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The Adequacy of Translation between Formal and Natural Languages

Introduction

THIS PAPER WAS written in 1970.[1] It arose out of a project led by Mr John Olney of the System Development Corporation in Santa Monica in which I had participated during part of the period 1966–8 when I was at the University of California at Los Angeles. This project tried to throw light on the semantics of English discourse by translating parts of English texts (all chosen from the *Scientific American*) into symbolic notation. The immense difficulties accompanying such an effort should be evident to anybody who has ever stopped to think about the complexity and variety of devices which we employ in ordinary speech, whatever it is that we wish to communicate. Indeed, the participants occasionally wondered whether anything would have come of the project if there had been a sufficiently vivid perception of these difficulties from the start. It soon became clear that the problem was not so much that of translating the English texts into an already existing formal system, such as first order or higher order logic, but rather that the available symbolic languages themselves had to be reappraised and extended at every turn. Almost every simple new English sentence required the introduction of new symbolic notation or else a carefully argued defence of the use of already existing symbolism to render a locution which had not yet been dealt with before.

Although this work was by no means useless, it had carried so strong a flavour of the ad hoc that we felt it was necessary to reflect upon the general methodological question of how and in what sense 'translations' of the sort we were attempting to give could contribute to linguistic theory. The present paper is the result of my own efforts to come to grips with this problem. It tries to provide a general and formally precise account of what should be understood by a translation from one language into another. It should be

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evident that such a formal account requires an underlying formal characterisation of what a language is. In view of the nature of the problem which led to the present investigation this characterisation had to encompass so-called 'formal' or 'symbolic', as well as natural, languages.

I presume that this identification of formal and natural languages will now meet with less resistance than it would have done in 1970. Even then, however, the view was not original. Indeed, it had at that time already been defended by the late Richard Montague. Montague's influence on the form and content of this paper is so deep and pervasive that it seems pointless and misleading to credit him separately with any specific ideas that will appear below. Yet some remarks concerning his influence seem in order. My notions of a formal syntax and an accompanying semantics differ only in details from a similar proposal of Montague's—such as that which can be found in his 1968 paper 'English as a Formal Language'—or in the article 'Universal Grammar' which was written at essentially the same time as the pages following this introduction (see Montague, 1974a). 'Universal Grammar' contains, moreover, a formal definition of the concept of translation; like everything else in that paper it is a very elegant statement, considerably more elegant than what the reader will find here. Indeed, I discovered after completion of my own paper that it paralleled 'Universal Grammar' in so many respects that its publication only seemed desirable if it were substantially expanded, preferably with some concrete applications. The great merit of Montague's work on grammar is that it provides absolutely precise accounts, along the lines of his theoretical persuasion, of actual parts of English. This has made him vulnerable to a good deal of criticism from linguists; but on the other hand, it is because of the painstaking rigour and lucidity of these concrete applications that his ideas have succeeded in capturing the imagination of so many linguists and philosophers over the past seven years.

The original purpose of this paper made the inclusion of such applications unnecessary. After all, the translations which the project produced were in principle available to anyone who requested them. However, the reader of this volume could hardly be expected to go through the trouble of obtaining them. In any case they are in a state so far removed from the ideal expounded in this paper that their effect would probably have been to destroy, rather than strengthen faith in the theory. I did consider adding to this article a concrete example of a translation from a fragment of one natural language into another; we now know more than we did five years ago about the syntax and semantics (of the sort which is explained and used in this paper) of fragments of natural languages, in particular languages other than

English, and also about the semantics of various extensions of the standard systems of symbolic logic, which might be used as intermediaries in translations between natural languages. Finally, however, I decided against such a project. It would have added another ten pages to an already lengthy paper; and, more crucially, I lacked the time to accomplish the task adequately. I realise, however, that in the absence of such an addition much of what this paper has to offer will appear schematic and divorced from the reality of translating actual languages.

Some readers will have doubts about my approach to the theory of translation which even a concrete example of the sort I envisaged would not dispel. I will try, in the remainder of this introduction, to give some indication of how those doubts might eventually be put to rest.

The problems with which the actual translator must cope often seem difficult beyond solution. Many of these difficulties are no doubt incidental. Yet some of them may well be the symptoms of a much deeper and more universal issue, much discussed in recent philosophical literature as the 'indeterminacy of translation'. It seems even now to be a matter of dispute exactly what the doctrine of indeterminacy amounts to. Since the appearance of Quine's 'Word and Object' various authors have proposed different and usually non-equivalent formulations of the doctrine; and as we try to render the arguments for and against these different versions more precise, ever finer distinctions will have to be drawn.

It would defeat the purpose of this introduction to go into these matters in detail. So let me offer just one possible statement of the indeterminacy thesis, which approximates, I believe, fairly closely what most would regard as the central claim: the meaning of many expressions of any natural language L is underdetermined by the verbal dispositions of the speakers of L; and in view of this, there may exist alternative schemes for translating L into some other language L′ such that there simply is no criterion that designates one of these as the correct scheme; moreover, this situation may arise even in a case where the translations s' and s'', according to the respective schemes, of the same sentence s of L are incompatible sentences of L′.

The theory developed here ignores the indeterminacy issue completely. Superficially it may even seem to contain the implicit denial that indeterminacy occurs. Those for whom indeterminacy is the fundamental problem of translation would probably regard such denial as sufficient reason to dismiss the theory as irrelevant. This obliges me to give, even at this early stage, a brief outline of the analysis whose details are spelled out in the main body of the paper.

A *translation* from a language L into a language L′ is defined as a certain function from the syntactic analyses of expressions (in particular, sentences) of L to syntactic analyses of expressions of L′ of the same semantic type; the *adequacy* of a translation is defined relative to a certain fixed association of a model for L′ with each of the models of L. Relative to such an association a given translation T is adequate if:

(1) it transforms each L-expression ϕ in its domain into an L′-expression $T(\phi)$ such that for each model $\mathcal{A}$ for L, $\mathcal{A}$ assigns to ϕ the same semantic value as the model for L′ associated with $\mathcal{A}$ assigns to $T(\phi)$; and
(2) the domain of T includes all (analyses of) sentences of L.

This characterisation may seem to be without empirical content insofar as it depends on a prior association of the models of the two languages. But this appraisal would be too harsh. Consider the case of Quine's linguist who joins a completely unknown tribe to learn their language L without any help from interpreters, earlier compiled dictionaries, or the like. Among the models for the language L there must be one in particular which represents the actual world. The natives' knowledge of L includes their ability to decide of (some of) the sentences of L whether they are true (either true absolutely, or true on particular occasions of use). Within the format of the grammar used in this paper, that ability comes to this. The speaker can identify, at least in principle, one of the models which belong to the semantics for L as representing the world surrounding him (from the perspective of his particular speech situation). Similarly, the linguist's knowledge of his own language L′ amounts to a parallel ability to identify the world surrounding him as some model of the semantics for L′. It is these two abilities which form the empirical basis of the association relation relative to which the adequacy of a translation scheme must be judged: when a speaker *s* of L and a speaker *s*′ of L′ are placed in the same situation, the model for L′ which *s*′ selects as corresponding to the situation must be the value which the association function assigns to the L-model selected in that situation by *s*.

What is the empirical significance of the claim that it is the model $\mathcal{A}$ which a speaker selects on a certain occasion as representing the world from that particular perspective? This is a difficult question; and it is, among other things, in the answer to this question that our views regarding the indeterminacy thesis will manifest themselves. If the language L were very simple, say a language of first order predicate logic whose primitive predicates all represent clearly recognisable and sharply demarcated qualities, and if, moreover, it were possible to point at objects without ambiguity, then the claim could be tested by asking the native speakers, pointing at various objects in turn, whether or not they belong to the extensions of the primitive predicates. This test could be pursued as far as desired, and if the claim is correct, this could in principle be so discovered.

With actual spoken languages the situation is bound to be much more complicated. The empirical evidence that can be gathered from the language behaviour of the speakers of L is in all probability not sufficient to determine which L-models correspond to particular contexts of use. It may well be argued therefore that the association relation is only partially defined. (The same considerations will of course apply to the correspondence between contexts of use and models of the translator's language L'.)

The indeterminacy of the association relation is not the only point where indeterminacy may manifest itself. The translator who starts from scratch, without any previous knowledge of the language L he is to translate into his own language L', will, if he is to proceed along the lines of the theory of this article, first have to formulate a syntax and semantics for L. And what criteria are there to decide whether that part of his task has been properly accomplished? In the last analysis the only criterion is the success of the translation scheme the translator manages to formulate on the basis of his syntax and semantics for L; and the success of a translation scheme can only be measured in terms of the quality of the explanation that it provides of the language behaviour of the speakers of L—or rather, of the explanation of this behaviour that is provided by a theory of which this scheme is part, but of which it needs not be the only part. It is an interesting, though complicated, question whether, or to what extent, the data about language behaviour will allow the translator to assess separately the adequacy of any one of the various components into which his theory of the language L breaks down (if it is set out along the lines followed in the present paper). I am afraid however that little about this can be said unless much more specific assumptions about L (as well as about L') are considered.

The rapidly expanding literature on model theoretic semantics for natural languages has to my knowledge been silent on the relation between the formal models its theories postulate and the phenomena of actual language use (such as assent to and dissent from sentential utterances, or language learning) to which we have more or less direct access. This is a gap which must ultimately be filled if we wish to develop model theoretic semantics into a comprehensive theory of language which is securely anchored to the linguistic facts we can observe. What follows does nothing to narrow that gap. I can only hope that the reader will have some idea of how my proposal could be applied to concrete cases; for without it he will find what follows a sterile exercise.

1. *Formation rules*

We will first develop a general, purely syntactical, notion of a *language*. A language will be essentially a set of well-formed expressions; to each expression is assigned some *grammatical category* of the language. Rules referring to these grammatical categories determine how given expressions may be combined into more complicated ones. These rules are called *formation rules*. The expressions of the language will all be strings of symbols that are drawn from a certain set which is given in advance.

Our first concern will be with the characterisation of what sorts of operations on strings we will allow. One might hope that only one operation might suffice, viz. that of concatenation of strings. But this hope is vain. Restriction to this sort of formation alone would lead to syntaxes for natural languages which, if they could be given at all, would be cumbersome and implausible (e.g. the number of grammatical categories they would have to contain would be very large).

Thus we will have to allow for other types of formation as well. Among the formation operations which are especially important is the one which gives for any strings x, y and symbol a, the string that results from replacing a in x everywhere by y. (We will henceforth refer to this operation as 'substitution'.)

On the other hand, the notion of a formation rule should not be too wide. In particular, formation rules should always be effective, in the sense that for any given rule there will be an effective procedure by means of which we can decide, for any expressions $e_1, \ldots, e_n, e_{n+1}$, whether e_{n+1} comes from applying the rule to $e_1, \ldots, e_n$ or not.

A notion of a formation rule that satisfies all these requirements can be developed in terms of the notion *formation language* defined below. This language is a system of first order predicate logic which contains only two non-logical constants, a 2-place predicate constant $\subset$ (where $\tau \subset \tau'$) is to be read as: 'τ is a substring of τ'') and a 2-place operation constant $\frown$ (where $\tau \frown \tau'$ is to be read as: 'the concatenation of τ and τ''). Formation rules will be characterised as operations which can be defined both by a purely existential, and by a purely universal formula of the system. This characterisation will on the one hand provide us with e.g. substitution, and on the other hand warrant the effectiveness of all formation rules.

Definition 1: let S be a set and F an n-place function. The *closure of* S *under* F (in symbols $cl_F(S)$) is defined as follows: Let

(i) $S_0 = S$;
(ii) for $n = 0, 1, 2, \ldots$ let
$S_{n+1} = S_n \cup \{x: \text{there are } x_1, \ldots, x_n \in S_n \text{ such that } x = F(x_1, \ldots, x_n)\}$.
$cl_F(S) = \bigcup_{n \in \omega} S_n$.

We say that F is *well-founded on* S iff $S = cl_F(S - \text{Range } F)$.

Definition 2: by the *formation language*, F1, we understand the first order language which is defined as follows:

Symbols: (1) variables: $v_0, v_1, v_2, \ldots$;
(2) logical constants: $\sim, \wedge, \vee, =$;
(3) a 2-place predicate constant $\subset$ and a 2-place operation constant $\frown$;
(4) parentheses (,).

Terms: (i) v_i is a *term*;
(ii) If τ, τ' are terms then $(\tau \frown \tau')$ is a *term*.

Formulae: (i) If τ, τ' are terms then $\tau = \tau'$ and $\tau \subset \tau'$ are *formulae*;
(ii) If ϕ, ψ are formulae, then $\sim\phi$, $(\phi \wedge \psi)$, $\vee v_i \phi$ are *formulae.*

Interpretations:

An *interpretation* for F1 is a pair $\langle D, P \rangle$ where

(i) D is a non-empty set;
(ii) P is a two-place function with domain D;
(iii) P is well-founded on D;
(iv) If $x, y, z, \in D$ then $P(x, P(y, z)) = P(P(x, y), z)$;
(v) If $x, x', y, y' \in D$ and $P(x, y) = P(x', y')$ then either ($x = x'$ and $y = y'$) or there is a $z \in D$ such that ($x = P(x', z)$ and $y' = P(z, y)$) or there is a $z \in D$ such that ($x' = P(x, z)$ and $y = P(z, y')$).

For any interpretation $\mathcal{A} = \langle D, P \rangle$ for F1, any sequence[2] d of members of D and any expression (i.e. term or formula) ϕ of F1, the *value assigned by* d *in* $\mathcal{A}$ *to* ϕ (in symbols: $[\phi]_{\mathcal{A}, d}$) is defined as follows:

(i) $[v_i]_{\mathcal{A}, d} = d_i$;
(ii) $[\tau \frown \tau']_{\mathcal{A}, d} = P([\tau]_{\mathcal{A}, d}, [\tau']_{\mathcal{A}, d})$;
(iii) $[\tau = \tau']_{\mathcal{A}, d} = \begin{cases} 1 \text{ if } [\tau]_{\mathcal{A}, d}, \text{ is equal to } [\tau']_{\mathcal{A}, d} \\ 0 \text{ otherwise} \end{cases}$

2. By a *sequence* we understand a function the domain of which is the set of natural numbers.

$$\text{(iv) } [\tau \subset \tau']_{\mathcal{A},d} = \begin{cases} 1 \text{ iff there are } a, b, \in D \text{ such that} \\ \text{either } [\tau']_{\mathcal{A},d} = P(a, [\tau]_{\mathcal{A},d}) \\ \text{or } [\tau']_{\mathcal{A},d} = P(P(a, [\tau]_{\mathcal{A},d}), b) \\ \text{or } [\tau']_{\mathcal{A},d} = P([\tau]_{\mathcal{A},d}, b) \\ 0 \text{ otherwise} \end{cases}$$

$$\text{(v) } [\sim\phi]_{\mathcal{A},d} = \begin{cases} 1 \text{ if } [\phi]_{\mathcal{A},d} = 0 \\ 0 \text{ otherwise} \end{cases}$$

$$\text{(vi) } [(\phi \wedge \psi)]_{\mathcal{A},d} = \begin{cases} 1 \text{ if } [\phi]_{\mathcal{A},d} = 1 \text{ and } [\psi]_{\mathcal{A},d} = 1 \\ 0 \text{ otherwise} \end{cases}$$

$$\text{(vii) } [\vee v_i\phi]_{\mathcal{A},d} = \begin{cases} 1 \text{ if there is an } a \in D \text{ such that} \\ [\phi]_{\mathcal{A},d_{(a/i)}} = 1^3 \\ 0 \text{ otherwise} \end{cases}$$

The *elementary formulae* of F1 are characterised by:

(i) $\tau = \tau'$ is an *elementary formula*;

(ii) If ϕ, ψ are elementary formulae then $\sim\phi$, $(\phi \wedge \psi)$, $\vee v_i(v_i \subset v_j \wedge \phi)$ and $\wedge v_i(v_i \subset v_j \rightarrow \phi)$[4] are *elementary formulae.*

Definition 3: a k-*place formation rule* of F1 is a pair $\langle\phi, \psi\rangle$ of elementary formulae of F1 such that $v_0, \ldots, v_k, v_{k+1}$ are all the free variables of ϕ and all the free variables of ψ, and for any interpretation $\mathcal{A} = \langle D, P\rangle$ and sequence d of elements of D, $[\vee v_{k+1}\phi]_{\mathcal{A},d} = [\wedge v_{k+1}\psi]_{\mathcal{A},d}$.

Let $\langle\phi, \psi\rangle$ be a k-place formation rule of F1. Let $\mathcal{A} = \langle D, P\rangle$ be an interpretation for F1 and let $d_0, \ldots, d_{k-1}, d_k$ be members of D. We say that $d_0, \ldots, d_{k-1}, d_k$ *satisfy* $\langle\phi, \psi\rangle$ *in* $\mathcal{A}$ iff for any sequence d of members of D, if for $i = 0, \ldots, k$, $d_i = d_i$, then $[\vee v_{k+1}\phi]_{\mathcal{A},d} = 1$.

It is obvious that concatenation itself corresponds to a formation rule. Indeed, it is characterised by the pair $\langle v_3 = v_3 \wedge v_2 = v_0 \frown v_1, v_3 = v_3 \rightarrow v_2 = v_0 \frown v_1\rangle$. To show that substitution can also be given as a formation rule is somewhat more involved. Since the construction of the proper pair $\langle\phi, \psi\rangle$ is both cumbersome and straightforward, we omit it.

The notion of formation rule developed here has the disadvantage that it is not recursive; i.e. the set of those pairs of elementary formulae ϕ and ψ with $v_0, \ldots, v_k$ free such that $\vee v_k\phi$ and $\sim\vee v_k\sim\psi$ are materially equivalent in all interpretations, fails to be recursive (in the ordinary sense of recursive). A more satisfactory notion of formation rule—i.e. one which is also suf-

3. If d is a sequence then by $d\frac{b}{i}$ we understand the sequence which is like d except that $\left(d\frac{b}{i}\right)(i) = b$.

4. We understand expressions like $\wedge v_i\varphi$ and $(\varphi \rightarrow \psi)$ as metalinguistic abbreviations.

ficiently powerful, but is such that the set of formation rules is itself recursive—could probably be developed by means of systems of equations analogous to those used to define the set of primitive recursive functions of natural numbers.

2. *Syntax*

We now proceed to develop a general notion of syntax.

Convention: throughout the remainder of this paper let E be a non-empty set and let P be a 2-place operation on E such that E and P satisfy the conditions (i), (ii), (iii) in the definition of an interpretation on page 281; and let $\mathscr{E} = \langle E, P \rangle$. Clearly E – Range P is non-empty. We refer to the members of E as the *expressions* of $\mathscr{E}$ and to the members of E – Range P as the *symbols* or *basic expressions* of $\mathscr{E}$.

Definition 4: by a *formal syntax* (for $\mathscr{E}$) we understand a pair $\langle \mathscr{B}, \mathscr{R} \rangle$, such that

(i) $\mathscr{B}$ is a function, the domain of which is the union of {0, 1} and a subset of E and the range of which consists of sets of symbols.

(ii) $\mathscr{R}$ is a set of triples such that:

(iii) For each triple $\langle r, \phi, \psi \rangle \in \mathscr{R}$ there is a number k such that r is a $k + 1$-place sequence of subsets of the domain of $\mathscr{B}$ and $\langle \phi, \psi \rangle$ is a k-place formation rule.

With any formal syntax $\mathscr{S} = \langle \mathscr{B}, \mathscr{R} \rangle$ for $\mathscr{E}$ we associate a function $\zeta_{\mathscr{S}}$, with the same domain as $\mathscr{B}$, as follows:

(i) For $\alpha \in \text{Dom } \mathscr{B}$ let $\zeta^{0}_{\mathscr{S}}(x) = \mathscr{B}(\alpha)$.

(ii) If n is a natural number and $\alpha \in \text{Dom } \mathscr{B}$, then $\zeta^{n1}_{\mathscr{S}}(\alpha) = \zeta^{n}_{\mathscr{S}}(\alpha) \cup$ $\{\gamma \in E$: there is a natural number k, and a member $\langle r, \phi, \psi \rangle$ of $\mathscr{R}$, where r is $k + 1$-place and there are $\gamma_0, \ldots, \gamma_{k-1} \in E$ such that for $i = 0, \ldots, k - 1, \gamma_i \in \zeta^{n}_{\mathscr{S}}(\beta)$ whenever $\beta \in r_i$, and $\langle \gamma_0, \ldots, \gamma_{k-1}, \gamma \rangle$ satisfies $\langle \phi, \psi \rangle$ in $\mathscr{E}$; and $\alpha \in r_k\}$.

(iii) For $\alpha \in \text{Dom } \mathscr{B}$, let $\zeta_{\mathscr{S}}(\alpha) = \bigcup_{n \in \omega} \zeta^{n}_{\mathscr{S}}(\alpha)$.

We think of the members of Dom $\mathscr{B}$ as names of grammatical categories. For each $\alpha \in \text{Dom } \mathscr{B}$, $\zeta_{\mathscr{S}}(\alpha)$ will be regarded as the category the name of which is α; the members of the $\mathscr{B}(\alpha)$'s will be referred to as the *basic expressions of* $\mathscr{S}$. We will call the members of $\bigcup_{\alpha \in \text{Dom } \mathscr{B}} \zeta_{\mathscr{S}}(\alpha)$ the *well-formed* *expressions* of $\mathscr{S}$; the members of $\zeta_{\mathscr{S}}{}^{(0)}$ will be the *formulae* of $\mathscr{S}$; and the members of $\zeta_{\mathscr{S}}{}^{(1)}$ the *variables* of $\mathscr{S}$.

As an example might serve the case where E – Range P consists of all words of English (and perhaps interpunction signs and blank space), one of the members of Dom $\mathscr{B}$ is the expression 'common noun phrase', $\mathscr{B}$ ('common noun phrase') is the set of all English common nouns (as 'tree', 'house', 'thought', 'noun'), and $\zeta_{\mathscr{S}}$ ('common noun phrase') consists of all common noun phrases, i.e. common nouns together with compound phrases like 'big tree', 'house with a garden', 'thought that is wrong', 'common noun phrase', etc.

The members of $\mathscr{R}$ are called the *rules of grammar* of the syntax $\langle \mathscr{B}, \mathscr{R} \rangle$. They tell us not only how to obtain compound expressions from expressions which belong to particular categories, but also to which categories the resulting compound expression belongs. It is perhaps often assumed (either explicitly or tacitly) that a simpler concept of syntax, in which no other types of formation play a role than concatenation, would be sufficiently general—and thus, in view of its simplicity, preferable to the notion defined above. Applying this simplification to the concept just defined we would obtain:

A *formal syntax* is a pair $\mathscr{S} = \langle \mathscr{B}, \mathscr{R} \rangle$, where $\mathscr{B}$ is as above and $\mathscr{R}$ consists of finite sequences of members of Dom $\mathscr{B}$. The categories $\zeta_{\mathscr{S}}(\alpha)$ of such a syntax would be defined by:

(i) $\zeta^{0}_{\mathscr{S}}(\alpha) = \mathscr{B}(\alpha)$;

(ii) $\zeta^{n+1}_{\mathscr{S}}(\alpha) = \zeta^{n}_{\mathscr{S}}(\alpha) \cup \{\gamma \in \mathrm{E}$: there is a number k, and $r \in \mathscr{R}$ and $\gamma_0, \ldots, \gamma_{k-1} \in \mathrm{E}$ such that:

(a) r is a $k+1$-place sequence;

(b) for $i = 0, \ldots, k-1$ $\gamma_i \in \zeta^{n}_{\mathscr{S}}(r_i)$;

(c) $\gamma = \mathrm{P}(\ldots(\mathrm{P}(\gamma_0, \gamma_1), \ldots), \gamma_{k-1})$;

(d) $\alpha \in r_k\}$;

(iii) $\zeta_{\mathscr{S}}(\alpha) = \cup\, \zeta^{n}_{\mathscr{S}}(\alpha)$.

Indeed, the syntaxes of all well-known finite languages of symbolic logic are usually given in this simpler form; and that syntaxes for natural languages, like English, can be developed in this same simpler fashion, is not impossible. It is, however, as was indicated before, unlikely that one would obtain a *natural* syntax for English in this way. Such a syntax would probably involve an unduly large number of categories and would also probably be incapable of resolving certain syntactic ambiguities (such as those related to personal pronoun anaphora) and thus push such problems back to the semantical level, where, in our opinion, they do not belong. The lack of success met by attempts to develop a formal syntax for English that employs no other formation rules than simple concatenation suggests that this approach is unsatisfactory. As Montague has pointed out—and as has since also come to be accepted by an increasing number of linguists—a particularly important formation rule is substitution. But there is no reason why formation rules other than concatenation and substitution should not be equally indispensable. A class of likely candidates is e.g. that of the various deletion rules that have been discussed by transformationalists. It may turn out that various other transformations will have to be adopted to achieve the simplest theory of language.

The reasons why we have allowed the members of r to be *sets* of category names, are similar to those which prompted us to introduce other means of formation besides concatenation. Allowing only single names of categories as members of the first components of formation rules would in practice lead to unnaturally large numbers of categories and would, moreover, make it more difficult to extend a formal syntax for a certain fragment of a natural language to one which would cover a larger part of that language.

For every well-formed expression of a given formal syntax there is a certain 'construction' which establishes its well-formedness. Such 'constructions' are essentially finite trees; and that is how we will represent them. We call such construction trees *analyses*.

The definition of the notion of an analysis is quite straightforward. It requires, however, some concepts related to the mathematical concept of a tree, the definitions of which we will give first.

Definition 5: (a) A tree is a set T of finite sequences of natural numbers, such that

(i) the empty sequence, 0, belongs to T;

(ii) if $s \smile \langle n \rangle$[5] belongs to T then s belongs to T and for all $m < n$, $s \smile \langle m \rangle$ belongs to T.

(b) If T is a tree and $t, t' \in$ T then we will say that t' *is below* t (in symbols: $t' < t$) iff t is a proper initial segment of t'. If $t \in$ T and there is no $t' \in$ T such that t' is below t then t is called an *endpoint of* T. If $t, t' \in$ T, $t' < t$ and there is no $t'' \in$ T such that $t' < t''$ and $t'' < t$, then we say that t' is a *successor* of t.

(c) Let T be a tree. T′ is a *subtree of* T iff T′ is a set such that for some $t \in$ T it is the case that

5. By $s \smile t$ we understand, in general, the concatenation of s and t.

(i) for all sequences t', $t' \in T'$ iff $t \smile t' \in T$.

If (i) holds for T, T′, t then we call t a *top* of T′ in T.

(d) A *decorated tree* is a function whose domain is a tree. We refer to the points, endpoints, etc. of a tree T also as the *points*, *endpoints*, etc. of any decorated tree whose domain is T.

(e) By a *subtree* of a decorated tree T we understand a function T′ such that Dom T′ is a subtree of Dom T and that if t is a top of Dom T′ in Dom T, $T'(t') = T(t \smile t')$.

Definition 6: (a) Let T be a tree. A subset B of T is called a *bar* of T if B satisfies the following conditions:

(i) If $t \in T$ then there is a $t' \in B$ such that either $t \leqslant t'$ or $t' < t$;
(ii) if $t, t' \in T$, and $t < t'$ and $t \in B$ then $t' \notin B$.

(b) If T is a decorated tree then B is called a *bar* of T iff B is a bar of Dom T.

Definition 7: (a) Let $\mathscr{S} = \langle \mathscr{B}, \mathscr{R} \rangle$ be a formal syntax. Let γ be a well-formed expression of $\mathscr{S}$. An *analysis of* γ *in* $\mathscr{S}$ is a finite decorated tree A such that:

(i) The range of A consists of pairs;
(ii) if t is an endpoint of A, then $[A(t)]_0$[6] belongs to the range of $\mathscr{B}$ and $[A(t)]_1 = \langle\langle\{\alpha : [A(t)]_0 \in \mathscr{B}(\alpha)\}\rangle, 0, 0\rangle$;
(iii) if t is not an endpoint of A and $t_0, \ldots, t_{k-1}$ are all the successors of t in A then there is a k-place rule $\langle r, \phi, \psi \rangle \in \mathscr{S}$, such that:

(a) for $i = 0, \ldots, k-1$, if $\alpha \in r_i$ then $[A(t_i)]_0 \in \mathscr{B}(\alpha)$;
(b) if $\alpha \in r_k$ then $[A(t)]_0 \in \zeta_{\mathscr{S}}(\alpha)$;
(c) $[A(t_0)]_0, \ldots, [A(t_{k-1})]_0, [A(t)]_0$ satisfy $\langle \phi, \psi \rangle$ in $\mathscr{E}$;
(d) $[A(t)]_1 = \langle r, \phi, \psi \rangle$;

(iv) $[A(0)]_0 = \gamma$.

(b) A is an *analysis in* $\mathscr{S}$ iff there is a well-formed expression γ of $\mathscr{S}$ such that A is the analysis of γ in $\mathscr{S}$.

Since the formation rules of $\mathscr{S}$ are recursive, the concept of an analysis in $\mathscr{S}$ will be recursive also provided $\mathscr{S}$ has a finite number of rules. Even then it does not follow, however, that the concept of a well-formed expression of $\mathscr{S}$ is recursive. For in the first place, definition 7 does not exclude the possibility of infinite analyses. But even where all analyses in $\mathscr{S}$ are

6. If s is a finite sequence then by $[s]_i$ we understand the ith component of s.

finite it may be impossible to determine in terms of some directly identifiable features of the string ϕ (such as e.g. its length) any upper bound of the analyses which ϕ would have if it were indeed well formed. In each of these cases it may well be impossible to formulate a decision procedure for well-formedness in $\mathscr{S}$. If, however, (i) we have a uniform way of computing for arbitrary expressions γ of $\mathscr{S}$, a tree size such that if γ has any analysis, then it has an analysis of at most that size; (ii) for every expression γ in E there are only finitely many pairs consisting of a k-place rule $\langle r, \phi, \psi \rangle \in \mathscr{R}$ and a sequence of k expressions $\gamma_0, \ldots, \gamma_{k-1} \in \mathrm{E}$ such that $\gamma_0, \ldots, \zeta_{k-1}, \gamma$ satisfy $\vee v_{k+1}\phi$ in $\mathscr{E}$; and (iii) we have a recursive method for finding, for any $\gamma \in \mathrm{E}$, all those pairs $\langle\langle r, \phi, \psi\rangle\langle\gamma_0, \ldots, \gamma_{k-1}\rangle\rangle$; then the concept of well-formedness in $\mathscr{S}$ is indeed recursive.

Condition (i) is obviously satisfied if $\mathscr{S}$ has no formation rules other than concatenation; but it is also a condition likely to be satisfied by syntaxes which also have certain other formation rules (such as e.g. substitution). That an expression could result in an infinite number of different ways from the application of a rule to a certain number of other expressions, should not be discarded automatically as absurd. For example, an expression P(1) results from substitution of 1 for any symbol x different from P, in P(x). However, here the pairs of a rule and a sequence of expressions to which the rule should be applied to yield the expression in question, though perhaps infinite in number, are very similar; if for the expression in question there exists an analysis involving one of those pairs, presumably a similar analysis will exist involving any of the other pairs. So it seems possible to modify condition (ii) in such a manner as to permit cases like the one just mentioned and still yield a recursive concept of grammaticality—provided that condition (iii) is also modified correspondingly.

Condition (iii) would probably hold in those syntaxes in which for each γ there are, as (ii) requires, only finitely many pairs. One can easily conceive of the way in which (iii) could be so adapted to more flexible alternatives of condition (ii), so that the recursiveness of the set of well-formed expressions remains guaranteed.

As a rule formal syntaxes for languages of symbolic logic are such that every well-formed expression has exactly one analysis. It is wrong however to demand this of formal syntaxes for natural languages. Indeed, syntactic ambiguity is a well-known phenomenon; and we could deprive ourselves of the means to account for this phenomenon if we insisted that each grammatical expression has a unique analysis.

In view of the possibility that a grammatical expression may have more than one analysis we will from now on talk almost exclusively about analyses even though our real interest will remain directed towards expressions. It is only the possible ambiguity of the expressions that excludes them as the immediate objects of the present technical development.

3. *Levels of analysis; translations*

The primary task of a translation (say, of one natural language into another) is to transform the principal vehicles of communication, i.e. sentences, of the first language into sentences of the second. Thus, if we were to regard only this basic function of translations we could characterise them simply as maps from sentences to sentences.

However, the way in which translations are defined and learned usually involves not only sentences, but other kinds of expressions as well. A translation from one natural language into another, for example, is normally given in the form of a dictionary (which usually pairs words with other expressions) together with certain stipulations which specify how one should render grammatical constructions of the first language within the second language. We take this to be an essential feature of translations. Therefore we will define a translation Tr (from a formal syntax $\mathcal{S}$ into a formal syntax $\mathcal{S}'$) as a pair consisting of a function Tr_0, which maps certain expressions of $\mathcal{S}$—henceforth referred to as the *elementary* expressions of Tr—onto expressions of $\mathcal{S}'$, and a function Tr_1, which maps n-place rules of grammar of $\mathcal{S}$ onto n-place rules of $\mathcal{S}'$.[7] The translation of an expression of $\mathcal{S}$ which results from applying the rule ρ of $\mathcal{S}$ to the elementary expressions $e_1, \ldots, e_n$ will then be the result of applying the rule corresponding to ρ by Tr_1 to the values of $e_1, \ldots, e_n$ under Tr_0; etc.

The elementary expressions of the translations can be of various levels of complexity: they can be words; but they also might be complex noun phrases, complex verb phrases, etc. A translation will be less 'revealing' about the relations between the respective structures of the languages which it links as its elementary expressions are more complex. Thus, from this point of view, the simpler its elementary expressions are, the better the translation is—the ideal being a translation the elementary expressions of which are the basic expressions of the syntax to which it applies. However, in practice one will often have to be content with translations, not all of the elementary expressions of which are basic.

Thus it is natural to develop a concept of translation which allows for other sets of elementary expressions than just the set of basic expressions of the

7. This is not quite correct, since it neglects the possibility that the ranges of Tr_0 and Tr_1 contain *schemata* (which will be defined later on).

syntax under consideration. But not every set of expressions is acceptable as the set of elementary expressions of a translation: if an expression e occurs in the analysis of e', then e and e' should not be elementary expressions of the same translation. In order to single out those sets which are acceptable as sets of elementary expressions, we introduce the notion of a *level of analysis*.

Definition 8: Let $\mathscr{S} = \langle \mathscr{B}, \mathscr{R} \rangle$ be a formal syntax.

(a) A *level of analysis in* $\mathscr{S}$ is a function $\mathscr{L}$ such that:

(i) the domain of $\mathscr{L}$ consists of analyses in $\mathscr{S}$;
(ii) for any analysis A in Dom $\mathscr{L}$, $\mathscr{L}$(A) is a bar of A;
(iii) if A is an analysis in $\mathscr{S}$, A′ is a subtree of A and t is a top of A′ in A, then if A is in the domain of $\mathscr{L}$ and there is a t' in $\mathscr{L}$(A) such that $t' \leqslant t$ then A′ is in the domain of $\mathscr{L}$ and $\mathscr{L}(A') = \{t' : (\exists t'' \in \mathscr{L}(A))\ t'' = t \smile t'\}$.

(b) If $\mathscr{L}$, $\mathscr{L}'$ are levels of analysis for $\mathscr{S}$ then we say that $\mathscr{L}$ is *at least as deep as* $\mathscr{L}'$ if (i) Dom $\mathscr{L}' \subseteq$ Dom $\mathscr{L}$; and (ii) if A $\in$ Dom $\mathscr{L}'$ and $t' \in \mathscr{L}'$(A) then there is a $t \in \mathscr{L}$(A) such that $t \leqslant t'$.

It is often the case that an expression of one language cannot be translated into any particular expression of some other language, but that its function can be rendered by some grammatical construction in that language. This phenomenon is sufficiently common to be accounted for in our formal characterisation of a translation. The notion of a schema, defined below, will serve this purpose. A more detailed explanation of the notion follows the definition.

Definition 9: (a) Let $\mathscr{S} = \langle \mathscr{B}, \mathscr{R} \rangle$ be a formal syntax. A k-place *schema in* $\mathscr{S}$ is a pair $\langle S, t \rangle$ such that

(i) S is a decorated tree;
(ii) t is a k-place sequence of distinct endpoints of S;
(iii) for all $t \in$ Dom S, S(t) is a pair;
(iv) if t is an endpoint of S which is not a member of t, then S(t) is a pair $\langle \gamma, \rho \rangle$ where $\gamma \in \bigcup$ Range $\mathscr{B}$ and $\rho = \langle\langle \{\alpha : \gamma \in \mathscr{B}(\alpha)\}\rangle, 0, 0\rangle$;
(v) for $i = 0, \ldots, k-1$, $[S(t_i)]_0 = 0$ and $[S(t_i)]_1$ is a triple the 0th member of which is a set of category names of $\mathscr{S}$ and the 1st and 2nd members of which are 0;
(vi) if t is a point of S, but not an endpoint, then $[S(t)]_1$ is a rule $\langle r, \phi, \psi \rangle \in \mathscr{R}$. Further, if $t_0, \ldots, t_{n-1}$ are the immediate successors

of t in S then $[S(t)]_1$ is an n-place rule; and if, for $i = 0, \ldots, n-1$, r_i' is the last component of $[[S(t_i)]_1]_0$ then $r_i \subseteq r_i'$. Finally, if for no $i < k$, $t_i < t$ then $[S(t_0)]_0, \ldots, [S(t_{n-1})]_0$, $[S(t)]_0$ satisfy $\langle r, \phi, \psi \rangle$; and if for some $i < k$, $t_i < t$ then $[S(t)]_0 = 0$.

(b) Let $\langle S, t \rangle$ be a k-place schema in $\mathscr{S}$ and let $A_0, \ldots, A_{k-1}$ be analyses of $\mathscr{S}$ such that, for $i = 0, \ldots, k-1$, if $\alpha \in [[S(t_i)]_1]_0$ then $[A_i(0)]_0 \in \zeta_{\mathscr{S}}(\alpha)$. A *result of applying* $\langle S, t \rangle$ *to* $A_0, \ldots, A_{k-1}$ (in symbols: $\langle S, t \rangle(A_0, \ldots, A_{k-1})$) is an analysis A of $\mathscr{S}$, such that

(i) Dom S $\subseteq$ Dom A;
(ii) if $t \in$ Dom S and for $i = 0, \ldots, k-1$, $t_i \neq t$ then $[A(t)]_1 = [S(t)]_1$;
(iii) for $i = 0, \ldots, k-1$, A_i is a subtree of A and t_i is a top of A_i in A;
(iv) if $t \in$ Dom S and for no $i < k$, $t_i < t$ then $A(t) = S(t)$.

It should be clear that for every k-place rule ρ of $\mathscr{S}$ there is a k-place schema S such that whenever $A_0, \ldots, A_k$ are analyses of $\mathscr{S}$ then A_k is the result of applying S to $A_0, \ldots, A_{k-1}$ iff $[A_k(0)]_0$ comes from $[A_0(0)]_0, \ldots, [A_{k-1}(0)]_0$ in the sense of definition 4(ii).

The notion of a schema, as it is defined here, should be regarded as a generalisation of the concept of a schema as it occurs in symbolic logic. There by a k-place schema one usually understands a formula with k free variables, e.g. a formula of sentential calculus with k free sentential variables $p_0, \ldots, p_{k-1}$. The result of applying the schema S to k formulae $\phi_0, \ldots, \phi_{k-1}$ is then simply the formula which we obtain if we replace in S p_0 everywhere by $\phi_0, \ldots, p_{k-1}$ everywhere by ϕ_{k-1}.

Schemata play an important role in translations. They appear indispensable when an expression γ of the language $\mathscr{L}$ from which we translate, does not correspond to any particular expression of the language $\mathscr{L}'$ into which we translate, even though every expression ϕ of $\mathscr{L}$ which is formed from γ and other expressions $\gamma_0, \ldots, \gamma_{k-1}$ does correspond, in a uniform way, to a complex expression ϕ' of $\mathscr{L}'$ formed out of the correspondents $\gamma_0', \ldots, \gamma_{k-1}'$ in $\mathscr{L}'$ of $\gamma_0, \ldots, \gamma_{k-1}$. In such a case we can usually represent the construction which gives us ϕ' from $\phi_0', \ldots, \phi_{k-1}'$ by a schema. As an example, consider the situation where $\mathscr{L}$ is a sentential calculus which contains the sentential connective $\wedge$ (representing conjunction), and $\mathscr{L}'$ is a sentential calculus, which has the same sentential constants as $\mathscr{L}$, but contains no other connectives than $\sim$ (negation) and $\rightarrow$ (material implication). A natural translation from $\mathscr{L}$ into $\mathscr{L}'$ will transform a formula $(\phi \wedge \psi)$ into, say, $\sim(\phi \rightarrow \sim\psi)$. A possible example from translations between natural languages is provided by the respective means of expressing the possessive in, say, Dutch and German, where Dutch expresses this relation with the help of the preposition 'van', and German by using the genitive. This means that 'of' will not be explicitly translated, but the (possessive) Dutch contexts in which it occurs are nonetheless systematically translatable into German equivalents. It should not be too hard to determine what schema will do this. (I speak here of a 'possible' example, as it is not excluded that the genitive ending in German should be treated as a separate lexical item. It is just conceivable that in this case we could translate Dutch 'van' into this item, and formulate the relevant recursive clauses of the translation definition in such a way that e.g. 'het boek van mijn vader' is converted into 'das Buch meines Vaters'.)

Definition 10: let $\mathcal{S} = \langle \mathcal{B}, \mathcal{R} \rangle$ and $\mathcal{S}' = \langle \mathcal{B}', \mathcal{R}' \rangle$ be formal syntaxes and let $\mathcal{L}$ be a level of analysis in $\mathcal{S}$. A *translation from* $\mathcal{S}$ *into* $\mathcal{S}'$ *down to* $\mathcal{L}$ is a pair $\mathrm{Tr} = \langle \mathrm{Tr}_0, \mathrm{Tr}_1 \rangle$ of functions Tr_0 and Tr_1 such that:

(i) the domain of Tr_0 consists of all those analyses A in $\mathcal{S}$ such that $\mathcal{L}(\mathrm{A}) = \{0\}$;

(ii) for each A in the domain of Tr_0, $\mathrm{Tr}_0(\mathrm{A})$ is either an analysis in $\mathcal{S}'$, or else a schema in $\mathcal{S}'$;

(iii) the domain of Tr_1 is $\mathcal{R}$;

(iv) if ρ is a k-place rule in $\mathcal{R}$ then $\mathrm{Tr}_1(\rho)$ is a k-place schema in $\mathcal{S}'$.

Definition 11: let $\mathcal{S}$, $\mathcal{S}'$, $\mathcal{L}$ be as above and let $\mathrm{Tr} = \langle \mathrm{Tr}_0, \mathrm{Tr}_1 \rangle$ be a translation from $\mathcal{S}$ into $\mathcal{S}'$ down to $\mathcal{L}$. We define a function Tr^*, whose domain consists of some (though not necessarily all) analyses A in the domain of $\mathcal{L}$, as follows, by recursion:

(i) if $\mathcal{L}(\mathrm{A}) = \{0\}$ then $\mathrm{Tr}^*(\mathrm{A}) = \mathrm{Tr}_0(\mathrm{A})$;

(ii) if $\mathrm{A} \in \mathrm{Dom}\,\mathcal{L}$, $\langle 0 \rangle, \ldots, \langle k-1 \rangle$ are all the one-place sequences in Dom A and for $i = 0, \ldots, k-1$, there are analyses A_i such that (i) $\langle i \rangle$ is a top of A_i in A, and (ii) $\mathrm{Tr}^*(\mathrm{A}_i)$ is defined, then

(a) if for all $i < k$ $\mathrm{Tr}^*(\mathrm{A}_i)$ is an analysis in $\mathcal{S}'$ and $[\mathrm{A}(0)]_1 = \rho$ then $\mathrm{Tr}^*(\mathrm{A}) = \mathrm{Tr}_1(\rho)(\mathrm{Tr}^*(\mathrm{A}_0), \ldots, \mathrm{Tr}^*(\mathrm{A}_{k-1}))$;

(b) if there is a $j < k$ such that $\mathrm{Tr}^*(\mathrm{A}_j)$ is a $k-1$-place schema of $\mathcal{S}'$ and for $i < k$, $i \neq j$, $\mathrm{Tr}^*(\mathrm{A}_i)$ is an analysis in $\mathcal{S}'$ then $\mathrm{Tr}^*(\mathrm{A}) = \mathrm{Tr}^*(\mathrm{A}_j)(\mathrm{Tr}^*(\mathrm{A}_0), \ldots, \mathrm{Tr}^*(A_{j-1}), \mathrm{Tr}^*(\mathrm{A}_{j+1}), \ldots, \mathrm{Tr}^*(\mathrm{A}_{k-1}))$.

It is clear that in general, Tr^* will *not* be defined for *all* analyses Dom $\mathcal{L}$. We say that Tr is *adequate* if the domain of Tr^* includes all those analyses in Dom $\mathcal{L}$ such that $[\mathrm{A}(0)]_0 \in \zeta_{\mathcal{S}}(0)$. Thus we are willing to call the translation adequate as long as all *formulae* of $\mathcal{S}$ (which do not fall 'below' our level of analysis $\mathcal{L}$) are translated by Tr. This criterion is to a certain extent arbitrary and may well be strengthened so as to include the requirement that other categories in $\mathcal{S}$ also be fully translated. Our actual choice of the criterion above reflects our viewpoint that the basic goal of a translation is the proper transformation of sentences.

Let $\mathcal{S}$ be a formal syntax. Then for any other formal syntax $\mathcal{S}'$ we can ask the question if $\mathcal{S}'$ is adequate for translation from $\mathcal{S}$ into it. This question seems, in this absolute form, rather meaningless. But we have just seen that, relative to a level of analysis $\mathcal{L}$ for $\mathcal{S}$, the question does make sense: $\mathcal{S}'$ can be regarded as adequate for translation from $\mathcal{S}$, relative to $\mathcal{L}$, if there is a translation Tr from $\mathcal{S}$ into $\mathcal{S}'$, down to $\mathcal{L}$, such that the domain of Tr* includes every analysis $A \in \mathrm{Dom}\,\mathcal{L}$ such that $[A(0)]_0 \in \zeta_{\mathcal{S}}(0)$. For given $\mathcal{S}$ and $\mathcal{S}'$ the answer is less likely to be positive as the level of analysis $\mathcal{L}$ is deeper. Indeed, if $\mathcal{L}$ is at least as deep as $\mathcal{L}'$ and there is an adequate translation from $\mathcal{S}$ into $\mathcal{S}'$ down to $\mathcal{L}$, then there is also an adequate translation from $\mathcal{S}$ into $\mathcal{S}'$ down to $\mathcal{L}'$.

The concept of a translation could be strengthened in various ways, even if we stay within the present, purely syntactic framework. A natural requirement would be, e.g., that the functions Tr_0 and Tr_1 be recursive. However, we will not further pursue the question whether the notion of a translation should or could be strengthened in such ways. A truly meaningful discussion of translations is possible only if the meanings of the translated expressions and of their translations are taken into account: a translation should preserve the meaning of the expressions which it translates.

4. *Types*

We will characterise the meanings of well-formed expressions of any formal syntax $\mathcal{S}$ in terms of intensional models for that syntax. Intensional models should be regarded as indexed 'collections' of possible worlds. (We hesitate to use the word 'collection', since cross-reference from one such possible world to others usually occurs in intensional models.) A well-formed expression will denote at each index of an intensional model an entity of the appropriate kind. By the *intension* which an intensional model assigns to a well-formed expression γ we understand the function which, for each index of the model, gives the entity denoted by γ at that index.

We said that a well-formed expression γ of a formal syntax $\mathcal{S}$ should denote at each index of an intensional model for $\mathcal{S}$ an object 'of the appropriate kind'. What we mean by this might be best elucidated by means of an example. Suppose that $\mathcal{S} = \langle \mathcal{B}, \mathcal{R} \rangle$ is a formal syntax for English, according to which proper nouns and common nouns are well-formed expressions. It is natural that any proper noun should denote in each intensional model for $\mathcal{S}$ at each index an individual of that intensional model; similarly a common noun should always denote a class of individuals. Thus the objects denoted by a proper and a common noun, respectively, are of different sorts: individuals as against classes of individuals. The question what *sort* of entity an expression denotes should be distinguished from the question *which* entities are denoted by the expression at particular indices in particular models.

It is somewhat problematic whether this question should be considered as belonging to semantics or to syntax. It is syntactic insofar as the type of object that an expression denotes ought to depend—it seems—only on the syntactic categories to which the expression belongs. Thus the question what sorts of objects are the denotata of well-formed expressions seems to be on a level which is intermediate between syntax and semantics. We will therefore treat it separately from, and prior to, our formal development of semantics itself.

Definition 12: for any natural number $n > 0$ the *types of n-sorted* logic are defined recursively as follows:

(i) For $i = 3, \ldots, n + 2$, $\langle i \rangle$ is a *type*;

(ii) if m is any natural number > 0 and $\tau_0, \ldots, \tau_{m-1}, \tau_m$ are types then $\langle 1 \rangle \smile \tau_0, \smile \ldots \smile \tau_{m-1} \smile \langle 0 \rangle$ and $\langle 2 \rangle \smile \tau_0 \smile \ldots \smile \tau_{m-1} \smile \tau_m \smile \langle 0 \rangle$ are *types*.

Definition 13: let S be a n-place sequence of sets.

(a) For any type τ of n-sorted logic the *realisation of* τ *in S* (in symbols: $\mathrm{Res}(\tau)$) is defined recursively by:

(i) for $i = 3, \ldots, n + 2$, $\mathrm{Res}(\langle i \rangle) = S_{i-3}$;

(ii) (a) if $\tau = \langle 1 \rangle \smile \tau_0 \smile \ldots \smile \tau_{m-1} \smile \langle 0 \rangle$, $\mathrm{Res}(\tau) = \mathscr{P}(\mathrm{Res}(\tau_0) \otimes \ldots \otimes \mathrm{Res}(\tau_{m-1}))$;[8]

(b) if $\tau = \langle 2 \rangle \smile \tau_0 \smile \ldots \smile \tau_{m-1} \smile \tau_m \smile \langle 0 \rangle$ then $\mathrm{Res}(\tau) = \mathrm{Res}(\tau_m)^{\mathrm{Res}(\tau_0) \otimes \ldots \otimes \mathrm{Res}(\tau_{m-1})}$.

(b) C is a *category connected with S* iff C is the realisation of some type of n-sorted logic in S.

8. If x, y are sets, then we understand by $x \otimes y$ the cartesian product of x; by x^y the set of all functions with domain y and range included in x; and by $\mathscr{P}(x)$ the power set of x.

The intensional models in terms of which we will characterise the semantics for formal syntaxes, will be *multi-valued*, i.e. we will not limit ourselves to models which are based upon two truth values, True and False, but in principle admit any non-empty set of truth values. Thus the basic constituents from which a model is built up are: (i) a set of indices (I); (ii) a set of individuals (D); and (iii) a set of truth values (V). The sorts of entities in the model are completely determined by these three constituents. Among them are—the set of individuals (D): the proper kind of denotata for proper names; the set of functions from I into D: the proper kind of intensions for proper nouns; the set of functions from I into $\mathscr{P}$(D): the proper kind of intensions for common nouns; the set of functions from I into V: the proper kind of intensions for sentences; etc. In general we will identify these sorts of entities of the model with the realisations of types in $\langle I, V, D\rangle$. Thus the set of possible models for a given formal syntax $\mathscr{S}$ is limited by the types which are associated with the well-formed expressions of $\mathscr{S}$. Each expression ought to have as its intension, in a model built from I, V, D, an entity in the realisation in $\langle I, V, D\rangle$ of the type of the expression. Thus the set of possible models for a formal syntax $\mathscr{S}$ is relative to a function which tells us for each of the well-formed expressions of $\mathscr{S}$ the type of that expression.[9] In view of the principle that the type of an expression ought to be determined entirely by the syntactic categories to which the expression belongs, such functions can be characterised as follows:

Definition 14: let $\mathscr{S}$ be a formal syntax.

(a) A *type function for* $\mathscr{S}$ is a function $\mathscr{T}$ such that:

(i) the domain of $\mathscr{T}$ consists of all non-empty sets S of category names of $\mathscr{S}$ for which $\bigcap_{\alpha \in S} \zeta_{\mathscr{S}}(\alpha) \neq \phi$;

(ii) the range of $\mathscr{T}$ consists of types of 3-sorted logic;

(iii) if Γ is a set of category names and there is a subset Γ' of Γ and a well-formed expression γ of $\mathscr{S}$ such that for any category name α of $\mathscr{S}$, $\alpha \in \Gamma'$ iff $\gamma \in \zeta_{\mathscr{S}}(\alpha)$, then $\mathscr{T}(\Gamma) = \mathscr{T}(\Gamma')$;[10]

9. In familiar symbolic languages, such as first and higher order predicate logic, or the theory of types as formulated by Church, the syntactic categories are themselves the types, so that no further specification of this function is necessary.

10. The significance of condition (iii) is this. I want to assume that the type of an expression is completely determined by the categories to which it belongs. The precise intention of this assumption is that whenever $\alpha_1, \ldots, \alpha_n$ are all the categories to which a certain expression γ belongs, and γ has type τ, then any other expression belonging to $\alpha_1, \ldots, \alpha_n$ must also be of type τ, irrespective of whether it belongs to yet some other categories. Thus any superset of $\{\alpha_1, \ldots, \alpha_n\}$ which is in the domain of τ must get the same value that $\{\alpha_1, \ldots, \alpha_n\}$ itself receives.

(iv) if $0 \in \Gamma$ then $\mathscr{T}(\Gamma) = \langle 2 \rangle \smile \langle 3 \rangle \smile \langle 4 \rangle \smile \langle 0 \rangle$.[11]

(b) Let $\mathscr{T}$ be a type function for $\mathscr{S}$. For any well-formed expression γ of $\mathscr{S}$ we understand by $\mathscr{T}(\gamma)$ the type τ such that if Γ is the set of α such that $\gamma \in \zeta_{\mathscr{S}}(\alpha)$ then $\tau = \mathscr{T}(\Gamma)$. If A is an analysis of $\mathscr{S}$, then by $\mathscr{T}(A)$ we understand $\mathscr{T}([A(0)]_0)$.

Type functions for formal syntaxes provide us with a further natural criterion for adequacy of translations: the type of the translation of an expression should be the same as the type of the expression itself. This seems perfectly plausible for certain types, such as the type of sentences or the type of singular terms. Whether the principle should hold for *all* types of expressions to which the translation function applies is perhaps not quite so obvious. However, the semantic criteria for adequacy discussed below will explain why it is indeed natural to demand that translations preserve type without exception.

Thus we arrive at the following definition of adequacy.

Definition 15: let $\mathscr{S}$, $\mathscr{S}'$ be formal syntaxes; let $\mathscr{T}$, $\mathscr{T}'$ be type functions for $\mathscr{S}$, $\mathscr{S}'$ respectively; and let $\mathscr{L}$ be a level of analysis for $\mathscr{S}$. A translation Tr of $\mathscr{S}$ into $\mathscr{S}'$ down to $\mathscr{L}$ is *adequate relative to* $\mathscr{T}$ and $\mathscr{T}'$ iff:

(i) Tr is adequate; and
(ii) for each analysis $A \in \text{Dom}\, \mathscr{L}$, $\mathscr{T}'(\text{Tr}^*(A)) = \mathscr{T}(A)$.

The principle that the type of an expression should be uniquely determined by the syntactic categories to which it belongs is always observed in the syntaxes and semantics of languages of formal logic. Moreover, presently existing grammars for natural languages are for the most part in agreement with it. For example, English expressions which, according to traditional English grammar, belong to the same category (categories) do as a rule denote objects of the same type. There are, however, exceptions to this rule. An example is the traditional category of adverbs. Certain adverbs (e.g. 'utterly') can be used both as modifiers of verbs and of adjectives, and as modifiers of other adverbs.

It seems to me that grammars whose categories do not always determine the type are unsatisfactory and should be replaced by more refined systems in which the type of an analysis only depends on the categories of its top.

11. The last condition warrants that the realisation of the type of a formula of $\mathscr{S}$ in $\langle I, V, D \rangle$ will always be a function from I into V.

5. *Interpretations*

Our last characterisation of adequacy for translations is still not satisfactory. A translation should preserve the meaning of the expressions it translates, not just the semantic *types*. In order to do justice to this stronger requirement we will now turn to the interpretations, or *models*, themselves. As we said above, our models are built from three basic sets: the set of indices, the set of truth values, and the set of individuals. In addition to these basic sets a model will include a function which assigns to each 'basic' expression an entity of the appropriate kind—the intension of the expression—and a function which assigns to each rule a function which will yield the intension of an expression formed by means of that rule when applied to the intensions of the component expressions.

We have spoken in the previous paragraph about 'basic' expressions. We do not want to restrict ourselves to the case where these 'basic' expressions are simply the members of the basic categories of the syntax $\mathscr{S}$ in question. We also want to consider cases where the 'basic' expressions—or, rather, their analyses—are those which are basic relative to some level of analysis for $\mathscr{S}$. The reason for this is the following. We want to say that a translation Tr from $\mathscr{S}$ into $\mathscr{S}'$, down to level $\mathscr{L}$, is adequate if it preserves the meaning of all the analyses translated. But only those analyses are translated which belong to the domain of $\mathscr{L}$. Thus we are interested only in *their* meanings; and it seems unnatural to demand in these circumstances that the meanings of analyses in Dom $\mathscr{L}$ be analysed further in terms of meanings of subtrees of these analyses which do not belong to Dom $\mathscr{L}$. Indeed, it may well be the case that we do not have such a semantical analysis at all, even though we do have a satisfactory semantical analysis of the meaning relations between the analyses which belong to Dom $\mathscr{L}$. On the other hand, the models for $\mathscr{L}$ which assign meaning only to those analyses which belong to Dom $\mathscr{L}$ should include those which we obtain when we 'restrict to Dom $\mathscr{L}$' any model for $\mathscr{S}$ relative to some level $\mathscr{L}'$ which is deeper than $\mathscr{L}$; i.e. for any model for $\mathscr{S}$ which assigns meanings to all analyses in Dom $\mathscr{L}'$ there should be a model which assigns meanings only to the analyses in Dom $\mathscr{L}$, and coincides with the former on all of Dom $\mathscr{L}$. This requirement causes a certain difficulty in connection with variables.

In general, some of the basic expressions of a language will function as variables. This is true not only of formal languages, but also of natural languages, where in particular the personal pronouns play such a role (in some, though not in all cases). Such expressions will not denote particular objects, but 'range' over classes of such objects. Expressions which contain variables will generally not denote particular entities either, for their denotations may vary with the denotations of the variables they contain. On the other hand, such complex expressions, unlike the variables themselves, will generally not range over the class of *all* entities of the appropriate type. For example, the phrase 'house owned by x' will generally denote different sets of individuals, according as x denotes different persons, but these sets will always be sets of houses. Thus among the models relative to a level of analysis $\mathscr{L}$ on which 'house owned by x' is a basic expression, there should be at least some in which this expression ranges over a subset of the appropriate category which is neither a singleton nor the whole category.

The question remains if we have any means to determine for any of the analyses which are basic in $\mathscr{L}$, whether it is a constant (i.e. ought to denote always a particular object), a pure variable (i.e. ought to range over the whole set of appropriate entities) or an expression which could range over other sets of entities of the appropriate sort. If we have no knowledge at all of that part of the syntactic analysis of an expression which lies 'below' $\mathscr{L}$, then we can distinguish among the expressions whose analyses are not below $\mathscr{L}$, only: (i) pure variables, (ii) basic expressions of the syntax which are not variables, and (iii) compound expressions; of the latter we would never know whether they 'contain free variables' or not, and so any subset of entities of the appropriate sort should be regarded as a possible range for such an expression. However, the natural situation in the present context seems to be one where we have, on the one hand, a complete syntactic analysis but, on the other hand, only a partial semantical theory, i.e. a semantic analysis which does not go down to the lowest level of our syntax. (This is—approximately—the present state of linguistic description of English.) In this situation we can, since we know the complete syntactic analysis of each expression, include among the constants at least those expressions the analyses of which contain no variables at all. If we could in addition recognise whether a variable occurs *free* in the analysis then we could include among the analyses which are to denote particular objects also those which, though containing variables, do not contain free occurrences of variables. Since, however, we have not discussed any syntactic characterisations of freedom and bondage, we will not go into this matter any further.

Thus far we have spoken of denotations as well as of intensions. In the technical development, however, it will be expedient to pay exclusive attention to intensions. In view of our stipulations about the connection between intensions and denotations it is clear that where denotations exist, the intension can always be retrieved from them. Thus if every expression did indeed have denotations (at all indices in each model) as well as an intension we could, in principle base the semantic account entirely on denotations, and then, if we so desired, introduce intensions by explicit definition. It is not clear however that every expression has denotation. The intension of a sentential connective, for example, seems to be a function from propositions (i.e. functions from indices to truth values) to propositions. But what is the denotation of a sentential connective at a particular index? We could stipulate that at each index the denotation of the connective is what we have just called its intension. But that procedure would seem to be rather artificial. And I do not know of any compensating advantages that might justify us in adopting it in spite of this artificiality.

Thus intensions will be our central semantic entities; and type functions for formal syntaxes should be understood as giving the type of the *intension* of an expression, and not of its denotations (clause (iv) of definition 14 should be understood in this perspective).

It will turn out that the objects that models assign to analyses are not themselves intensions, but functions from assignments to intensions. This unpleasant complication arises because, on the one hand, the intensions of expressions that 'contain free variables' will vary with the assignments and, on the other hand, the intension of a compound expression under a given assignment will depend not only on the intensions of the components under that particular assignment, but also on the intentions of the components under other assignments. (This is, in general, the case if one of the component expressions is a variable binding operator—as are, for example, the quantifiers in predicate logic.)

Definition 16: let $\mathscr{S} = \langle \mathscr{B}, \mathscr{R} \rangle$ be a formal syntax. Let $\mathscr{L}$ be a level of analysis for $\mathscr{S}$ and $\mathscr{T}$ a type function for $\mathscr{S}$. Let $\mathscr{Y} = \langle \mathrm{I}, \mathrm{V}, \mathrm{D} \rangle$ be a triple of non-empty sets.

(a) An *assignment range for* $\mathscr{S}$, $\mathscr{T}$ *in* $\mathscr{Y}$, *down to* $\mathscr{L}$, is a function U such that:

- (i) the domain of U consists of the analyses A of $\mathscr{S}$ such that $\mathscr{L}(\mathrm{A}) = \{0\}$;
- (ii) if $\mathrm{A} \in \mathrm{Dom}\ \mathrm{U}$ and $[\mathrm{A}(0)]_0 \in \zeta_{\mathscr{S}}(1)$ then $\mathrm{U}(\mathrm{A}) = \mathrm{Re}_{\mathscr{Y}}(\mathscr{T}(\mathrm{A}))$;
- (iii) if $\mathrm{A} \in \mathrm{Dom}\ \mathrm{U}$ and for all $t \in \mathrm{Dom}\ \mathrm{A}$, $[\mathrm{A}(t)]_0 \notin \zeta_{\mathscr{S}}(1)$ then there is an $x \in \mathrm{Re}_{\mathscr{Y}}(\mathscr{T}(\mathrm{A}))$ such that $\mathrm{U}(\mathrm{A}) = \{x\}$;
- (iv) if $\mathrm{A} \in \mathrm{Dom}\ \mathrm{U}$, $[\mathrm{A}(0)]_0 \notin \zeta_{\mathscr{S}}(1)$, but for some $t \in \mathrm{Dom}\ \mathrm{A}$, $[\mathrm{A}(t)]_0 \in \zeta_{\mathscr{S}}(1)$ then $\mathrm{U}(\mathrm{A}) \subseteq \mathrm{Re}_{\mathscr{Y}}(\mathscr{T}(\mathrm{A}))$.

(b) Let U be an assignment-range for $\mathscr{S}$, $\mathscr{T}$ in $\mathscr{Y}$, down to $\mathscr{L}$. The *set of assignments in* U (in symbols: As(U)) is the set of all functions F such that:

(i) Dom F = Dom U; and

(ii) for all A $\in$ Dom F, F(A) $\in$ U(A).

(c) A *model for* $\mathscr{S}$, $\mathscr{T}$, *down to* $\mathscr{L}$, *based upon* $\mathscr{Y}$ is a quadruple $\langle I', V', U, G\rangle$, where

(i) $I' \subseteq I$ and $V' \subseteq V$;

(ii) U is an assignment range for $\mathscr{S}$, $\mathscr{T}$ in $\mathscr{Y}$, down to $\mathscr{L}$;

(iii) G is a function the domain of which is $\mathscr{R}$;

(iv) for each *k*-place rule $\rho = \langle r, \phi, \psi\rangle$ in $\mathscr{R}$, $G(\rho)$ belongs to $(\mathrm{Re}_{\mathscr{Y}}(\mathscr{T}(r_k))^{As(U)})^{\mathrm{Re}_{\mathscr{Y}}(\mathscr{T}(r_0))^{As(U)} \otimes \ldots \otimes \mathrm{Re}_{\mathscr{Y}}(\mathscr{T}(r_{k-1}))^{As(U)}}$.

(d) Let $\mathfrak{A} = \langle I', V', U, G\rangle$ be a model for $\mathscr{S}$, $\mathscr{T}$, down to $\mathscr{L}$, based upon $\mathscr{Y}$.

(i) For any analysis A in Dom $\mathscr{L}$, the *value of* A *in* $\mathfrak{A}$ (in symbols: $\mathfrak{A}^*(A)$) is defined as follows:

(a) if A $\in$ Dom U, then $\mathfrak{A}^*(A) = \{\langle F, F(A)\rangle : F \in As(U)\}$;

(b) if $\mathfrak{A} \in$ Dom $\mathscr{L}$ − Dom U, $[A(0)]_1 = \rho$, ρ is *k*-place, and for $i = 0, \ldots, k-1$, $\langle i\rangle$ is a top of A_i in A, then $\mathfrak{A}^*(A) = G(\rho)(\mathfrak{A}^*(A_0), \ldots, \mathfrak{A}^*(A_{k-1}))$.

(ii) For any analysis A $\in$ Dom $\mathscr{L}$ we understand by the *assignment range* of A *in* $\mathfrak{A}$ (in symbols: $U^*_{\mathfrak{A}}(A)$) the range of $\mathfrak{A}^*(A)$.

(iii) Let A be an analysis in Dom $\mathscr{L}$ and let $[A(0)]_0$ be a formula of $\mathscr{S}$. We say that A *holds in* $\mathfrak{A}$ iff for all F $\in$ U and for all $i \in I'$, $\mathfrak{A}^*(A)(F)(i) \in V'$.

As is apparent from the previous definition V′ should be regarded as the set of truth values which each correspond to some form of truth in the theory of many valued logics; the members of V′ are usually referred to as the 'designated truth values'. It is important to allow for the possibility that the set I′ be a *proper* subset of I. The need for this arises for example within certain accounts of necessity and possibility, where we want to say that A holds in a given model $\mathfrak{A}$ if what (the sentence) A (represents) is true at some one particular index of $\mathfrak{A}$ which represents the actual world. The remaining indices are nonetheless required for the recursive definition of intension.

Definition 17: (a) A *semantics for* $\mathscr{S}$, $\mathscr{T}$, *down to* $\mathscr{L}$, is a class of models for $\mathscr{S}$, $\mathscr{T}$, down to $\mathscr{L}$.

(b) Let $\mathscr{C}$ be a semantics for $\mathscr{S}$, $\mathscr{T}$, down to $\mathscr{L}$. Let A be an analysis in Dom $\mathscr{L}$ and suppose that $[A(0)]_0 \in \zeta_{\mathscr{S}}(0)$. We say that A is *analytic in* $\mathscr{C}$ iff whenever $\mathfrak{A} = \langle I, V, U, I', V', U, G\rangle \in \mathscr{C}$, $F \in As(U)$, and $i \in I'$, then $\mathfrak{A}^*(A)(F)(i) \in V'$.

Models, as defined here, are both intensional and multi-valued. If we were only interested in the characterisation of sets of analytic formulae, a simpler definition would have been sufficient. Indeed, in that case it would have been sufficient to consider either 'two-valued' intensional models (i.e. models in which $V = \{0, 1\}$ and $V' = \{1\}$) or else multi-valued 'extensional' models (i.e. models in which I is a singleton and $I' = 1$). To be precise, if $\mathscr{C}$ is any semantics for $\mathscr{S}$, $\mathscr{T}$, down to $\mathscr{L}$, and Γ is the set of analyses in Dom $\mathscr{L}$ which are analytic in $\mathscr{C}$, then there is a semantics $\mathscr{C}'$ for $\mathscr{S}$, $\mathscr{T}$, down to $\mathscr{L}$, consisting of two-valued models only, such that Γ is the set of analyses in Dom $\mathscr{L}$ which are analytic in $\mathscr{C}'$. Similarly there is a semantics $\mathscr{C}'$ consisting of extensional models only, such that Γ is the set of analyses in Dom $\mathscr{L}$ which are analytic in $\mathscr{C}''$. I have nonetheless characterised models in the general way of Definition 16, since I believe that the construction of intuitively natural semantics for natural languages may well require both a variety of possible worlds, and a wide range of truth values. (Indeed, it appears now that vagueness—a phenomenon so pervasive in, and, it seems, so essential to, languages which serve the needs of normal communication—can best be treated in a model theory employing a large truth-value space which, however, has the structure of a Boolean algebra, rather than that of a linear ordering: see e.g. Fine, 1975; Kamp, 1975.)

Our definition of a model says hardly anything about the semantic interpretation of the various rules of the syntax $\mathscr{S}$. Indeed, our conditions only warrant that the semantic entity assigned to a compound expression will be of the right type. However, it seems natural to impose further restrictions on the semantic operations that correspond to the rules of grammar. For example, it is plausible that, like the formation rules themselves, these operations should be recursive. Another natural restriction would be given by the condition that whenever $\langle I, V, D, I', V', U, G\rangle$ and $\langle I, V, D, I', V', U, G'\rangle$ are models for $\mathscr{S}$, $\mathscr{T}$, down to $\mathscr{L}$, then $G' = G$. A stronger limitation than this last one would result if we were to characterise the interpretations of the grammar rules as formulae of some appropriate language which define operations of the appropriate sort. The intension of an expression γ in a model $\mathfrak{A}$, based upon $\langle I, V, D\rangle$, where γ results from applying the rule ρ to $\gamma_0, \ldots, \gamma_{k-1}$, would then be the result of applying to the intensions in $\mathfrak{A}$ of $\gamma_0, \ldots, \gamma_{k-1}$ respectively the operation defined in $\langle I, V, D, As(U)\rangle$ by the formula which interprets ρ. Furthermore, by admitting only formulae of certain forms as interpretations of rules of grammar we could guarantee recursiveness.[12]

12. Montague's general recursion theory, would be suited to this purpose.

I realise that many questions in this area are left unanswered, but will not go into this matter any further.

Definition 18: Let $\mathcal{S}$, $\mathcal{T}$, $\mathcal{L}$ be as above and let $\mathcal{L}'$ be a level of analysis for $\mathcal{S}$ such that $\mathcal{L}$ is at least as deep as $\mathcal{L}'$.

(a) Let $\mathfrak{A} = \langle I, V, D, I', V', U, G \rangle$ be a model for $\mathcal{S}$, $\mathcal{T}$, down to $\mathcal{L}'$. The *restriction of* $\mathfrak{A}$ *to* $\mathcal{L}'$ is the septuple $\langle I, V, D, I', V', U', G \rangle$ where U′ characterised as follows:

(i) The domain of U′ consists of all those analyses A of $\mathcal{S}$ such that $\mathcal{L}(A) = \{0\}$;
(ii) if $A \in \text{Dom } U'$ and $A \in \text{Dom } U$ then $U'(A) = U(A)$;
(iii) if $A \in \text{Dom } U'$ and $A \notin \text{Dom } U$ then $U'(A) = U^*_{\mathfrak{A}}(A)$.

(b) Let $\mathscr{C}$ be a semantics for $\mathcal{S}$, $\mathcal{T}$, down to $\mathcal{L}$. The *restriction of* $\mathscr{C}$ *to* $\mathcal{L}'$ is the class of restrictions to $\mathcal{L}'$ of members of $\mathscr{C}$.

It should be clear that if $\mathfrak{A}'$ is the restriction to $\mathcal{L}'$ of a model $\mathfrak{A}$ for $\mathcal{S}$, $\mathcal{T}$, down to $\mathcal{L}$, then $\mathfrak{A}'$ is a model for $\mathcal{S}$, $\mathcal{T}$, down to $\mathcal{L}'$; that for every assignment F in the assignment range of $\mathfrak{A}$ there is an assignment F′ in the assignment range of $\mathfrak{A}'$ such that for all $A \in \text{Dom } \mathcal{L}'$, $\mathfrak{A}'^*(A)(F') = \mathfrak{A}^*(A)(F)$; and that the restriction to $\mathcal{L}'$ of a semantics for $\mathcal{S}$, $\mathcal{T}$, down to $\mathcal{L}$ is a semantics for $\mathcal{S}$, $\mathcal{T}$, down to $\mathcal{L}$.

The concept of a semantics for a formal syntax enables us to formulate various new adequacy criteria for translations. I find it difficult to decide, within the present general framework, which of these criteria should be preferred and will therefore present the various possibilities that have occurred to me. These semantic adequacy criteria will all be variations of the principle, from which we started, that translations should preserve the meaning of the expressions translated. This principle can be stated only if with the expressions translated as well as with their translations there is associated some kind of meaning. This will indeed be the case if, for the syntax $\mathcal{S}$ from which we translate and for the syntax $\mathcal{S}'$ into which we translate, there are models $\mathfrak{A}$ and $\mathfrak{A}'$, respectively, down to sufficiently deep levels of analysis for $\mathcal{S}$ and $\mathcal{S}'$. If $\mathfrak{A}$ and $\mathfrak{A}'$ are based on the same sets, and if their sets of 'true' truth values and relevant indices coincide, the principle that translations ought to preserve meaning can be formulated without too much difficulty. If the models do not correspond in this way, however, the meanings of a translated expression and its translation can in general no longer be easily compared. Even in such situations certain, more complicated, formulations of the principle could be given; but we will not pursue this. We will consider the principle only in connection with models which correspond in the manner just explained.

I have said already it is not the task of linguistic theory to specify the intended models uniquely. We must deal with classes of models. Let us suppose that $\mathscr{S}$, $\mathscr{S}'$ are formal syntaxes, that $\mathscr{T}$, $\mathscr{T}'$ are type functions for $\mathscr{S}$, $\mathscr{S}'$, that $\mathscr{C}$ is a semantics for $\mathscr{S}$, $\mathscr{T}$, and that $\mathscr{C}'$ is a semantics for $\mathscr{S}'$, $\mathscr{T}'$. In what sense can a translation from $\mathscr{S}$ into $\mathscr{S}'$ preserve the intensions that the expressions to which the translation applies are being given by the various members of $\mathscr{C}$? There is perhaps no unique answer to this question. But one possible answer is the following: for each model $\mathscr{A}$ in $\mathscr{C}$ there ought to be a model $\mathscr{A}'$ in $\mathscr{C}$ such that any analysis A to which the translation applies has the same intension in $\mathscr{A}$ as its translation has in $\mathscr{A}'$. Consistent with the limitations which we set ourselves above, we will require that $\mathscr{A}$ and $\mathscr{A}'$ are based upon the same sets and have the same sets of true designated truth values and relevant indices.

Thus the adequacy of a translation will be relative to a function from $\mathscr{C}$ into $\mathscr{C}'$ which gives us for each $\mathscr{A}$ in $\mathscr{C}$ a thus corresponding $\mathscr{A}'$ in $\mathscr{C}'$. (In case $\mathscr{C}'$ is a semantics for a natural language, one might perhaps require that the function be *onto* $\mathscr{C}'$, but in the case where $\mathscr{C}'$ is a semantics for a formal language, this requirement would be counter-intuitive. To see this it suffices to think of the translations from English into first-order logic which, under the name 'symbolisations', play an important part in any introductory course of formal logic. The 'schemes of abbreviations' (see e.g. Kalish and Montague, 1964) on which such translations are usually based, can be regarded at least in part as devices to determine which of the possible models for first-order logic correspond to models for English, and thus which models for first-order logic should be disregarded in this connection.

A translation, as defined in section 3, is always from a syntax $\mathscr{S}$ into a syntax $\mathscr{S}'$, *relative to a level of analysis* $\mathscr{L}$ for $\mathscr{S}$. Only analyses in Dom $\mathscr{L}$ are translated, and thus a semantics $\mathscr{C}$ for $\mathscr{S}$ *down to* $\mathscr{L}$ will suffice for a proper formulation of the new adequacy criteria for such a translation. But what kinds of semantics for $\mathscr{S}'$ does such a formulation require?

We could demand that the semantics for $\mathscr{S}'$ always consist only of models which assign meanings to all analyses in $\mathscr{S}'$. Indeed, if $\mathscr{S}'$ is a syntax for a formal language, this condition normally will be satisfied. But if $\mathscr{S}'$ is a syntax for a natural language, this requirement seems unduly severe. On the other hand, it is clear that certain semantics $\mathscr{C}'$ for $\mathscr{S}'$ will not yield complete adequacy criteria for a given translation from $\mathscr{S}$ into $\mathscr{S}'$, simply because the models in $\mathscr{C}'$ may well fail to assign intensions to some of the translated analyses. We could in such cases weaken the principle and demand only that *whenever* a model in $\mathscr{C}'$ assigns an intension to the translation of an analysis A in $\mathscr{C}$, this intention must be the same as the intention assigned to A by the corresponding model(s) in $\mathscr{C}$. We will, however, not consider this possibility and discuss only the situation where the models in $\mathscr{C}'$ do indeed give intensions to all analyses in the range of the translation function.

Let us return for a moment to the case in which $\mathscr{C}$ and $\mathscr{C}'$ are singletons. Let $\mathscr{C} = \{\mathfrak{A}\}$ and $\mathscr{C}' = \{\mathfrak{A}'\}$, and let us assume that $\mathfrak{A}$ and $\mathfrak{A}'$ are based upon the same sets and that their sets of relevant indices and true truth values coincide. Let further Tr be a translation from $\mathscr{S}$ into $\mathscr{S}'$, down to $\mathscr{L}$, and let $\mathfrak{A}'$ assign intensions to all members of Range Tr*. Tr is adequate, we have said, if 'the intension assigned by $\mathfrak{A}$ to any analysis A in Dom Tr* is the same as the intension that $\mathfrak{A}'$ assigns to Tr*(A). This statement, however, is in need of explanation, as a model assigns to an analysis not simply an intension but rather a function from assignments to intensions. As a matter of fact, for many an analysis this function is constant. This will be the case whenever the analysis in question is of an expression which does not 'contain any free variables'. One may, in many cases, be interested only in expressions of this sort, and thus prepared to regard the translation as adequate as long as it preserves *their* intensions. This condition will be satisfied in particular if the translation preserves the assignment ranges of *all* analyses it translates.

We will adopt a condition which is even stronger and demand that to each assignment F(F′) in the assignment range of $\mathfrak{A}(\mathfrak{A}')$ should correspond to an assignment F′(F) in the assignment range of $\mathfrak{A}'(\mathfrak{A})$ such that for all $A \in$ Dom Tr*, $\mathfrak{A}'^*(A)(F') = \mathfrak{A}^*(A)(F)$. Unfortunately there is no obvious way of pairing assignments in $\mathfrak{A}$ with assignments in $\mathfrak{A}'$, and thus the procedure of defining such a correspondence will be slightly involved.

Definition 19: let $\mathscr{S}$, $\mathscr{S}'$ be formal syntaxes; let $\mathscr{T}$ and $\mathscr{T}'$ be type functions for $\mathscr{S}$ and $\mathscr{S}'$, respectively; let $\mathscr{L}$ be a level of analysis for $\mathscr{S}$, $\mathscr{C}$ a semantics for $\mathscr{S}$, $\mathscr{T}$ down to $\mathscr{L}$, Tr a translation from $\mathscr{S}$ into $\mathscr{S}'$, down to $\mathscr{L}$, and $\mathscr{C}'$ a semantics for $\mathscr{S}'$, $\mathscr{T}'$ down to some level $\mathscr{L}'$, such that for all $A \in$ Dom Tr*, Tr*(A) $\in$ Dom $\mathscr{L}'$.

(a) A *weak interpretation of* Tr, *relative to* $\mathscr{C}$, $\mathscr{C}'$ is a function In from $\mathscr{C}$ into $\mathscr{C}'$, such that for all $\mathfrak{A} \in \mathscr{C}$, $\mathfrak{A}$ and In($\mathfrak{A}$) are based upon the same sets and have the same sets of relevant indices and designated truth values.

(b) Let In be a weak interpretation of Tr relative to $\mathscr{C}$, $\mathscr{C}'$. We say that In is *adequate* if for all $\mathfrak{A} \in \mathscr{C}$ and $A \in$ Dom Tr* $U^*_{\mathfrak{A}}(A) = U^*_{\mathrm{In}(\mathfrak{A})}(\mathrm{Tr}^*(A))$.

(c) Tr is called *weakly adequate relative* to $\mathscr{C}$, $\mathscr{C}'$ iff there is an adequate weak interpretation for Tr, relative to $\mathscr{C}$, $\mathscr{C}'$.

(d) A *strong interpretation of* Tr, *relative to* $\mathscr{C}$, $\mathscr{C}'$, is a pair $\langle \text{In}, \text{Co} \rangle$ such that:

(i) In is a weak interpretation of Tr, relative to $\mathscr{C}$, $\mathscr{C}'$;
(ii) Co is a function, with domain $\mathscr{C}$;
(iii) for $\mathscr{A} \in \mathscr{C}$, Co($\mathscr{A}$) is a many-many correspondence[13] between the assignment ranges of $\mathscr{A}$ and of In($\mathscr{A}$).

(e) Let $\langle \text{In}, \text{Co} \rangle$ be a strong interpretation for Tr, relative to $\mathscr{C}$, $\mathscr{C}'$. We say that $\langle \text{In}, \text{Co} \rangle$ is *adequate* if whenever $\text{A} \in \text{Dom Tr}^*$, $\mathscr{A} \in \mathscr{C}$ and $\langle \text{F}, \text{F}' \rangle \in \text{Co}(\mathscr{A})$ then $\mathscr{A}^*(\text{A})(\text{F}) = (\text{In}(\mathscr{A}))^*((\text{Tr}^*(\text{A}))(\text{F}'))$.

(f) We say that Tr is *strongly adequate relative to* $\mathscr{C}$, $\mathscr{C}'$, iff there is an adequate strong interpretation of Tr relative to $\mathscr{C}$, $\mathscr{C}'$.

Of course, various other notions of semantic adequacy could be introduced as well. In particular we could limit the requirement that a translation preserve the intensions of the expressions it translates to expressions of a certain kind: to the class of all formulae; to the class of all expressions not containing free variables; to the class of all sentences (provided that the notions of freedom and bondage have been given); or others. In each such case we may distinguish between weak and strong adequacy; however, in some cases, such as the last two of the three mentioned above, the two notions will coincide.

The notions of semantic adequacy defined above have the following properties. Let $\mathscr{S}$, $\mathscr{S}'$, $\mathscr{T}$, $\mathscr{T}'$, $\mathscr{L}$, $\mathscr{C}$, $\mathscr{C}'$, Tr, $\mathscr{L}'$, be as in definition 19.

(1) Let $\mathscr{C}''$ be a semantics for $\mathscr{S}'$, down to $\mathscr{L}'$, and let $\mathscr{C}' \subseteq \mathscr{C}''$. If Tr is weakly (strongly) adequate, relative to $\mathscr{C}$, $\mathscr{C}'$ then Tr is weakly (strongly) adequate relative to $\mathscr{C}$, $\mathscr{C}''$.

(2) Let $\mathscr{C}''$ be a semantics for $\mathscr{S}$, down to $\mathscr{L}$ and let $\mathscr{C} \subseteq \mathscr{C}''$. If Tr is weakly (strongly) adequate relative to $\mathscr{C}''$, $\mathscr{C}'$ then Tr is weakly (strongly) adequate relative to $\mathscr{C}$, $\mathscr{C}'$.

(3) Let $\mathscr{L}''$ be a level of analysis for $\mathscr{S}$ such that $\mathscr{L}$ is at least as deep as $\mathscr{L}''$ and let $\mathscr{C}''$ be the restriction of $\mathscr{C}$ to $\mathscr{L}''$. Then if Tr is weakly (strongly)

13. We call a binary relation R a *many-many correspondence* if whenever x R u, y R u and y R v, then x R v. If Dom R = A and Range R = B then we say that R is a *many-many correspondence between A and B*.

adequate relative to $\mathscr{C}, \mathscr{C}'$, there is a translation Tr′ from $\mathscr{S}$ into $\mathscr{S}'$, down to $\mathscr{L}''$ which is weakly (strongly) adequate relative to $\mathscr{C}''$, $\mathscr{C}'$.

Thus it is no more difficult to give a semantically adequate translation when the semantics of the language *into* which one translates is *less* specific (i.e. contains more models) or the semantics for the language *from* which one translates is *more* specific (i.e. contains fewer models). And if a level of analysis $\mathscr{L}$ for a syntax $\mathscr{S}$ is at least as deep as some other level $\mathscr{L}''$, then it will be easier to give a semantically adequate translation from $\mathscr{S}$ into $\mathscr{S}'$ down to $\mathscr{L}''$ than it is to give an adequate translation from $\mathscr{S}$ into $\mathscr{S}'$ down to $\mathscr{L}$—provided that the semantics for $\mathscr{S}$ down to $\mathscr{L}''$ is indeed the restriction of the semantics down to $\mathscr{L}$.

7. *Translations as a means of formulating semantics*

So far our discussion of translations has been based on the assumption that the languages they link are characterised as formal syntaxes and semantics of the sorts defined in the previous sections. However, at the present moment no such characterisation is available for any natural language, in particular not for English, the language with which the project was concerned.[14] One may therefore wonder how this paper could have any significance for the particular translations which were produced on this project and the usefulness of which we promised to explain. For what would the claim that such a translation is adequate amount to, in the term 'adequate' is meaningful only with respect to a semantics for English of which at best fragments are available?

In answer to this question one might reply that, even though at the present time we have no complete description of English in terms of a formal syntax and semantics, such a description nevertheless 'exists', in some abstract sense of 'exist'. Thus our claim that the translations are adequate is meaningful, insofar as it refers to this unknown, but yet existing, description.

From this point of view it is difficult to see how we could ever be *justified* in claiming a particular translation to be adequate. For such a justification would undoubtedly require knowledge of the description to which this claim implicitly refers.

We can, however, interpret the claim that a translation is adequate as a claim *about* the semantical structure of English: the semantics must be such that relative to it and the semantics of the formal language the translation is

[14] Cf the introduction of this paper.

adequate. Indeed, given the semantics for the latter language and a formal syntax $\mathscr{S}$ for English, the translation uniquely specifies a semantics for English, consisting of interpretations down to the same level of analysis as that to which the translation itself goes. Each model of this semantics corresponds to a model belonging to the semantics for the language into which we translate; it is based upon the same sets as this latter model, and assigns to the elementary expressions of the translation and to the rules of $\mathscr{S}$ what the latter model assigns to the translations of these expressions and rules.

In fact, one of the most natural ways to formulate semantics for natural languages (at least in the present state of semantical theory) may be just this: to develop a formal language (in the sense of this paper) and then to give translations from the natural language into it. This is essentially what we have tried to do in the translation part of this project. The formal language developed there is by no means complete, and in particular only fragments of its semantics have as yet been developed. But the general lines along which the details of the theory should be worked out are clear enough to lend substance to the claim that the translations are correct.