ of L in M is essentially the same as for L itself. The only difference is that we now have new clauses such as

$$[R(c,d)]^{\tau, V}_{M,t,a} = 1 \text{ iff } < F_t(c), F_t(d) > \varepsilon R_t^M$$

(Notice that this clause only gives the satisfaction value of R(c,d) at t by a if the sentence belongs to L_t. Thus, if we put $L' = \cup_{t \varepsilon \tau} L_t$, the satisfaction definition yields $[\varphi]^{\tau}_{M,t}$ only as a partial function on the sentences L′. However, one easily verifies that if $\varphi \varepsilon L_t$ then $[\varphi]^{\tau, V}_{M,t,a}$ is always defined for any assignment a which maps the variables into $\bar{U}_t^M$. It follows that for any sentence φ of L_t, $[\varphi]^{\tau}_{M,t}$ is also defined.

In the light of this new development we have to re-examine the concept of validity. We can no longer define the validity of an arbitrary sentence of the language L′ as its being true at each time in each model; for L′ itself is meaningful only in relation to the particular model M. The old definition will do, however, for that part of L′ which is independent of M, viz L; and for that fragment we will retain it. As regards sentences in L′ — L we may wonder if there is any sense to the question whether they be logically true or not. There *is*, I believe, a sense to the question, but that sense is parasitic upon the concept of logical truth for sentences in L: A sentence in L′ — L can be regarded as a logical truth if it can be obtained from a logically true sentence in L of the form $(\forall v_{i1}) \ldots (\forall v_{in})\ \varphi\ (v_{i1}, \ldots,$

v_{in}) by dropping the quantifiers $(\forall v_{i1}), \ldots, (\forall v_{in})$ and replacing all the free occurrences of $v_{i1}, \ldots, v_{in}$ in $\varphi(v_{i1}, \ldots v_{in})$ by individual constants $c_{j1}, \ldots, c_{jn}$ of L' respectively.

The new semantics generates a distinct different set of valid sentences of L. A striking example is provided by the sentence $(\exists v_1)(E(v_1) \wedge \neg PE(v_1)) \rightarrow (\exists v_1)PF(E(v_1) \wedge \neg P\,E(v_1))$

In the old semantics this sentence was valid. But according to our new theory it is not. Instead we have now that $\neg(\exists v_1)PF(E(v_1) \wedge \neg E(v_1))$ is valid. Thus the assertion that there is something of which it was going to be the case that it exists for the first time, instead of being entailed by the assertion that something exists now for the first time, is a logical falsehood.

I will make no attempt to axiomatize or otherwise characterize, for any time structure or class of such structures, the set of valid sentences according to the new semantics. One of the most important new aspects of the new semantics is its emphasis on the sensitivity of the past tense operator to explicit references made inside its syntactic scope. The new clause for formulae of the form $P\,\varphi$ expresses the view that this sensitivity takes the simple form of a limitation in temporal reach to that portion of the past during which all the mentioned individuals have been in existence. But this is not the whole story. In the first place we observe in ordinary discourse a similar influence on the temporal reach of the future tense operator. Moreover, here, as in certain past tense sentences, the influence is of a more complicated sort, expressions other than names playing their part in it as well. Compare for example the sentences: ‚Jones will always be famous‘, and ‚Jones will always remain a heavy smoker‘. The first sentence clearly is *not* true if there will be a time, be it after Jones‘ death, at which he is no longer famous. The second, however, is generally regarded as true if Jones will combust his two packets of cigarettes a day until the moment it will have killed him; what is or is not the case after his death is irrelevant to the truth value of the sentence. Thus in the first sentence the temporal reach of the future tense operator extends beyond Jones‘ death, in the second sentence – at least as we would normally understand it – it does not. Clearly this difference is explained by the difference between the predicates ‚is famous‘ and ‚is a heavy smoker‘. The former can be meaningfully asserted both of the living and the dead, but the latter only of the living (barring the context of the ghost story). Generally the temporal reach of a tense operator is limited to those phases of the individuals explicitly mentioned during which the predicates can be meaningfully asserted of them.

Similar influence is sometimes exerted on the past tense operator. ‚Jones has always had a stammer‘ is generally understood to mean that Jones has stammered since the time he started speaking. And anyone who would say of Jones, who was born with, and has always retained, a left leg slightly shorter than his right one, ‚There was a time when Jones didn't limp‘, would certainly be considered a sophist if he defended his statement by referring to the period of Jones's infancy when he didn't walk at all.

A more precise elaboration and formal account of these points would require an adequate theory of applicability of predicates. I will make no attempt at such an analysis here.

Another difficulty arises if we rescind the unnatural condition that a name of a thing is introduced into the language the very moment that the thing comes into being. Let us return to the sentence PQ(c); let us assume again that c was introduced into the language at t_0 and that Q(c) is false throughout the interval $[t_0, t_1]$; but in contrast with our earlier assumption we now suppose that the object $F'^M(c)$ did exist at times before t_0, and that at least one such time t_2 it had the property Q. Thus intuitively PQ(c) should of course be true at t_1. However, both the old and the new clause for the truth value of P φ give us either no truth value for PQ(c) at t or else the wrong one; for what is the truth value of Q(c) at t_2? Clearly the question makes no immediate sense, for at t_2 there was no sentence Q(c) of which the question could be asked.

Of course we can evade the issue by agreeing that we will always *symbolize* an English sentence of the form ‚c once had the property Q' as $(\exists v_1)(v_1 = c \wedge PQ(v_1))$. This is a device which has often been used in modal and tense logic to solve problems in connection with definite descriptions which are meant to refer to *actual* objects even though they occur within the scope of a modal or tense operator.

SECTION 5

There are however other ways to solve this particular problem about descriptions in modal and tensed contexts. I believe it is useful to give a short overview of the possibilities which are known to me.

But let me first state what the problem is. Sentences such as ‚Jones' mistress has always been a blonde' and ‚It is possible that Jones' mistress would have been a brunette' are ambiguous. The first for example, can be understood as an assertion about her who is now Jones' mistress — that she was born with blond hair and never had it dyed a different colour. But it can also be understood as saying that whoever was, at any time in the past, Jones' mistress was a blonde (at least while she was his mistress). (Admittedly this latter context would probably be more naturally expressed by the sentence ‚Jones' mistresses have always been blondes'.) Similar ambiguity can be observed in the second sentence. It could say that Jones could have had another mistress than he in fact has, viz. a brunette; but also that Jones' actual mistress could have had hair of a colour different from the one it is.

Suppose we add to our languages of tense predicate logic a description operator T, an operator which forms a singular term $(T\, v_i)\, \varphi$ out of one formula, φ, binding one variable, v_i. In any model M relative to $\mathcal{T}$ $(T\, v_i)$

φ will *denote at* t *relative to* the assignment a the object u iff u is the only element of $\bar{U}_t^M$ such that $[a]\,^u/_{v_i}$ satisfies φ in M at t; if there is no such u then $(T \vee i)$ φ will not denote at all. In such cases the satisfaction value of $Q((T \vee i)\varphi)$ at t by a will be undetermined. This makes our truth definition truly partial (in the sense that even some sentences belonging to L_t will be assigned no truth value at t); but for the purpose of the present discussion this seems the most natural decision to take about descriptions which do not denote properly. (N. B. The introduction of complex singular terms leads to the usual complications of the satisfaction definition. The recursion can no longer be on the satisfaction relation alone; instead we have to define by simultaneous recursion the satisfaction value of a formula by an assignment and the denotation value of a singular term, given an assignment. Both values may be indicated by the usual notation $[\alpha]^{\tau,v}{}_{M,t,a}$, the grammatical category of α indicating what sort of value is represented. The only recursive clause in the definition which defines a denotation, rather than a satisfaction, value is $[(Tv_i)\varphi]^{\tau,v}{}_{M,t,a} = u$ iff u is the only element of $\bar{U}_t^M$ such that $[\varphi]^{\tau,v}{}_{M,t,[a]u/vi} = 1$. The slight disadvantage of this extended formalism is that it offers no straight forward way of adequately symbolizing one of the two senses of ambiguous sentences of the form ‚the φ has never failed to be a ψ'. Consider a special case: ‚the object v_1 which stands in the relation R to all v_2 has never failed to be Q'. One obvious symbolization of this sentence is: $\neg P \neg Q((Tv_1)(\forall v_2)R(v_1,v_2))$.

But this renders only the sense: ‚it never failed to be the case that what stood in the relation R to all things at the time had then the property Q'. The other sense is adequately expressed by:

$$(\exists v_3)(v_3 = (Tv_1)(\forall v_2)R(v_1, v_2) \wedge \neg P \neg Q(v_3));$$

but one might feel that the syntactic structure of this formula departs so much from the English expression it symbolizes that it fails to uncover some of the features of semantic structure of English that determine the sense of the expression.

So we may ask if a different formalism would not allow a more direct symbolization which nonetheless has the proper truth conditions.

Perhaps the most elegant solution of this problem is obtained if we add abstraction operators to the language.

For the present purposes the best way of obtaining such an extension of a language of tense predicate logic is through adding an infinite set of abstraction operators, $\wedge^0, \wedge^1, \wedge^2, \wedge^3, \ldots$; $\wedge^n$ forms an n-place predicate $(\wedge^n v_{i1} \ldots v_{in}\,\varphi)$ out of a formula φ binding the variables $v_1 \ldots v_{in}$. The satisfaction value of a formula

$(\wedge^n v_{i1} \ldots v_{in}\,\varphi)\,(v_{j1} \ldots v_{jn})$ *in* M *at* t, *relative to* $\mathcal{T}$ and $\vee$ *by* a is defined by the clause

$$[(\wedge^n v_{i1} \ldots v_{in}\,\varphi)\,(v_{j1} \ldots v_{jn})]^{\tau,v}{}_{M,t,a} = 1 \text{ iff } [Q]^{\tau,v}{}_{M,t,a'} = 1, \text{ where } a' = [..[a]^{\alpha(vj1)}/_{vj1}] \ \ldots]^{\alpha(vjn)}/_{vjn}$$

and similarly in case the argument places are filled by singular terms other than variables. Within this formalism we can symbolize the expression mentioned above either as $\neg P \neg Q((T v_1)(\forall v_2) R(v_1, v_2))$ or as

$$(\wedge^1 v_3 \neg P \neg Q(v_3))((T v_1)(\forall v_2) R(v_1, v_2)).$$

The second symbolization has the same truth conditions as $(\exists v_3)(v_3 = (T v_1)(\forall v_2) R(v_1, v_2) \wedge \neg P \neg Q(v_3))$.
An analogous extension can of course be formulated for modal predicate logic.

An alternative solution is implicit in unpublished notes by David Kaplan on demonstratives. He introduces among other notions a description operator to which he refers as ‚demonstrative that'. Analogously let us introduce, in addition to our description operator T, a second description operator T_d; it too forms a singular term out of one formula, binding one variable. However, the denotation of a term $(T_d v_i)\ \varphi$ is not the unique object which satisfies φ at the time to which the evaluation at t of the truth value of a sentence in which the term occurs leads us, but rather the unique object satisfying φ at the time of utterance of the sentence. Formalization of this idea requires that we keep track of that moment of utterance during the evaluation — i.e. what we should define in general is the *satisfaction value* a formula φ *in* a model M *relative to* $\mathcal{T}$ and V, *at* t, *when part of an utterance made* at t', *by a*, in symbols $[\varphi]^{\tau,V}{}_{M,t,t',a}$. The recursive definition of this notion is just like our old definition, with the second subscripted time variable appended everywhere. Typical clauses of the definition are e.g.

$$[\neg\varphi]^{\tau,V}{}_{M,t,t',a} = \text{iff } [\varphi]^{\tau,V}{}_{M,t,t',a} = 0$$
$$[P\varphi]^{\tau,V}{}_{M,t,t',a} = 1 \text{ iff there is a } t'' < t \text{ such that for all } u \in V,\ u \in \bar{U}_{t''}{}^{M} \text{ and } [\varphi]^{\tau,V}{}_{M,t'',t',a} = 1.$$

The relevance of the second time index becomes apparent only in the clause for the denotation value of $(T_d v_i)\ \varphi$ which is defined by

$$[(T_d v_i)\varphi]^{\tau,V}{}_{M,t,t',a} = u \text{ iff } u \text{ is the unique element of } \bar{U}_{t'}{}^{M} \text{ such that } [\varphi]^{\tau,V}{}_{M,t',t',[a]^{u}/v_i} = 1$$

The *truth value of* a sentence φ *in* M at t *relative to* $\mathcal{T}$ can now be defined as $[\varphi]^{\tau,V}{}_{M,t,t,a}$, where *a* is any assignment into $\bar{U}_t{}^M$, and V is the set of individuals mentioned by individual constants in φ; thus the truth value of φ at t is, roughly speaking, the truth value that φ has at t when (part of) an assertion made at that same moment t.

This definition raises a special problem, however, in those cases where $t < t'$ and the object that uniquely satisfies φ at t' has not yet come into existence at t. For the moment I will simply assume that in such a case the denotation value of $(T_d \vee_i)\ \varphi$ relative to the given parameters — and thus also the satisfaction value of any formula whose evaluation leads to the evaluation of $[(T_d \vee_i)\varphi]^{\tau,V}{}_{M,t,t',a}$ — are undefined. I will return to this question later.

Within this formalism the two different senses of our English sentence are adequately formalized as

$\neg P \neg Q((T \vee_1)(\forall \vee_2)\ R(\vee_1, \vee_2))$ and
$\neg P \neg Q((T_d \vee_1)(\forall \vee_2)\ R(\vee_1, \vee_2))$ respectively.

A related solution to our problem can be obtained by introducing instead of the description operator T_d a one-place sentential operator N, meaning (it is) now (the case that): The essential function of the word ‚now' is to refer that what stands in its scope to the time t of the assertion of which it is part, even if in that assertion it stands itself within the scope of another tense operator whose evaluation leads to the consideration of times other than t. Compare for example the sentences: ‚It was predicted that there would now be a rainstorm' and ‚It was predicted that there would be a rainstorm'. Clearly there sentences are not equivalent. In fact the first sentence entails the second, but not conversely. For while the second statement locates what is predicted only in the future of the moment of prediction, the first locates it at the moment of assertion.

‚Now' 's counterparts in the realm of modal logic are the expressions ‚actually', ‚really' and ‚in fact'. Other words with such a function are ‚I' and ‚you' (‚Jones says that I am a bastard' said by Kamp means that Jones says that Kamp is a bastard, not Jones, etc.); and also ‚here'.

The (satisfaction & denotation) value definition for a language containing N must again take two temporal indices into account, one indicating the time at which the expression is being evaluated, and the other the time of utterance. The only new clause is that for formulae of the form $N\varphi$:

$$[N\varphi]^{\tau,V}{}_{M,t,t',a} = \text{iff } [\varphi]^{\tau,V}{}_{M,t',t',a} = 1$$

(i.e. the value of $N\varphi$ at t when part of an assertion made at t′ is just the value of φ at the time t′).

This new system with N displays a feature which has no analogue in the systems of tense predicate logic discussed earlier. It contains sentences which, although logically equivalent (in the sense that for any model M and time t their truth values will coincide) are not interchangeable salva veritate as subformulae of larger sentences. The most obvious example is *provided by any pair of* sentences of the forms φ and $N\varphi$ respectively, e.g. Q(c) and NQ(c); for while $Q(c) \longleftrightarrow NQ(c)$ is a logical truth, $PQ(c) \longleftrightarrow PNQ(c)$ is not. Of course there are many formulae which although they contain N in an essential way are not only equivalent but also interchangeable in all contexts. Typical cases of these are the following pairs:

$(\neg N\varphi, N \neg \varphi)$, $(N\varphi, NN\varphi)$, $(N(\varphi \vee \psi), (N\varphi \vee N\psi))$, $(N(\varphi \wedge \psi), (N\varphi \wedge N\psi))$.

The introduction of N into tense predicate logic requires yet another modification, viz. in the recursive clauses for formulae of the forms $(\exists v_i)\varphi$ and $(\forall v_i)\varphi$. For consider the sentence

$$F(\exists v_1)(Q(v_1) \wedge N \neg Q(v_1)).$$

According to the standing truth definition this sentence is true at t if there is a later time t′ at which there is an object that has been in existence by the time t′, that has Q at t′ but of which it is the case at t that it does not have Q. In this way the definition may force us to ask at t questions about objects which have not yet come into existence then. The best solution to this dilemma involves I believe, two independent alterations. (1) We should introduce into our language besides the operator N one or more operators which behave essentially like the word ‚then' in English, i.e. which may take us outside the scope of another tense operator to a time indicated by a third tense operator of even wider scope. Thus in the statement ‚there will be something such that everything now existing will be different from that thing then' the word ‚then' will carry us from the moment of assertion to which we were referred by ‚now' back to the future time indicated by ‚there will be' even though ‚then' stands inside the narrower scope of ‚now'. (A comprehensive analysis of ‚then' has been given by F. Vlach in his doctoral dissertation, UCLA 1970). (2) We should regard formulae of the form $N\varphi(v_i)$ always as abbreviations of $N(\exists v_j)$ (then $(v_j = v_i) \wedge \varphi(v_j)$), where v_j does not occur in $\varphi(v_i)$ and ‚then' refers back to the time at which the quantifier is evaluated that binds v_i. Then the sentence $F(\exists v_1)\ (Q(v_1) \wedge N \neg Q(v_1))$ becomes, in full expansion: $F(\exists v_1)(Q(v_1) \wedge N(\exists v_2)(\text{then } v_2 = v_1 \wedge \neg Q(v_2))$; and ‚there will be something which now neither exists nor has existed' becomes $F(\exists v_1)(N\forall v_2)\ (E(v_2) \vee PE(v_2) \rightarrow \text{then } v_2 \neq v_1))$; etc. However, an adequate formalization of ‚then' leads to complications into which I do not want to get into here. Moreover for the difficulty we are having here we need the ‚then' operator only in front of formulare of the forms $\alpha = \beta$ and $\alpha \neq \beta$, where α and β are either variables or individual constants. Thus for our purpose it will be sufficient to introduce a new ‚identity' relation $\approx$, such that in a system which contains the adequate counterpart(s) of ‚then' a formula $\alpha \approx \beta$ can always be regarded as short for Th $\alpha = \beta$, where Th is the appropriate ‚then' operator. This implies that the subformula $\alpha = \beta$ of the formula φ should be regarded as satisfied by *a* at t if $\alpha = \beta$ is satisfied by *a* at a sufficiently late time (i.e. a time where the objects denoted by α and β or assigned to them by *a* both have come into being. In all applications of $\approx$ that I have here in mind, however, at least one of α, β will be a variable bound by a quantifier which itself is evaluated at the time where $\alpha \approx \beta$ is evaluated.

Rather than expanding formulae of the form N $\varphi(v_i)$ in the way suggested under (2) we may formulate the satisfaction clause for formulae of this form so as to be in agreement with the results this expansion would give if we were to adopt also the clause for formulae of the form $\alpha \approx \beta$ I just indicated. The direct satisfaction condition becomes:

$$[N\ \varphi]^{\tau,v}{}_{M,t,t',a} = 1 \text{ iff for each } v_i \text{ which occurs free in } \varphi,\ a(v_i)\ \varepsilon\ \bar{U}_{t'}{}^{M} \text{ and } [\varphi]^{\tau,v}{}_{M,t',t',a} = 1$$

One of the consequences of this is that the formluae $\neg N\ \varphi$ and $N \neg \varphi$ are no longer interchangeable salva vertitate in all contexts. For $[N\ \varphi]^{\tau,v}{}_{M,t,t',a}$ and $[N \neg \varphi]^{\tau,v}{}_{M,t,t',a}$ will be both o in cases where for some v_i free in φ $_a(v_i)$ does not belong to $\bar{U}_{t'}{}^{M}$. However it remains that any sentence of the form $\neg N\ \varphi \longleftrightarrow N \neg \varphi$ is a logical truth.

We can now symbolize the two senses of our standing sentence as

$\neg P \neg Q((Tv_1)(Vv_2)\ R(v_1,v_2))$ and
$\neg P \neg Q((Tv_1)\ N(Vv_2)\ R(v_1,v_2))$.

Here again, though, we meet the difficulty mentioned before, that the thing satisfying $(Vv_2)\ R(v_1,v_2)$ uniquely at the time of utterance may not have existed at some earlier times. We will turn to this shortly.

SECTION 6

Let us now return to the problem we encountered in connection with the sentence PQ(c). The solution to that problem lies in the observation that the behaviour of proper names shows an important similarity with that of the word ‚now' whose logical function we just analyzed. For in a way proper names, too, refer back to the moment of assertion: they point at the object they name at the time the assertion is made whether the name was part of the language at earlier times which the evaluation of the assertion leads us to consider is irrelevant. This aspect can be rendered adequately within the semantical definitions that we designed in order to account for T_d and N. What has to be modified in these definitions is the clause(s) for atomic formulae containing proper names. I will just consider one special case. The definition for the atomic sentence Q(c) now becomes:

$$[Q(c)]^{\tau,v}{}_{M,t,t',a} = 1 \text{ iff } F_{t'}{}^{M}(c)\ \varepsilon\ Q_t{}^{M}$$

These last developments also enable us to see how we should analyse such sentences as: ‚London once did not exist.' Clearly one cannot formalize this sentence as $P \neg E(l)$ – where E is our earlier existence predicate – even though it is possible to formalize its future tense analogue, viz. ‚London once will no longer exist', as $F \neg E(l)$. The difficulty with $P \neg E(l)$ is that according to our semantics it can not be true (at least, if we make the correct assumption that London did not first come into being, then cease to exist for a while; to re-emerge eventually as what it is now). For the temporal reach of the initial P is restricted by the existence of London and throughout that existence $\neg$ E(l) was of course false. But the following symbolization I believe to be adequate:

$$P(\forall v_1)(E(v_1) \rightarrow Nv_1 \neq l)$$

However, adequate it will be only if once again we modify the satisfaction definition. For *any* symbolization in which l occurs inside the scope of P (and as far as I can see this is an unavoidable feature of such a symbolization) can be adequate only if we allow the temporal reach of P to extend backwards beyond the beginning of London's existence. In the sentence under consideration, however, it is plausible that we should allow this, as the individual constant l, though occurring inside the scope of P, occurs also within the *narrower* scope of N, which refers us back to the present. Formally this means that we must redefine $[\varphi]^{\tau}_{M,t,a}$ the truth value of the sentence φ in M at t, relative to τ, as $[\varphi]^{\tau,V}_{M,t,t,a}$, where a is any assignment and (i)V is the set of objects named by individual constants which occur in φ in positions where they do not stand within the scope of N. This, however, forces us to modify also the clause for formulae of the form N φ:

$$[N\,\varphi]^{\tau,V}_{M,t,t',a} = 1 \text{ iff } [\varphi]^{\tau,\, V \cup V'}_{M,t',t',a} = 1$$

where V' is defined as is V in (i) above.

It is interesting to observe that according to our last formulation of the truth conditions ‚London once will not exist' can not be adequately symbolized in a completely symmetrical fashion, i.e. by $F(\forall v_1)(E(v_1) \rightarrow Nv_1 \neq 1)$. For this sentence will be false in the likely situation that by the time that London will no longer exist new objects will have come into being. What we really *should* have is the symbolization $F(\forall v_1)(E(v_1) \rightarrow N\,v_1 \not\approx 1)$; and similarly $P(\forall v_1)(E(v_1) \rightarrow Nv_1 \not\approx 1)$ as symbolization for the sentence ‚London once did not exist'; but in the formal system on which we have settled $\approx$ does not occur. That $P(\forall v_1)(E(v_1) \rightarrow Nv_1 \neq 1)$ works is due to the fact that that sentence is equivalent to $P((\forall v_1)(E(v_1) \rightarrow Nv_1 \not\approx 1)$, (or rather: would be in an extended system containing $\approx$. $F(\forall v_1)(E(v_1) \rightarrow Nv_1 \neq 1)$, on the other hand, is not equivalent to $F(\forall v_1)(E(v_1) \rightarrow Nv_1 \not\approx 1)$.

There are, however, apart from the formula $F \neg E(1)$ already mentioned, various other adequate symbolizations of ‚London once will not exist', e.g. $F(\forall v_1)(E(v_1) \rightarrow \neg Nv_1 = 1)$, or $F(\forall v_1)(E(v_1) \rightarrow v_1 \neq 1)$.

This may be the right point to mention some fundamental criticism against the whole approach towards intensional logic on which these notes are based. In this approach we have tried to express the truth conditions in a given context of a sentence of the form O φ, where O is some nontruth functional operator in terms of the truth conditions of φ in other contexts.

It is sometimes objected that to state that the sentence FQ(c) is true now iff the sentence Q(c) will be true at some future time or that PQ(c) is true now iff Q(c) was true at some past time is rather misleading. What, so the objection goes, does the truth value of the sentence Q(c) (or off some possible utterance of the sentence Q(c)) at some future or past time have to do with the truth *now* of FQ(c) or PQ(c); in fact the human race could disappear or adopt a completely new system of communication; or our present language could gradually shift its truth conditions, it may well be that we should regard all these possibilities as implicit by excluding the theoretical notion of a sentence's being *true at* a time (or *in* a context) of which tense (or, generally, intensional) logicians try to give a formal analysis: but then the notion becomes so abstract and so far removed from what we ordinarily *understand* by truth that the analysis can hardly be regarded as an explanation of the semantics of tensed language as it is actually in use.

I still do not believe that this criticism is wholly justified. I believe that the understanding of e.g. past tense sentences through the understanding of what it means for the corresponding present tense sentence to be true at any time, together with what it means for certain possible situations of assertion to be connected with the actually present situation through that relation which refer to as ‚temporal predicession' is an important and essentially correct insight. However, I am more sympathetic now to some elements of the critism than I have been until quite recently. And the above treatment of proper names within tense logic constitutes in fact a concession on my part of an aspect of the criticism which I now regard as right: the proper name indicates to us as it were what object we should pick up here and now to consider of *it* whether in past or future it had or will have certain properties.

The question arises whether a similar analysis would not be also appropriate with regard to other grammatical categories, say, predicates. Can't we maintain,for example, that when I tell you ‚Jones will be bald' I invite you not just to pick up, right now, the referent of ‚Jones' and to wonder whether it will satisfy at some future time the sentential function ‚x is bald', but rather to pick up also the property expressed by ‚bald' and then to consider whether at some time in the future that property will obtain of that object? Such a view seems much in line with what has recently been said by a number of people (e.g. Kripke, Putnam, Ishiguro, Wiggins, ...) about certain property expressions – those to which Kripke refers as *rigid* designators of properties. Typical examples are common nouns which refer to natural kinds. But other examples may well be found among adjectives which stand for qualities, as well as common nouns standing for artifacts, and even perhaps certain verbs. In these latter cases that to which the predicate refers is often something like a *procedure* which we may use to verify, for any object, whether it satisfies the predicate or not. An adequate formalization of these ideas would require a proper representation within the formal semantics of the entities (be they clusters of features, procedures of verification or what have you) that these rigid designators refer to. This I will not attempt to do here. Yet it is clear that once that is accomplished, the techniques here developed for proper names can easily be extended to cover rigid predicates as well. (Within such a system it will also be possible to account elegantly for the kind of ambiguity which is displayed in the sentence ‚this insect will always be coloured like the leaf it is sitting on', which of course can mean either that the insect will always have that colour, which is now also displayed by its present support, or that it will always adopt its colour to the leaf upon which it takes position.)

The sentence ‚London once did not exist' has its obvious modal counterpart ‚London could have failed to exist'. On first sight this sentence may appear equally puzzling. And on second sight it can be analyzed in the same way as its temporal analogue. The analysis requires an extension of modal predicate similar to the one we obtained by supplementing our system of tense logic with N. Here we add a one place sentential operator Ac; Ac φ is to read ‚it is actually the case that φ'. From this intended reading it is evident that Ac has very much the same function within modal contexts that N fulfills in tensed discourse: Ac brings us back not to the time, but to the world of assertion.

We could give an adequate formulation of the semantics for this extended system of modal predicate logic using the same technique we applied previously to tense logic, viz. by making the truth definition dependent not upon one, but upon two worlds. However, within the context of modal logic we may proceed in a simpler manner and stipulate that a model M is to provide, in addition to the various items mentioned earlier, a particular member w_0^M of W^M, the *actual world* of the interpretation. We are interested primarily only in the truth values of sentences in *this* world. We have to consider the ‚truth values' of sentences in other worlds as well in order to make the truth definition work for complex sentences in this world, but it is *only for this purpose* that they need to be considered. This position suggests a slightly different definition of logical truth (and correspondingly for logical consequence). We say that a sentence φ is *logically true relative to* a class of models of the new type if for each of these models M, φ is true in M in w_0^M.

It is also appropriate to adapt our treatment of individual constants. Proper names in modal discourse can roughly be divided into two categories – those which name actual objects, objects which exist in the actual world; and those which name beings and things from fiction or mythology (of course there may be some overlaps.) For the moment I will just be concerned with names for actual objects. So I will assume that the individual constants of our formal system all stand for such ‚ordinary' proper names. As all these constants name, in any given model M, things which belong to $u^M_{w_0M}$, it is enough that M specify for each of them simply its actual denotation. Thus, rather than specifying c_w^M for each w M will just specify c^M.

In any other world than w_0^M c will name this same object, provided that it belongs to U_w^M – but as, because of the form of the satisfaction definition, we will, when evaluating at w_0^M a sentence φ in which c occurs, never come to worlds in which c^M does not exist, the limitation is of no practical import. From what I have said in connection with N, it might appear that we could give an analogous satisfaction clause for formulae of the form Acφ, viz.

$[Ac\varphi]^v_{M,w,a} = 1$ iff for each $\vee_i$ free in φ, $_a(\vee_i)\ \varepsilon\ \bar{U}^M{}_{w_0}{}^M$ and $[\varphi]^v{}_{M,w_0{}^M,a} = 1$ Adopting this clause we can give an adequate symbolization of the modal analogue of ‚London once did not exist', viz. the sentence ‚London might not have existed'. The symbolization is:

$$\Diamond\ (\forall \vee_1) \neg\ Ac \vee_1 = 1$$

However, the justification of this clause for Acφ can be not quite the same as that for the clause for Nφ in tense logic. There the justification depended essentially on the assumption which underlies all versions of the semantics that I have here presented; that for any two different times t and t′ either all the individuals mentionable at t′ can also be mentioned at t′ or vice versa. But can such an assumption be made about possible worlds?

The whole point of our original modification of the semantics of modal logic was to avoid awkward and probably non-sensical questions about whether or not this (actual) object exists in that possible world. Does it make any more sense to ask of a possible object – I mean one which belongs to some non-actual world – whether it exists in our world? Probably not. If we assume that it does not, however, any attempt to formally represent ‚London might not have existed' along the lines which I have just tried, is doomed to failure. And thus a quite different way of understanding the English sentence would be required. Perhaps the appropriate account for sentences like this is one in which we accept that for any given actual object a there are not only alternative a-worlds, but also alternative non-a-worlds – i.e. not only can we get to another possible world by modifying the world ‚around' a, but we can also arrive at a new world by throwing a out. I will not try, however, to work this suggestion out in any detail here.

SECTION 7

Related to the last English sentence we considered are similarly puzzling statements such as ‚Pegasus does not exist'. The fact that such a sentence can be meaningful at all has often been taken as a proof that its grammatical subject cannot be a logically proper name but must be a concealed description, so that we may read the sentence as ‚there is no such thing as Pegasus' or – quite horribly – as ‚there is no Pegasizer'. I do not believe that this argument is correct, and will argue instead that the sentence in question can be understood even on the premise that ‚Pegasus' is a proper name. The central observation to be made in this connection about names of fictional or mythological beings is that they transfer us automatically to *their own* world. Thus a sentence in which ‚Pegasus' occurs will generally be understood to be about the world of Greek Mythology, where Pegasus belongs. If we want to say something about Pegasus in relation to our world, rather than to his own, we have to make this evident through special devices. Expressions such as ‚actually' or ‚in reality' serve this purpose; and so does, in certain contexts, the verb ‚exist'.

These ideas lead to the following formal systems. Let L be a language of modal predicate logic which may contain individual constants; all of these stand for proper names, but they may stand for fictional, as well as ordinary, proper names. A model M for the language specifies the same items that were mentioned earlier. Now, however, the objects it assigns to proper names need not all belong to $U^{M}{}_{w_0}{}^{M}$. Moreover, the model assigns to each constant c a world $w_c{}^M$, *the world of* c.

For ordinary names, which denote actual objects, this world will always be $w_0{}^M$, the actual world. But for a fictional name c $w_c{}^M$ will be a different world. Of course $w_c{}^M$ is generally not the only world in which the thing denoted by c exists. This is evident for nonfictional names. As we have seen earlier it is only because the objects they denote belong also to worlds other than the actual world, that counterfactual assertions in which the names occupy the subject position can be true. And there is no reason why this should be otherwise for fictional names. Even though counterfactuals about fictional objects are somewhat more tenuous than counterfactuals about actual objects, there are some at least which seem to be true e.g. ‚If Hamlet's uncle, rather than Polonias, had been behind the curtain, Hamlet himself would not have been killed.' And there is no reason why we should want to account for the truth of such a counterfactual differently from what appears to be the correct analysis of counterfactuals about actually existing things.

In the present system we should define the *truth value of* a sentence φ *in* M as follows:
(i) If there is no individual constant c in φ such that $w_c{}^M \neq w_0{}^M$, then $[\varphi]^v{}_M = [\varphi]^v{}_{M,\, w_0{}^M}$;
(ii) If there is no individual constant c in φ such that $w_c{}^M \neq w_0{}^M$ and moreover for each other individual constant c′ in φ $w_{c'}{}^M = w_c{}^M$ then $[\varphi]^v{}_M = [\varphi]^v{}_{M,\, w}$
Thus in the present system our primary interest in truth values is not restricted exclusively to the actual world; for certain sentences we want to know the truth values in other, fictional, worlds; for these sentences, moreover, it doesn't even make sense to ask whether they are true in the actual world.

It should be evident that the last definition of the truth value of φ in M is incomplete. The definition says nothing about sentences in which names occur for beings whose main worlds are different. This, however, may well be a virtue rather than a drawback. I think that we *are* puzzled by sentences like ‚If Gilgamesh had smashed Loki, Baldur would have made off with either Isis or Artemis‘ and that we are rightly puzzled. Our inability to make sense of such claims derives, I believe, from our baffle at the conflation of two or more totally unrelated worlds of fiction.

However, it may well be that this throws away too much. In particular there are many tales in which names of actual things appear in perfect harmony with what are obviously names of fictitious beings: Baker Street and Sherlock Holmes, Dreyfuss and Jean Barrois, Selim the Great and Nathan the Wise. In other words some fictional worlds contain actual objects which moreover bear, within their context, the same names as in reality. Thus with names c of actual things at least we should associate a set w_c^M of fictional worlds – all those in which the referents figure under their usual names. We may then stipulate that the truth value of a sentence in which both fictional and actual names occur is defined as the truth value of the world of all the fictional names occurring in φ provided that the worlds of these fictional names are the same and moreover that this world belongs to the set of worlds assigned to each of the ordinary names.

Let us return at last to the sentence which provoked the formulation of our latest system. ‚Pegasus does not exist‘ can now be formalized as $(\forall \vee_1)$ $(Ac(\exists \vee_2)(\vee_2 = \vee_1) \rightarrow \vee_1 \neq c)$

This sentence will be true in every model M such that (i) $w_c^M = w_o^M$ and (ii) w_c^M does not belong to $U^M{}_{w_o}{}^M$.

One plausible objection against the theory outlined here is that it implies that every factual assertion about a fictional object must be either true or false, even assertions like ‚Pegasus has a dark spot over his left eye‘ or ‚Pegasus prefers wheat to oats‘. As Greek mythology does not deal with such touching domestic details about its protagonists there is just no sense even in *trying* to find out whether these claims are true. One way to cope with this objection without departing foo far from the present framework is to allow for ‚partial‘ worlds in which e.g. a 1 place predicate does not have simply an extension – so that it is true of everything in that extension and false of everything outside it – but rather a *positive* as well as a *negative* extension, which are mutually exclusive but not jointly exhaustive. The predicate will be true of the things in its positive extension, false of those in its negative extension, while of the objects which are in neither extension it can neither be said to be true nor be said to be false. More-place predicates can be handled in a similar way.

An alternative, but essentially equivalent approach would be to replace the single fictional worlds by clusters of worlds. A sentence about a fictional object would then be regarded as true if it were true in all the worlds of the cluster, false if it were false in all the worlds of the cluster, and undetermined otherwise. This approach, however, would make it slightly more difficult to handle counterfactuals about fictional entities. A precise elaboration of either suggestion would, however, lead us too far a field.

I have left to the very end the question whether names can refer to what has not yet come into being. I earlier brushed this question aside by simply assuming that this is not possible. This is indeed what I believed for some time. But I do now think that this is not right. I was lead to this new conviction by some remarks made to me in conversation by Mr. Paul Durham of UCLA. His point, which seems to me to be conclusive, draws attention to the ways in which names are introduced. The most basic way of introducing a name is often held to be the one in which someone points at a thing saying, ‚That is . . .', or ‚That will be called . . .', where the expression in the place of the dots is the name introduced. Unfortunately this paradigm is like so many others which philosophers make the starting points of their doctrine quite exceptional in practice. Usually names are introduced by means of speech acts of the form ‚. . . is the so and so', or ‚„. . ." will be the name of the so and so', e.g. ‚„John" will be the name of our next child'. The interest of such speech acts is that they introduce a name, i.e. a rigid designator, via a description which itself may vary its reference with time and other contextual aspects. Now the object designated by the description may exist at the time when the speech act is performed. Such is the case when an explorer confers new names (usually inspired by his home-town or royal superior; why are those who absorb so much food for fantasy generally so dull when it comes to naming?) upon newly discovered islands, bays or mountains. In these cases the naming, even though it may take the form of a pronouncement such as: ‚This island, situated at . . . and . . ., shall be called St. Edward's Island', could also have been achieved by just saying ‚This is St. Edward's Island' while stamping on the ground, or hoisting a flag. Cases where the pronouncement of the phrase could not be replaced by an actual pointing at the named object simultaneous with pronouncing the name occur for example when it is not the explorer who confers the name but rather some authority in his homeland which bases its action on the charts the explorer has brought back from his voyage. Similar namings occur in astronomy where often names are attached to celestial bodies which are not — and often even cannot — be seen directly but whose existence can be indirectly inferred from other observations. Such namings can of course go wrong, as the descriptions used may not properly denote. A typical and well-known example is that of Vulcan, the planet which was supposed to cause such and such disturbances in the paths of other observed planets. What happens when we realize that the naming was a misfire? Well, in the first place we recognize that all our previous statements which involved the name are nonsensical. It is interesting, however, that even after such a discovery the name itself doesn't disappear entirely from the language. We do still say things like: ‚It was eventually discovered that Vulcan never existed'. Now, one might argue that although the word ‚Vulcan' occurs in this last statement, it isn't used there any more as a name but truly as an abbreviated description. But, even if I see no way to refute this view conclusively, I am not sure that it is right. The reason why it *appears* to be right is that we can find out the truth of the statement only by applying the procedure for finding the bearer of ‚Vulcan' which comes with the name – the procedure, that is, which is suggested by the description through which the name was introduced. Thus, one might be tempted to conclude, the only reason why the statement can be said to be true is that with the word ‚Vulcan' is associated a definite sense, the one conferred by the introducing descriptive phrase. Here, however, it becomes important to ask if in this sense of ‚sense' not all names have a sense, as well as, in the ordinary case, a referent. In principle there is always *some* procedure for finding out what the referent is of a name which one encounters for the first time: one tries to trace the use of the name back to its origin, i.e. the act through which it was introduced. Usually that act will have been a speech act of the form: ‚the so and so shall be called . . .'. So in the end one will have to use the description ‚the so and so' in order to determine the referent. (In practice there are of course almost always alternatives to, or shortcuts through, this procedure available, but that does not change the fact that the procedure just suggested is the basically correct one and that, consequently the others can be regarded as correct only *in so far as* they give the same result as this one. So I see no reason why we should look upon the use of ‚Vulcan' after the discovery that there is no such thing as the word purports to denote, as the use of something which is not a name.

This discussion shows that the commonly drawn distinction between expressions which ‚have sense' – such as e.g. definite descriptions – on the one hand, and proper names which have *only* denotation, on the other is really more complicated than it is generally believed to be. It seems to me that all that can be said about the difference is that a complex description makes its sense explicit through its structure – that is why its referent can be determined *even though* it has never been associated with the description by an explicit act of baptism; and also why its referring function can be understood even though it varies with the circumstances. This can of course also be the case for certain expressions which though apparently simple, really *are* understood as short for complex expressions whose structures make their senses manifest. But I think it is at least dubious and probably false, that ‚Vulcan' *ever* functioned as such an abbreviation, even after the discovery that no such planet could exist.

Everything I have said so far about introductions of names through speech-acts of the form ‚. . . shall be the name of the so and so', applies without exception to cases where ‚the so and so' denotes something which does not yet exist, even the cases where it *follows from the form* of ‚the so and so' that whatever it refers to cannot yet have come into being – as with e.g. ‚The Jones's second next pair of twins'. In fact the very bulk of name introductions is for objects which do not exist at the time of the introduction, viz. those which introduce names for children which are yet to be born. (Of course, there is a slight difficulty here concerning namings after conception, for some might maintain that what is being named on such occasions is not the future child but rather the actual foetus; however, I do not believe that this is what future parents do in general intend. In any case the insistence on this becomes spurious from a theoretical standpoint as soon as one accepts that similar acts of naming can be performed before conception. Perhaps, though, these aren't quite the same; for parents who name their future child after conception have, something present that appears to give some concrete justification to the naming which is missing in preconceptual acts of name giving. I think, however, that the intuition I have here tried to express is misleading. Its proper explanation is this: once conception has taken place the chance that the naming will in the end turn out to have been a successful one (viz. when the child is indeed born and thus renders the description used in the introduction a properly denoting phrase) increases substantially. Now, even if we accept that there are non-denoting names, we may still subscribe to the doctrine that namings are really *proper* only if the names they introduce denote. Non-denoting names, even if they are names, really *are* clutter that impedes language's proper functioning as a vehicle for communication (it should be obvious from what I have said earlier that here I do not aim at fictional names, but only at names which purport to denote actual things but fail to do it). Improper names should be shunned; and post-conceptional namings are safer – i.e. less likely to turn out as improper- than pre-conceptional ones.

In incorporating these observations in our formal system for tense logic let us first assume that no names are non-denoting. Then we may formulate our system in virtually the same way as before, except that now we do no longer insist that the denotation in M of a constant c of L_t should belong to $\bar{u}^M{}_t$. The most important consequence of this is that the truth value in M at t of a sentence φ of L_t may now well be undetermined: This will always be so if φ contains an individual constant whose denotatum has not yet come into existence at t (unless we make the arbitrary and unwarranted decision that all atomic formulae containing c will be satisfied at t by (say) no assignment whatever). The situation is here different from the one we found ourselves in earlier; now we can no longer salvage the principle that every sentence of L_t has a definite truth value in M at t by adapting the temporal reach of the operator P to the past histories of the objects named by the individual constants mentioned in its scope (or context). On the other hand it is now plausible that we should modify the satisfaction clause for formulae F φ, so that the reach of F only begins at a time where all the objects referred to by constants in φ have come into being:

$$[F\ \varphi]^{\tau,v}{}_{M,t,t',a} = 1 \text{ iff there is a } t'' > t \text{ such that } \textit{for} \text{ every c in } \varphi \; c^M \varepsilon\, \bar{U}_{t''}{}^M \text{ and } [\varphi]^{\tau,v}{}_{M,t'',t',a} = 1$$

It appears to me that future tense statements involving names that refer to things which do not yet exist, are very strong witnesses to the adequacy of the doctrine which analys what is true now in terms of what will be true later (or has been true in the past): I cannot conceive of any other way in which we might understand the parents' musings about what their John, who won't be born for at least another six months, will become when he will be a grown man. For the name, even though it is already there, will come to do its *normal duty* only by the time John will be there.

Let us now allow also names which fail to denote altogether. This forces us to proceed in a slightly different way. As I have said, names can be introduced either by direct attachment to an object or through some description. Introductions of the former type may as before be represented in the model by a simple assignment of an object c^M to the constant c. But again we should insist, as I did earlier on, that if $c \varepsilon L_t$ then $c^M \varepsilon \bar{u}_t^M$. An introduction of the latter kind at a time t may be represented formally by adding to L_t a pair consisting of (1) the individual constant which represents the name introduced and (2) the description through which the introduction takes place. If the description fails to denote properly, so will the name. But in that case while all or some sentences containing the description may still be regarded as meaningful (this depends on the particular theory of descriptions we eventually adopt) the corresponding sentences with the name should *all* be considered as meaningless. This is one way in which the name's ‚senselessness' manifests itself. Thus, suppose I introduce the expression ‚François II' by stipulating that it is to be the name of the next king of France, then even if ‚the next king of France will be bald' may be false simply because France will never have a king again, ‚François II will be bald' will be meaningless. We may know enough about a name to know how we should go about finding its referent. But if we follow that route and don't find anything, we have discovered an anomaly of language – even if, as I argued, it is not the discovery that a certain expression belongs to a different syntactic category from where we previously believed it to belong. If we arrive at the same dead end when trying to find the referent of the corresponding description, what we discover is certainly *not* an anomaly of language. If it is an anomaly at all, it is at best one of our, or others', or everyone's beliefs.