# Time, Tense, and Quantifiers

Proceeding of the Stuttgart Conference on the Logic of Tense and Quantification

Edited by Christian Rohrer

Max Niemeyer Verlag
Tübingen 1980

CIP-Kurztitelaufnahme der Deutschen Bibliothek

**Time, tense, and quantifiers** : proceedings of the Stuttgart Conference on the Logic of Tense and Quantification / ed. by Christian Rohrer. – Tübingen : Niemeyer, 1980.
(Linguistische Arbeiten ; 83)
ISBN 3-484-10374-4

NE: Rohrer, Christian [Hrsg.]; Conference on the Logic of Tense and Quantification ⟨1979, Stuttgart⟩

ISBN 3-484-10374-4 / ISSN 0344-6727

# SOME REMARKS ON THE LOGIC OF CHANGE, PART I

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*I.* I knock the egg against the rim of the frying pan. This produces a change in the egg: a moment ago it was in one piece; now it is kaput. Whenever there is change there is a succession of incompatible states. And whenever there is such a succession there is a change.

Suppose a given state p is followed by an incompatible state q.* Then a change occurs. But when? Answer: Not before p has ended and not after q has

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* The terminology of this paper will in certain respects be quite loose. Thus I will often speak of states, or conditions, and of the sentences which characterize these states or conditions without distinguishing between them. I will also speak of predicates as characterizing conditions or events, meaning thereby that the condition or event is characterized by a sentence which consists of the predicate followed by the name(s) of individual(s) whose identity is clear from the context. It will become explicit in section 3 that I am only concerned with conditions, states and events which are characterizable by means of sentences of a given language; so this kind of sloppiness does not affect anything of substance that is discussed in this paper.

In view of the intimate relationship between linguistic expressions on the one, and such entities as states, conditions etc., on the other hand it is also harmless to talk, as I shall sometimes do, of two conditions or states as being incompatible, or contradictories, or of one state or condition implying another. I also sometimes speak of a predicate Π as implying, being incompatible with, or being the contradictory of, another predicate R . Π implies R iff any sentence of the form Π(a) implies any sentence of the form R(a), and analogously for the other two relations mentioned. So this way of speaking should not lead to confusion either.

Truly problematic is my use of the terms 'incompatible', 'contradictory' and 'implies' (irrespective of the nature of the terms that they are made to stand between). My basic intention has been to use these expressions in accordance with what I take to be their accepted meanings in logic; roughly: sentences $s_1$ and $s_2$ are incompatible iff of necessity $s_1$ is false whenever $s_2$ is true; they are contradictories iff it is logically necessary that $s_1$ is true iff $s_2$ is false; and $s_1$ implies $s_2$ iff by logical necessity $s_2$ is true whenever $s_1$ is. In the course of our investigations we shall have to abandon some of the familiar connections between truth, falsehood and negation, however, and this means that the concepts of incompatibility and contradictoriness cannot retain the significance that they have in classical logic. Contradictoriness in particular is a notion that will have to be reassessed and redefined after some other logical issues have been discussed - this in fact touches upon the most difficult issues to which our investigations will lead us.

begun. If p and q are incompatible without being contradictories this may leave a lot of room for the time of change. But if p and q are contradictories the situation is different. Let us consider this case first and refer, while discussing it, to q as not-p. When does the change from p to not-p occur? What answer we are prepared to give to this question is tied up with various logical issues. In fact there are two logical principles, one of long standing and the other plausible enough, which in conjunction leave no room for the change at all. The first of these is the *Principle of Bivalence* (PB), which says that at any time t it must be either that the condition p obtains or else that the contradictory condition not-p obtains.

The second, which I shall call the *Principle of Incompatibility* (PI), asserts that at the time of change from p to q neither p nor q obtains. Clearly these two principles exclude the possibility of there being any time of change from p to not-p at all. For according to PB either p or not-p should hold at such a time; and according to PI neither of them should hold.

Since there are no times of change from a given state to its contradictory, a present tense report of such a change cannot possibly be true. It does not prohibit the truth of reports of such changes in the past, however, nor of predictions that such changes will happen in the future.

What seems to me the natural setting for the study of the logical implications of this theory of change is provided by recent (and as yet unpublished) work of J. Stavi. Stavi has shown that four binary temporal connectives S,U,S',U' suffice for the characterization of all temporal properties of and relations between or among propositions that can be specified in the first order theory of the earlier later relation (assuming only that this relation is a linear order). The four are semantically defined by the following clauses (where $\chi$ is any formula I write: $[\chi]_t = 1$ for "$\chi$ is true at t", and $[\chi]_t = 0$ for "$\chi$ is false at t"; $t < t'$ means "t is earlier than t' ").

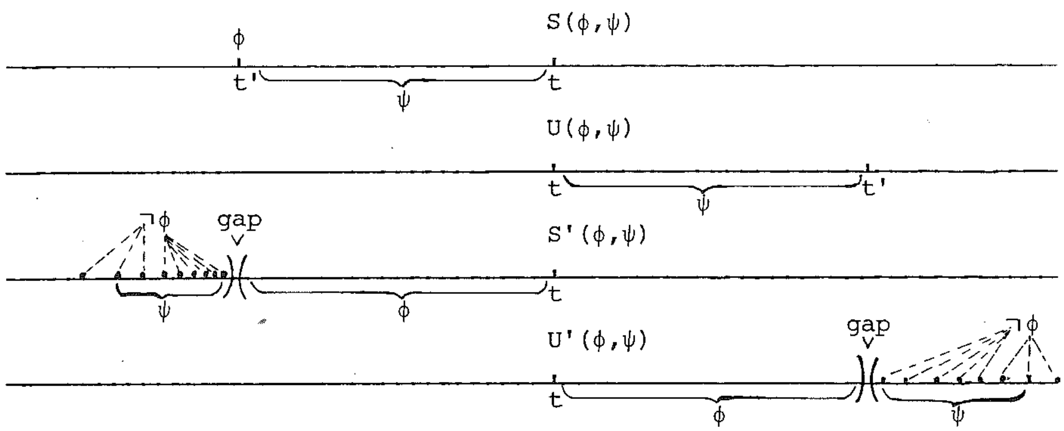
$$[S(\phi,\psi)]_t = 1 \text{ iff } (\exists t')(t' < t \;\&\; [\phi]_{t'} = 1 \;\& \\ (\forall t'')(t' < t'' < t \rightarrow [\psi]_{t''} = 1))$$

$$[U(\phi,\psi)]_t = 1 \text{ iff } (\exists t')(t < t' \;\&\; [\phi]_{t'} = 1 \;\& \\ (\forall t'')(t < t'' < t' \rightarrow [\psi]_{t''} = 1));$$

$$[S(\phi,\psi)]_t = 1 \text{ iff } (\exists t')(t' < t \;\&\; (\forall t'')(t' < t'' < t \rightarrow [\phi]_{t''} = 1)) \;\& \\ \neg(\exists t')(t' < t \;\&\; (\forall t'')(t' < t'' < t \rightarrow [\phi]_{t''} = 1) \;\& \\ (\forall t'')(t'' < t' \rightarrow (\exists t''')(t'' < t''' < t' \;\& \\ [\phi]_{t'''} = 0))) \;\&\; (\exists t')([\phi]_{t'} = 0 \;\& \\ (\forall t'')((t' < t'' < t \rightarrow ((\exists t''')(t'' < t''' < t \;\& \\ [\phi]_{t'''} = 0) \rightarrow [\psi]_{t''} = 1)));$$

$$
\begin{aligned}
[U'(\phi,\psi)]_t = 1 \text{ iff } & (\exists t')(t < t' \;\&\; (\forall t'')(t < t'' < t' \to [\phi]_{t''} = 1)) \;\& \\
& (\exists t')(t < t' \;\&\; (\forall t'')(t < t'' < t' \to [\phi]_{t''} = 1) \;\& \\
& (\forall t'')(t' < t'' \to (\exists t''')(t' < t''' < t'' \;\& \\
& [\phi]_{t'''} = 0))) \;\&\; (\exists t')([\phi]_{t'} = 0 \;\& \\
& ((\forall t'')(t < t'' < t' \to ((\exists t''')(t < t''' < t'' \;\& \\
& [\phi]_{t'''} = 0) \to [\psi]_{t''} = 1))).
\end{aligned}
$$

Thus $S(\phi,\psi)$ is true at t provided $\phi$ is true at some earlier time t', since which time $\psi$ has been true uninterruptedly until t, and $U(\phi,\psi)$ is true at t provided $\psi$ is true from t uninterruptedly until some later time, at which $\psi$ is true. $S'(\phi,\psi)$ says, at t, that there is a *gap* before t between which and t $\phi$ is uninterruptedly true, while there are times arbitrarily close before the gap at which $\phi$ is false; moreover there is a stretch of time of which the gap is the least upper bound throughout which $\psi$ is true. $U'(\phi,\psi)$ says the same as $S'(\phi,\psi)$, but with past and future reversed (just as $U(\phi,\psi)$ says the same as $S(\phi,\psi)$ with reversal of past and future).




With the help of these operators we can express "there was a change from p to not-p" as

$$P(S'(\neg p,p) \vee S(\neg p \;\&\; S(p,p),\neg p) \vee S(p \;\&\; S(p,p),\neg p)$$

(where $P\phi$ ("it was the case that $\phi$") is short for $S(\phi,\phi \to \phi)$ ).
"There will be a change from p to not-p" and "there has been a change from p to not-p" (where this last phrase is to be understood as implying that the condition not-p into which the change resulted has continued until the present) can be expressed similarly. And the same is true of more complicated temporal relations that involve change from some condition into its contradictory. Methods related to those used in Stavi's proof that S,U.S' and U' are capable of expressing all first order topological relations between propositions, establish the deductive completeness, relative to a variety of linear order-

ings and classes of linear orderings of appropriate axiomatizations of the language containing these four connectives. All these results are quite complicated, however, and this is not the place to present them.

We often say such things as: "It is now becoming dark", "The tree is changing its colour", "He is turning pale", etc. Moreover there are many verbs, such as e.g. 'fade', 'mature', 'die', 'appear', 'mellow', which convey change of a particular sort in their subjects and which are nevertheless used perfectly naturally in the present progressive. And an even larger class of transitive verbs (sometimes classified as 'accomplishment verbs'), which imply a change in their direct objects, also occur quite naturally in the present progressive. How could our present theory of change account for these uses? The reply to this must be that in all these cases there is a transition from one state p into another state q which is incompatible with p but not its contradictory. These are separated by a number of intermediate states. Each such intermediate state begins and subsequently ceases, and thus is associated with two changes of the kind that our theory deals with directly.
Such an account of protracted change raises a number of questions: What changes between contradictory states must be contained within a period of time in order that a certain protracted change can be said to be going on during that period? Is it possible to spell out in a systematic way, for any expressions of protracted change conditions on sequences of changes between contradictions that are necessary and sufficient for the applicability of the expression? Assuming such a systematic correlation could be given, should we consider such expressions as shorthands for the statements of the corresponding conditions on sequences? There are complex and difficult problems. Before tackling any of them *, however, we must, I think, consider a more fundamental issue: Is it really correct to hold that changes between contradictories do not, strictly speaking, ever occur?

*II.* It is common to represent time as a linearly ordered structure of instants, e.g. as a structure isomorphic to that of the real numbers, the representation employed in theoretical physics. Such a representation of time is also presupposed by the vast majority of results in what has become known as 'tense logic', (and in particular in the results of Stavi which I mentioned in the previous section). Such representations are quite abstract. For anything that can be *recognized* as standing in some temporal relation to some-

* In fact none of them could be dealt with in the first part of the paper published here.

thing else must, unlike the elements which these representations postulate, have some finite duration - this is so whether the relation is a relation between experiences, or between events which need not be experiences themselves but which must be experienced in order to be recognized as standing in the relation.

In the early years of this century Norbert Wiener showed how from such items of finite duration as experiences or other events, and the temporal relations by which they can be recognized as being connected, one can construct a linear ordering of instants;[1] the original items reemerge as convex sets[2] of such instants, and the original temporal relations between them are easily defined in terms of the single relation which linearly orders the instants. Formally Wiener's construction comes to this. We start from a certain set E, the members of which (think of them as events of some sort of another) enter into two relations, that of complete precedence $\propto$, and that of overlap, O. Intuitively $e_1 \propto e_2$ if $e_1$ has come to an end by the time $e_2$ begins; and $e_1 O e_2$ iff $e_1$ and $e_2$ are (at least in part) simultaneous.

Suppose we want to construct a linear structure of instants such that each member of E can be made to correspond to a convex set of these instants - the instants at which one might say that the event 'goes on'. This requires:

R1. that each event goes on at at least one instant;

R2. that whenever $e_1 O e_2$ then there is an instant at which they both go on; and, more generally, if E' is any finite subset of E such that for any two $e_1, e_2 \varepsilon$ E' $e_1 O e_2$ then there is an instant at which all members of E' go on; and

R3. that whenever $e_1 \propto e_2$ then if $t_1$ and $t_2$ are any instants at which, respectively, $e_1$ and $e_2$ go on then $t_1$ is earlier than $t_2$.

It is intuitively plausible that one way to satisfy these three requirements is to define instants as maximal sets of pairwise overlapping elements of E:

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1 One finds references to and descriptions of this in many works of Russell's, (who may well have been the one to have conceived the idea on which Wiener's paper is based.)

2 By a *convex xet of* an ordering $\langle X,<\rangle$ I understand any subset Y of X such that for all x,y,z $\varepsilon$X if $x < y < z$ and x,z $\varepsilon$Y then y $\varepsilon$ Y. An *interval of* $\langle X,<\rangle$ is a convex subset of a special kind, vz. a subset Y such that there are x,z $\varepsilon$ X such that Y is the set of elements of X bounded below by x and above by z. In this last definition 'bounded' is intentionally ambiguous between < and <. Thus with the particular boundaries x and z are associated four distinct intervals, vz. $\{y: x < y < z\}$ , or [x,z]; $\{y: x < y < z\}$,or (x,z]; $\{y: x \leq y < z\}$, or [x,z); and $\{y: x < y < z\}$, or (x,z). In my related [ ] I unwisely used the term 'interval' to refer to what I have here, consistently with common mathematical terminology, termed convex sets.

> t is an *instant* (of the structure $\mathcal{E} = \langle E, \propto, O\rangle$ ) iff $t \subseteq E$; for all $e_1, e_2 \in t$ $e_1 O e_2$; there is no t' such that $t \subset t' \subseteq E$ which also satisfies the condition that for any $e_1, e_2 \in t'$ $e_1 O e_2$;

while putting moreover, for any two such instants $t_1$, $t_2$:

> $t_1 < t_2$ iff there are $e_1 \in t_1$ and $e_2 \in t_2$ such that $e_1 \propto e_2$.

Clearly these definitions meet R1. - R3. It is not difficult, moreover, as Wiener was the first to point out, to show that < linearly orders the set of instants of $\mathcal{E}$ provided that $\mathcal{E}$ satisfies the following postulates:

A1. $e_1 \propto e_2 \rightarrow e_2 \not\propto e_1$ ;

A2. $e_1 \propto e_2 \ \& \ e_2 \propto e_3 \rightarrow e_1 \propto e_3$ ;

A3. $e_1 O e_2 \rightarrow e_2 O e_1$ ;

A4. $e_1 O e_1$ ;

A5. $e_1 \propto e_2 \rightarrow \neg e_1 O e_2$ ;

A6. $e_1 \propto e_2 \ \& \ e_2 O e_3 \ \& \ e_3 \propto e_4 \rightarrow e_1 \propto e_4$ ;

A7. $e_1 \propto e_2 \vee e_1 O e_2 \vee e_2 \propto e_1$ .

Of these postulates A1. - A6. are readily justified in terms of the intended meanings of < and O. We shall presently discuss the acceptability of A7.

We shall denote the set of instants of $\mathcal{E}$ as $T(\mathcal{E})$, the order between these instants as $<(\mathcal{E})$ and the pair $\langle T(\mathcal{E}), <(\mathcal{E})\rangle$ as $\mathcal{T}(\mathcal{E})$ is the minimal structure which satisfies the requirements R1. - R3.[4]

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3 Actually A2. is redundant as it follows from A7. and A4.

4 There is a difficulty here which relates to certain infinite sets of pairwise overlapping events. Suppose e.g. that E contains for each natural number n two elements $e_n, f_n$ such that

i) for any n, m $e_n$ and $e_m$ overlap; $e_n$ and $f_m$ overlap iff m > n, and if $m \leq n$ then $e_n \propto f_m$. An example of this situation is provided by a set E whose members are intervals of the real number line and where $e_n$ is the interval $(0, \frac{1}{2n})$ and $f_n$ the interval $(\frac{1}{2n}, 1)$. In this situation there will be an instant t which contains all the $e_n$, and at which therefore each $e_n$ 'goes on'. If moreover E contains no other members than $e_n$ and $f_n$ then there is no need of t for the satisfaction of R1. - R3., so that $\mathcal{T}(\mathcal{E})$ is not exactly the minimal structure to satisfy these requirements. I will ignore the complication in what follows, and continue to speak as if $\mathcal{T}(\mathcal{E})$ were in fact the smaller structure satisfying R1. - R3. There is an additional problem with my earlier stipulation that if $e \in t$ then e can be said to 'go on' at t. Intuitively it would seem more appropriate to say, in the case we are contemplating, that t is the starting point of each of the $e_n$ than that $e_n$ goes on at t. Thus in general the membership relation $e \in t$ is less restricted than what is normally understood by 'e goes on at t'. For this latter phrase to be applicable we should presumably demand not only that $e \in t$ but in addition that t be internal to some interval such that $e \in t$ for all t belonging to that interval.

However, there may be considerations other than those incorporated in R1.-R3. which favour a richer instant structure than $\mathcal{T}(\mathcal{E})$. In physics for instance one normally insists that time is like the real numbers, rather than e.g. the rationals. The only rationale for this is, it seems, provided by the consideration that the application of physical laws, such as e.g. those of classical or relativistic mechanics, becomes highly artificial (though not strictly impossible) if time is assumed to have a structure other than that of the real numbers. Of course nothing that has been said indicates whether $\mathcal{T}(\mathcal{E})$ might not itself have the structure of the reals; but the commitment that is made in physics does not seem to depend on this. The question of the structure of temporal order is thus a fairly involved one to which that of $\mathcal{T}(\mathcal{E})$ might provide a partial, but can hardly be expected to give a complete answer.

In any case, however, little if anything can be ascertained about the structural properties of $\mathcal{T}(\mathcal{E})$ before we clarify the precise nature of E. What I have said about $\langle E, \propto, O\rangle$ so far still leaves room for a wide range of interpretations; and different interpretations are bound to lead to very different structures of instants. For instance, it is compatible with my earlier remarks about the nature of E to interpret E as consisting of the experiences of a given individual X at some particular time t together with those past experiences of his of which he is still aware at t. On this interpretation of E $\mathcal{T}(\mathcal{E})$ will be absurdly limited. It will contain no times in the future of (its times corresponding to) t; and its times in the past of t are unlikely to allow him to incorporate adequately new information about events preceding t which he might acquire subsequently. Without being told more about X's experiences it is impossible to say much about the instant structure derived from this set E. Not that this matters much. For the features of this instant structure will have only the remotest bearing on what would be regarded an adequate representation of time. What X himself will accept as an adequate interpretation, in particular, will depend more on certain general inductively based beliefs, e.g. that he has had more experiences than he can now remember; that he will have experiences in the future; that others have experiences too, many of which are not matched by his own experiences; or that of the vast variety of the world's events only some are experiences, and only some of the remaining ones are experienced.

Like the considerations which lead to real-number-like time in physics these hypotheses lead to richer instant structures. But unlike those physical considerations they can be interpreted, more directly, as assumptions of the existence of richer *event* structures - which will lead to richer instant structures when subjected to Wiener's construction. In fact each of the hypotheses I listed corresponds to a different way of interpreting E:

as the set of X's present as well as all of his past experiences; as the set of all of X's experiences whether past, present or future; as the set of all experiences ever had by anyone; or as the set of all (observer-independent) events.

Saying that any of these interpretations is possible would seem to imply that it induces, or at least can be combined with, interpretations of < and O so that the resulting structure $<E, \propto, O>$ satisfies A1. - A7. I already remarked that the verification of A1. - A6. is unproblematic. Indeed, each of the interpretations mentioned, in conjunction with any interpretations of $\propto$ and O that are consistent with the informal descriptions of their meanings on p. verifies these postulates. A7. on the other hand is not so easily seen to be valid, and here it might make a difference which particular interpretations of E, $\propto$ and O we consider. If for instance we take E to consist of X's experiences up to t, and $\propto$ and O to be such that $e_1 \propto e_2$, or $e_1Oe_2$, is to hold only if X is aware at t that $e_1$ precedes, or overlaps, $e_2$, then there is no reason whatever why A7. should be true: X may be perfectly well aware of $e_1$ and $e_2$ separately without being aware of the temporal relation in which they stand to each other. If on the other hand we stipulate that, say $e_1 \propto e_2$ iff $e_1$ *was* entirely before $e_2$, and $e_1Oe_2$ iff they did overlap, whether X is aware of these facts or not, then A7. would seem a good deal more plausible. Similarly if E is the set of all observer-independent events and $\propto$ and O are the relations of actual precedence an overlap A7. might appear to be satisfied. (I am consciously ignoring the difficulties that might be thought to arise here in connection with the theory of relativity.). I believe, however, that even these last two interpretations of E, and O do not render A7. true. The grounds for this belief derive from the following two convictions:

1. The concept of an event is not absolute. What can be singled out as a separate event depends on the tools available for singling it out. There are no good reasons to suppose these tools, - the components of some particular conceptual scheme, which enables its users to distinguish one thing from another, to see it as different from others in various ways, and to see it as different now from what it was before, - to be completely determined by the intrinsic features of the phenomena for whose structural perception they are employed. Nor if we restrict attention to human users, are there grounds for supposing them to be fully determined by a combination of the intrinsic features of nature on the one hand and biologically fixed characteristics of homo sapiens on the other. Nor, finally, are we at present entitled to the

belief, that there is one optimal conceptual scheme to which any other could be reduced. It appears unwarranted therefore to speak of 'the class of events' as if this were a totality that was fixed absolutely. All we can meaningfully refer to is the class of events relative to a given conceptual scheme $\Sigma$ ; and where $\Sigma$ is not mentioned explicitly it is the conceptual scheme of the speaker which must be understood as presupposed.

2. The concepts which enable us to discern difference and change in the empirical world are typically vague: there always may be, and generally will be, cases where the criteria associated with the concept fail to determine whether it applies. I suspect that vagueness is inevitable - that it follows from certain general premisses concerning the ways in which concepts are used and can be learned by creatures such as ourselves. I shall not attempt to work out such an argument here, however. For my present purpose it is enough that many of the concepts we do employ are in fact vague; and that there exists at present no viable prospect of replacing them by concepts that are fully precise. We often treat, it appears, vague predicates as if they were 'potentially sharp', i.e. as if they had sharp boundaries which could be in any one of a certain range of possible locations. In doing so we manage to hold on to the principles of classical logic, - such as, in particular, the law of excluded middle - in spite of the circumstance that certain sentences (e.g. some sentences of the form Q(a).) are neither true nor false.[5]
Vagueness in the concepts used to individuate events may lead to certain indeterminacies in the relations of temporal overlap and precedence. This may happen in particular where the events are changes involving vague concepts. Suppose a screen shows two light spots, one, $s_1$, changing from red to orange, and the other, $s_2$, changing from green to blue. Are the changes simultaneous or does one of them precede the other? There are at least two ways in which we may approach this question:

a) We treat the colour predicates involved as potentially sharp. Suppose first, just for the sake of discussion, that events other than the two in

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5 That ordinary discourse does treat vagueness in this way is suggested for instance by our tendency to treat such statements as "a is red and a is not red" as contradictory, and also, though perhaps not quite as readily, statements of the form "a is red or a is not red" as tautologous, even where we admit the possibility that a lies in the truthvalue gap of 'red'. [5] contains a formal analysis of this use of vague predicates, a variant of which will play a role in later parts of this paper. See also [3] .

question already determine a time structure $\langle T,<\rangle$ and that we are asking with respect to this time structure at what instant, or during which interval, each of these changes took place. Then we can reason as follows: Each of the possible ways in which we can resolve the ambiguity between orange and red will determine the exact time $t_1$ where $s_1$ ceases to be red and starts being orange. Similarly each of the ways of resolving the ambiguity between green and blue determines a precise instant $t_2$ for the transition of $s_2$ from green to blue. It is possible that no matter how the ambiguities are resolved $t_1$ is always before $t_2$. In that case it is reasonable to maintain that the change in $s_1$ *is* indeed before that in $s_2$. By the same token, if no matter how we resolve the ambiguities in the four colour predicates $t_1$ and $t_2$ coincide then this justifies the claim that the changes are simultaneous; and if $t_2$ is in all cases before $t_1$ then we may say that the second change precedes the first. It is perfectly possible, however, that on some resolutions $t_1$ precede $t_2$ while on others $t_2$ precedes $t_1$ while on yet others they coincide. In that case the temporal order of the two changes must be regarded as undetermined. As this last possibility cannot be excluded we have no reason for believing A7. to be satisfied.[6]

To assume that the structure of time has already been determined by events whose mutual temporal relations are completely fixed is, within the present context, to put the cart before the horse. For it is precisely the question whether the structure of events can be assumed to yield a suitable - in particular linear - instant structure which is at stake. We cannot permit ourselves, therefore, to describe the situation by referring, in the way we have just done, to the 'times' of the two changes. It ought to be evident, however, that this does not affect the essence of our conclusion: the temporal relations between events need not be fully determined.

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6 There is a possible source of misunderstanding here. According to the theory of supervaluations that is discussed in [3] and [5] A7. *would* be satisfied, in as much as it comes out true in each of the structures that result from resolving the ambiguities. Our present concern, however, is whether $\langle E, \propto, O\rangle$, taken as a model in the usual sense satisfies A7. directly, i.e. if the union of $\propto$, O and $\breve{\propto}$ equals $E^2$. Only this will guarantee that Wiener's construction yields a linear, and not only a partial, ordering.

Moreover, the intuition that each way of resolving the ambiguities results in an instant of change can still be expressed, but now rather as the proposition resolutions yields an event structure in which the change, call it $e_0$, is instantaneous in the precise sense of not being divided by other events:

$$(1)\quad \neg(\exists e_1, e_2)(e_1 O e_0 \; \& \; e_2 O e_0 \; \& \; e_1 \propto e_2)$$

b) It is tempting to try to avoid indeterminacy of temporal relation by analyzing the changes as lasting for as long as $s_1$ and $s_2$ belong to the truthvalue gaps of the relevant predicates. But to think that temporal determinacy could thus be restored is an illusion. Talk of truthvalue gaps easily leads us to think of a vague predicate Q as dividing its range of application into three, rather than two, parts: positive extension, negative extension and truthvalue gap. But we must be careful here. An object a belongs to the truthvalue gap of Q if it is undetermined whether the criteria associated with Q are satisfied by a or not. To treat the gap as on a par with positive and negative extension is to suggest that there is an other set of criteria which decide whether (i.e. are satisfied iff) satisfaction of the former criteria is undetermined. In general, however, there are no grounds for supposing that there is such a second set of criteria. If there isn't then no object a can be regarded as *definitely* belonging to the truthvalue gap of Q; and, by the same token, if a changes in the respects relevant to Q, there is no time which can be counted as definitely belonging to the period of transition. The following parallel with intuitionistic mathematics may be of some help to get this point across.

Where a is in the truthvalue gap of Q the proposition Q(a) is in a position similar to that of a proposition p of intuitionistic mathematics which is neither provable nor refutable. No proof may be possible that p is undecidable by mathematical argument, and thus the proposition that p is undecidable would, even if it could be expressed, never be assertable.[7]

This is not to say that it is never possible to ascertain that an object belongs to the truth value gap of a predicate. In fact it is precisely in the context of change that we find the best evidence that this possibility is sometimes realized. There are occasions, for instance, where it seems right to say: "I was getting angry" to report a gradual transition from a state of non-anger to a state of anger, passing through a number of intermediate states which are recognizable as being neither states of anger nor states of non-anger. Here there is a period of some finite duration during which I was in the truthvalue gap of 'angry'. But note that this does not help us at all to establish that the temporal relations between events are determinate. For let us suppose that the account I just gave of "I was getting angry" also applies to the changes in the spots $s_1$ and $s_2$ which we discussed above; i.e. there is a period during which $s_1$ is definitely in the truthvalue gaps of 'red' and 'orange' and also a period during which $s_2$ is definitely in the truthvalue gaps of 'green' and 'blue'. In order that it be

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7 I cannot say that I find this account of the nature of truthvalue gaps a great deal more satisfactory than others with which I have been aquainted (including earlier attempts of my own). Such accounts run into an apparently fundamental difficulty. Truthvalue gaps constitute a phenomenon for which there is no room in scientific languages as we have come to conceive of them, largely under the influence of the development of modern logic during the past century. As it is only of such languages that we understand the logical properties reasonably well it is a sound methodological canon to insist on their use in all scientific exposition. It seems, however, that when we describe truthvalue gaps in such a language we cannot help distorting the phenomenon that we are trying to capture. This is particularly true of model-theoretic analyses of vagueness, such as can be found e.g. in [3] and [5] , which are formulated in a language which builds the theory of sets on a basis of classical predicate logic. The only proper way to give an account of vagueness is to formulate the logic of languages which contain vague predicates *directly* (i.e. without reducing it, as one does in modeltheoretic treatments, to that of some other type of language). This is a problem that is still largely unsolved. (It might be thought that the supervaluation theory of vagueness has resolved this problem by showing that the logic of languages with vague predicates is the same as that of languages all of whose predicates are sharp. But note that all the supervaluation theory asserts is that the laws governing those operators on which modern logic has primarily concentrated (vz. truthfunctions and quantifiers) are not affected by vagueness. So if the supervaluation account is at all correct, this only means that to show the distinctive logical features of vagueness we would have to formulate the logic of languages which contain additional logical notions besides these familiar operators.

determined whether the first change completely preceded the second, it must be determined where the first of these periods ends and where the second of them begins. But there is no reason why the boundary between the truth value gap of, say, 'orange' and its positive extension should be sharply defined and thus no reason why it should be determined exactly when $s_1$ leaves the truthvalue gap of 'orange' and enters the positive extension. Similarly the time at which $s_2$ moves from the positive extension of 'green' into its truth-value gap need not be precisely fixed. Thus temporal indeterminacy can no more be excluded on this second account of change than it could on the first.[8]

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8 The same point might also be made in a slightly different way: As soon as there is the possibility of ascertaining that a belongs to the truth-value gap of Q there arises therewith the possibility of replacing the single predicate Q by three incompatible predicates (we might call them 'definitely Q', 'non-Q' and 'neither Q nor non-Q') whose extensions are, respectively, the positive extension of Q, its negative extension, and its truthvalue gap.In fact we achieve precisely this effect by introducing, as we often do in ordinary discourse, two distinct negation-operators, roughly those which are familiar from the theory of three-valued logic as "strong",or "inner",and "weak",or "outer",negation. I did make this very distinction just now when I used both the "strong" 'non-' and the "weak" 'neither ... nor'. These predicates are ideally both mutually exclusive and jointly exhaustive. This means that e.g. the negative extension of 'definitely Q' coincides with the union of the (positive) extensions of non-Q and 'neither Q nor non-Q'. For each of these predicates, however, exactly the same question arises that we earlier felt obliged to ask about Q: What reasons could we have for believing that, say, 'definitely Q' doesn't have any truthvalue gap; or, in other words, that the boundary between the positive extension of Q and its truthvalue gap is sharp? In general there are no such reasons.

It should be possible in principle to adopt criteria that decide whether an object lies in the truthvalue gap of some such predicate as 'definitely Q'. Then the same procedure could be repeated, now introducing three new predicates to replace the single 'definitely Q'. [3] suggests a number of ways in which we might analyse languages in which this process is repeatable ad libitum. Whether all vagueness could be eliminated through an infinite number of iterations though not through any finite segment thereof is a matter of speculation.

*III.* The theory of change that I wish to explore starts from the idea that it is the changes themselves which are constitutive of time. It seems to me that we generally take for granted that changes are observer-independent events; and I shall assume from now on that they are. In the sequel of this paper therefore, it is only event structures whose members are such observer-independent events that will be of importance. Henceforth event structures shall be understood to be of this sort. We have seen, however, that even such event structures cannot be assumed to satisfy A7. Only in ideal modifications of such structures, where all vagueness has been removed, will A7 be valid.

For what is to follow it will be helpful to have at hand a more formal representation of the ideas that have been discussed in the last section. Such a representation will be somewhat simplified if we assume, as I will do from now on, that an event-structure is determined by a language L (of which we should think in this connection as embodying the conceptual scheme used in individuating the events). Such a language will contain, in particular, for each of the concepts used in the individuation of events a simple or complex predicate which expresses that concept. I shall make further assumptions about L as we go along.

The event structure determined by L must incorporate, besides the three components E, $\propto$, O which we have already discussed at some length, also a component to which I shall refer as $\mathcal{A}_1$, which conveys what sort of event each of the members of E is. That is, it must specify for each $e \in E$ the conditions, as expressed in L, in terms of which e is singled out. It is natural to represent this component as a function from E to appropriate expressions of L. Which expressions are appropriate? Let us assume that L has at least the structure of a language of 1st order predicate logic, i.e. contains predicates, names for and variables ranging over individuals, truth functions and quantifiers; and further that L is *tensed*, in the sense that its expressions may take different semantic values at different times. In particular, the sentences of L may be true at one time while false at another. The sentence $Q(a)$, e.g., will be true at those times where a satisfies Q and false at those times where it does not satisfy Q. In such a tensed language the changing conditions that serve to individuate events are naturally expressed by complete sentences. So we shall assume that the fourth component is a function from E to sentences of L. There are, however, several ways in which a condition can be used to individuate an event. The event might be the presence of that condition during some limited period; or it might be the event of that condition's absence; or it might be the event of that condition coming to obtain; or of its ceasing to do so. (Especially these last two types of events are important here, of course). It is therefore not enough for $\mathcal{R}_1$ to associate with an event e from E just a sentence of L, it must also make clear in which of the four mentioned ways that sentence is related to e. I shall assume therefore that $\mathcal{R}_1$ maps each e onto a pair consisting of a sentence and one of the symbols P (for Present), F (for Failing), B (for Becoming) and C (for Ceasing).

The next question we must ask is: Which sentences of L serve for the individuation of events? One is tempted to answer to this: Any meaningful sentence of L. For let s be any such sentence and suppose for instance that s holds at one time and fails at some later time. Then there ought to be somewhere between these two times an event which is the ceasing of the condition expressed by s. There are two difficulties with this answer. The first is that until we know more of the language L we cannot be quite so sure that each of its meaningful sentences expresses a condition change in which determines an event. This is a question to which I shall return later.

There is, however, another, and more fundamental,difficulty. Suppose s is a grammatically complex sentence. Then the conditions for its obtaining or failing at any particular time are determined by the semantic values, at various times, of its constituents - this is what grammatical complexity means! It is therefore not possible that the complex sentence alters its truthvalue without any accompanying change in the value of at least one of its atomic components.

I shall assume that such changes in the values of atomic constituents can always be reduced to changes in the satisfaction of atomic *predicates*. If one also assumes, as I shall do, that L contains a name for each of the individuals it talks about[9], then it follows that all events are determined by atomic sentences (i.e. sentences consisting of an atomic predicate followed by an appropriate number of names.[10]

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9 The assumption that every individual has a name, which will also be convenient later, can be eliminated by methods familiar from standard developments of symbolic logic. I will not dwell on this any more.

10 Where L is assumed to have the structure of a language of first order predicate logic (possibly with some additional apparatus) the notion of an atomic sentence is unproblematic. When we apply the present analysis to any natural language, however, we shall have to consider very carefully which expressions are to be regarded as playing the role I have here attributed to atomic sentences.

In addition to the information, provided by $\mathcal{A}_1$, as to how the members of E are individuated we shall also want further facts concerning what was the case when a given event occurred. Some such information could be extracted from the quadruple $\langle E, \propto, O, \mathcal{A}_1\rangle$ as it stands. E.g. we may infer that $s_2$ remained green while $s_1$ turned orange from the fact that the event $e_1$ of $s_1$'s turning orange is included in some event $e_2$ which is identified as the event of $s_2$'s being green - where inclusion of $e_1$ in $e_2$ can be expressed as

$$(2) \quad (\forall e)(e \propto e_2 \rightarrow e \propto e_1) \ \& \ (\forall e)(e_2 \propto e \rightarrow e_1 \propto e) \ \& \ (\forall e)(eOe_1 \rightarrow eOe_2)$$

To assume that all such information can be so extracted is to put a heavy strain on the notion of an event, however. It would require that even conditions which, although contingent, nevertheless happen to hold forever would determine "events" characterized by their presence; and such unbounded "events" would be very different from what normally counts as an event. As I do not wish to commit myself to such requirements I shall assume that event structures come with yet a fifth component, $\mathcal{A}_2$, which supplies the information about what was the case during each of the members of E. This may still not encompass all information about the members of E that we shall need. For it may happen that a given event e is individuated as the change from, say, not-Q(a) to Q(a) but that it is also the change in some other condition, e.g. from R(b) to not-R(b). It is therefore desirable to let $\mathcal{A}_2$(e) map atomic sentences of L to one of the four values P,F,B,C. $\mathcal{A}_2(e)(s)$ = P will mean that s is true throughout e; $\mathcal{A}_2(e)(s)$ = F that it is false throughout e; $\mathcal{A}_2(e)(s)$ = B that e is a change from not-s to s; and $\mathcal{A}_2(e)(s)$ = C that e is a change from s to not-s.

The sets $\mathcal{A}_2$(e) will have to satisfy certain consistency requirements. In the first place there must be consistency between the assignments to any two overlapping events: If $e_1$ and $e_2$ overlap and the atomic sentence s belongs to the domains of both $\mathcal{A}_2$(e) and $\mathcal{A}_2(e_2)$ then it must yield the same value for these two functions. Furthermore the information provided by $\mathcal{A}_2$ must not contradict that provided by $\mathcal{A}_1$. The simplest, and most natural,way to ensure this is to stipulate that the first member of $\mathcal{A}_1$(e) always belong to the domain of $\mathcal{A}_2$(e), and that the values agree (i.e. if $\mathcal{A}_1(e) = \langle s,B\rangle$ then $\mathcal{A}_2(e)(s)$ = B etc.). This renders the component $\mathcal{A}$, strictly speaking superfluous for the semantic considerations that follow. Yet conceptually it seemed better to include it as a separate component into our representations.

I shall refer to quintuples $\langle E, \propto, O, \mathcal{A}_1, \mathcal{A}_2\rangle$ which satisfy the mentioned conditions (i.e. $\langle E, \propto, O\rangle$ satisfies A1. - A6., and the functions $\mathcal{A}_1$ and $\mathcal{A}_2$

satisfy the consistency requirements just listed) as *event models*.

I suggested earlier that event structures $\langle E, \propto O\rangle$ are typically underdetermined in that A7. may fail, and that this indeterminacy is due to vagueness in the predicates used in individuating the events in question. Let us assume that such vagueness is the only source of indeterminacy of temporal relations. Then in each (ideal) situation where all gaps have been removed the temporal structure will be fully determinate (i.e. A7. will hold). One might also expect that the model $\mathfrak{M}' = \langle E' \propto ', O' \mathcal{A}_1', \mathcal{A}_2'\rangle$ corresponding to such an ideal situation is determinate in yet another respect, vz. that at each of the instants which result when Wiener's construction is applied to the structure $\mathcal{E}' = \langle E', \propto', O'\rangle$ every atomic sentence of L has a definite value. Suppose, however, that the instant contains an event e which is a change from not-s to s. Then, according to the Incompatibility Principle, s can be neither true nor false at t.

It seems reasonable, in view of the assumptions we have made already, to assume in addition that these are the only cases where an atomic sentence lacks a value at an instant:

(3) unless t contains an e such that $\mathcal{A}_2(e)(s) = B$ or $\mathcal{A}_2(e)(s) = C$
$[s]_t = 1$ or $[s]_t = 0$.

Let us make the intuitively obvious stipulation that the truthvalue of any atomic sentence s at an instant t is determined by $\mathcal{E}'$ via the condition:

(4) $[s]_t = {1 \atop 0}$ iff $(\exists e)(e \in t \;\&\; \mathcal{A}_2(e)(s) = {P \atop F})$

Thus any structure $\mathfrak{M}'$ corresponding to a situation where vagueness has been resolved must satisfy the further condition that, given definition (4) of "$[s]_t$", (3) holds. This leaves us with the pairs $\langle t,s\rangle$ where t contains an event e that is a change with respect to s. The dilemma we meet here is the one already signaled in section 1: if there are genuine times of change (from a given condition into its contradictory), then either Bivalence of Incompatibility must be given up.

There are various options which we can pursue at this point. The simplest of these, in the sense that it allows us to stay as closely as possible to familiar metamathematical terrain, succeeds in maintaining in a fashion, both PB and PI.

The means of expressing change will be quite limited; this can be done only by applying to a primitive predicate of L either the operator B ("-- is becoming ..." or the operator C ("-- is ceasing to be ..."). (BQ) and

(CQ) are again predicates[11] and (BQ)(a), for instance, is to be read "a is becoming Q", or "a is changing from non-Q to Q" - similarly for (CQ)(a). As always where we deal with truthvalue gaps the real difficulty arises over negation. Here it presents itself in the following guise. We have already admitted that a's becoming, or ceasing to be, Q is incompatible both with its being Q and with its 'failing to be' Q. This may sound incoherent. For if a's becoming Q, say, is incompatible with a's being Q does that not imply that if a is becoming Q then a fails to be Q? In an informal setting it is difficult to escape this appearance of incoherence; a consistent description requires two distinct concepts of negation, whereas prevailing logical doctrine has made us prone to overlooking what differences in meaning between various expressions for negation we might otherwise have been able to discern. What words we use in our attempt to capture the difference between these two negation concepts may not matter all that much. What does matter is that our present commitments force a bifurcation of the concept of negation upon us - into what corresponds, roughly, to the "strong", or "inner", and the "weak", or "outer", negation that one finds in the theory of three-valued logic, and especially in the theory of presupposition.
Here, as in certain systems of three-valued logic, one of these negations becomes definable as soon as the other is introduced as a primitive. Thus if we represent the "strong" concept of negation, at which I gestured above when using the expression 'fails to be Q', by a predicate operator F, then the disjunction

$$(FQ)(a) \lor (BQ)(a) \lor (CQ)(a)$$

exhausts the conditions under which it is not the case that a is Q, and thus represents the negation of 'Q(a)' in a second ("weak") sense. I shall represent this second negation as a sentential operator, $\neg$ .

So far we have only defined $\neg$ as applying to sentences of the form Q(a). However, the same thesis that led to its definition in those contexts, vz. the view that Q,BQ,CQ, and FQ are pairwise incompatible and jointly exhaustive, suggests similar definitions for it where it stands in front of sentences of the forms (BQ)(a), (CQ)(a), and (FQ)(a):

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11 Strictly speaking the operators B and C, as well as the operator F that is introduced below, are systematically ambiguous, in that they transform 1-place predicates into 1-place predicates, 2-place predicates into 2-place predicates, etc. In what follows I shall only deal with 1-place predicates; but what I have to say will invariably apply also to predicates of more places.

$$\neg(BQ)(a) \equiv_{df} (Q(a) \vee (CQ)(a) \vee (FQ)(a),$$

etc.

In all these contexts $\neg$ has the properties of classical negation, and, in particular, obeys the law of excluded middle. Note, however, that we have only accounted for the operator as applying to atomic sentences. The truth definition below will make it behave like classical negation also in the context of complex formulae. If we introduce $\neg$ as a primitive then F can be contextually defined by:

$$(FQ)(a) \equiv_{df} \neg Q(a) \ \& \ \neg(BQ)(a) \ \& \ \neg(CQ)(a)$$

We shall adopt both F and $\neg$ as primitives, identifying $\neg$ with the negation operator which L already contains (in as much as it encompasses first order predicate logic). Let L' be the extention of L which we obtain by adding the predicate operators B, C, and F. (We shall later differentiate between a number of languages $L_i$; for each of these we will then have a corresponding extention $L_i'$ ).
Sentences of the form Q(a) can be negated in two ways. One of these - that involving F - behaves in accordance with the principle of Incompatibility, the other, involving $\neg$ , preserves Bivalence. Thus, in a sense both principles have been saved, but at the cost of giving up a unified account of negation.

In conjunction with any event model $\mathcal{M} = \langle E, \propto, O, \mathcal{A}_1, \mathcal{A}_2 \rangle$ we must consider the various models which describe the ideal situations in which all indeterminacies of $\mathcal{M}$ have been resolved. We shall refer to such models as *completions* of $\mathcal{M}$, and usually refer to them as '$\mathcal{M}'$' or $\langle E', \propto', O', \mathcal{A}_1', \mathcal{A}_2' \rangle$. Each such $\mathcal{M}'$ leads in turn to a model $\mathcal{N} = \langle T', <', \mathcal{F}' \rangle$ where $\langle T', <' \rangle$ is the instant structure derived by Wiener's construction from $\mathcal{E}' = \langle E', \propto', O' \rangle$. We shall refer to this last model as the *instant model derived from* $\mathcal{M}'$. $\mathcal{F}'$ is to convey the truthvalues of the atomic sentences of L at the instants $t \in T'$. In the context of the option we are now pursuing this is best done in the format familiar from ordinary model theory, i.e. by specifying the extensions of the primitive predicates of L, as well as of the predicates that can be formed out of these by application of the operators B,C and F. Thus $\mathcal{F}'$ will map each pair consisting of a $t \in T'$ and one of these predicates onto the set of individuals which satisfy the predicate at that time. More specifically:

(5) i) $\mathcal{F}'(t,Q) = \{a: (\exists e)(e \varepsilon t \ \& \ \mathcal{A}_2'(e)(Q(a)) = P\}$ ;
ii) $\mathcal{F}'(t,FQ) = \{a: (\exists e)(e \varepsilon t \ \& \ \mathcal{A}_2'(e)(Q(a)) = F\}$ ;
iii) $\mathcal{F}'(t,BQ) = \{a: (\exists e)(e \varepsilon t \ \& \ \mathcal{A}_2'(e)(Q(a)) = B\}$ ;
iv) $\mathcal{F}'(t,CQ) = \{a: (\exists e)(e \varepsilon t \ \& \ \mathcal{A}_2'(e)(Q(a))) = C\}$ ;

On the basis of this information we can then give a (two-valued) truth definition for the sentences of L, as follows. The truthvalues of the atomic sentences of L' are given by the obvious clauses:

(6) i) $[Q(a)]_{\mathfrak{N}',t} = 1$ iff $\mathcal{F}'(t, Q(a)) = P$
ii) $[(BQ)(a)]_{\mathfrak{N}',t} = 1$ iff $\mathcal{F}'(t, Q(a)) = B$
iii) $[(CQ)(a)]_{\mathfrak{N}',t} = 1$ iff $\mathcal{F}'(t, Q(a)) = C$
iv) $[(FQ)(a)]_{\mathfrak{N}',t} = 1$ iff $\mathcal{F}'(t, Q(a)) = F$

For the truthvalues of complex sentences we shall adopt clauses familiar from standard developments of the model theory of tense logics.[12]
In particular we have for sentence negation simply:

(7) $[\neg \phi]_{\mathfrak{N}',t} = 1$ iff $[\phi]_{\mathfrak{N}',t} = 0$

(Precisely what further clauses are required depends on the resources of L. We shall come back to this). The supervaluation theory (as it is developed e.g. in [3] and [5]) suggests that the values which the sentences of L receive in the instant models derived from the completions of $\mathfrak{M}$ be used to characterize truth and falsehood relative to $\mathfrak{M}$ itself. There are at least two ways in which we might go about this. First we might stipulate, for any t belonging to the (not necessarily linear) instant structure $\langle T,<\rangle$ derived from $\mathfrak{M}$, that a sentence s is true (or false) at t iff s is true at t in each model $\mathfrak{N}'$ derived from a completion $\mathfrak{M}'$ of $\mathfrak{M}$. This cannot be the correct specification, however, for in general the "instants" of T need not occur in the instant structures derived from completions of $\mathfrak{M}$; nor need they correspond to the instants of such a derived structure in a one-to-one fashion. It is possible for instance that two sets $t_1$ and $t_2$ of pairwise overlapping events which are both maximal relative to the temporal relations of $\mathfrak{M}$ will collapse into a single instant t' of $\mathfrak{N}'$. This happens, to be precise, if for any $e_1 \varepsilon t_1$ and $e_2 \varepsilon t_2$ $\mathfrak{M}$ does not specify the temporal relation between them, while in $\mathfrak{M}'$ any two such events overlap. In this way several "instants" of $\mathfrak{M}$ may correspond to a single instant of $\mathfrak{N}'$.

12 see e.g. [2],[6]

Formally this difficulty could be overcome by the stipulation that s be true, or false, at t iff it is true, or false, in each $\mathcal{N}'$ at each t' which corresponds to (i.e. settheoretically includes) t. But conceptually this remains somewhat unsatisfactory in as much as it is dubious whether the so-called "instants" of T can always be regarded as genuine instants at all.

The second alternative is to define truth and falsity directly with respect to events of $\mathcal{M}$:

(8) s is true (false) in $\mathcal{M}$ at e iff for each $\mathcal{N}'$ and each instant t' of $\mathcal{N}'$ such that $e \in t'$ s is true (false) at t' in $\mathcal{N}'$.

This definition is actually a quite natural explication of the central insight of tense logic - and more generally of the theory of indexicals - that the central task of semantics is to characterize what it is for a sentence (which contains context-sensitive elements) to be true at the time at which it is uttered. Utterances are events of usually relatively short, but nevertheless finite, duration. It is thus possible that a sentence lacks a truth-value at the time of its utterance in virtue of the fact that that time can be subdivided into smaller portions during some of which the sentence is true while during others it is false. (8) succeeds in capturing that possibility.[13]

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13 In actual discourse indeterminacy of this sort tends to be resolved by a form of the principle of charity: where X utters s and s is true only during some part of the time that it takes X to make his utterance there is a certain conversational presumption that one should take, as the time of evaluation for s precisely that part of the duration of the utterance during which s *is* true. It might be noted in this context that the notion 'time of utterance' is not as clear-cut as some discussions in which it plays a role may seem to imply. There is a problem of interpretation, in particular, with written utterances, such as one finds e.g. in letters. What is the time of utterance of a particular sentence s occuring somewhere in a letter of some length? is it the period that it took to write the entire letter; or just the time it took to write that particular sentence; or some period of intermediate duration, during which the passage was written in which the sentence figures? Usually these questions do not matter as the sentence does not fluctuate in truthvalue even over the longest period that could qualify as the time of utterance. There is, however, enough indeterminacy here to provide the principle of charity with scope for action. Evidently these problems are intimately connected with those, usually discussed in a different context, relating to the so-called "specious present". Indeed, once one accepts that "the present" can only be a certain *event* (which is recognized as being simultaneous with its recognition), the interminacy of its extent becomes, if not inescapable, at least very plausible.

Precisely how we solve the problem of characterizing truth and falsehood relative to $\mathfrak{M}$ should not affect the characterization of validity (of sentences) and logical consequence. These notions seem to require unversal quantification both over possible completions and over the instants of the models derived from these completions. Thus we could define

(9) s is valid in $\mathfrak{M}$ iff s is true at each instant of each instant model derived from a completion of $\mathfrak{M}$.

(a similar definition could be given for the notion of logical consequence. I shall ignore this second concept in the discussions that will follow. It will in all cases relevant here be connected with the concept of validity in the manner familiar from classical logic - i.e. via the Deduction Theorem). This is admittedly not a very natural notion. We are not so much interested in what has to come out true at all times of a particular model - which after all represents a particular way in which the world is or could have been - but rather in what has to come out true always in *any* such model. Earlier work in tense logic has taught us, however, that there is an important source of complications here. As soon as the object language contains even the simplest tense operators (such as Prior's P ('it was the case that') or F ('it will be the case that')) the notions of validity and logical consequence become sensitive to the properties of temporal order. For instance the set of valid sentences will be different depending on whether time is supposed to be discrete or dense; and even depending on whether we take its ordering to be like that of the rationals or that of the reals. This leads naturally to the relativization of these notions to classes of models - where it is natural to think of any such class as containing precisely those models whose time structures possess all those features of temporal ordering which, in the context of the characterization of validity in question, are thought of as logically necessary aspects of time.

In the work on tense logic to which I referred above it was possible, and natural, to specify various intended classes of models by simply listing the conditions (such as density, having no beginning and no end point, etc.) that the temporal orderings of their models must satisfy. But here that doesn't seem to be the most satisfactory way to proceed. For the members of such a model class should now be event models and not instant models. They should, moreover, be allowed to be partially indeterminate. It is by no means clear that the conditions with which tense logic has been traditionally concerned can be captured as conditions on such partial event models. How e.g. is one to state a condition on $\mathfrak{M}$ which is necessary and sufficient therefor that each instant model derived from a completion of $\mathfrak{M}$ is discretely, or densely, ordered?[14] I should add immediately at this point that even where such conditions on partially determinate event models can be found, it is not necessarily desirable that we insure that the instant models have a feature such as density or discreteness by imposing this condition on the underlying event models. For, as I already observed in section 2, our reason for believing time to have a certain property need not be our conviction that the structure of actual events (under some scheme of individuation) yields such an instant structure under Wiener's construction; but that it may be of a quite different sort. If it were, then it would be wrong to try and insure that the instant structures have the property by imposing on the underlying event models a condition that warrants it. Rather the intended instant models should result from embedding the instant models that Wiener's procedure produces from the completions of the event models in the class $\mathcal{K}$ into certain time structures that do have the desired property. We shall not pursue. this issue any further here, however.

Besides conditions which concern the structure of temporal order there are others which also could, and in some cases should, be imposed on the models of our present theory. These conditions relate to the temporal connections between the predicates Q, BQ, CQ, FQ. The first set consists of different versions of what I shall call the *Principle of Separation* (PS). According to PS any two events $e_1, e_2$ which are such that $e_1 \propto e_2$, and during which two contradictory conditions obtain, are separated by an event which is a change from the first condition to the second. We shall formulate the principle as applying to atomic sentences only:

PS1: Suppose $e_1 \propto e_2$, $\mathcal{A}_2(e_1)(Q(a)) = F$, and $\mathcal{A}_2(e_2)(Q(a)) = P$; then there is an e such that $e_1 \propto e \propto e_2$ and $\mathcal{A}_2(e)(Q(a)) = B$.

PS2: Suppose $e_1 \propto e_2$, $\mathcal{A}_2(e_1)(Q(a)) = P$ and $\mathcal{A}_2(e_2)(Q(a)) = F$; then there is an e such that $e_1 \propto e \propto e_2$ and $\mathcal{A}_2(e)(Q(a)) = C$.

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14 Some such problems have commanded a certain interest from those who were involved with the reduction of instants to events which I have here primarily associated with Wiener. See e.g. Russell's [8] , where a number of such questions are discussed. (Of course Russell's problem was different from the present one in as much as he only considered temporally determinate event structures.).

Equally plausible, though, as we shall see, reducible to these conditions in the presence of certain other principles to be stated below, are the following.

PS3: Suppose $e_1 \propto e_2$, $\mathcal{A}_2(e_1)(Q(a)) = C$, $\mathcal{A}(e_2)(Q(a)) = P$;
then there is an e such that $e_1 \propto e \propto e_2$ and $\mathcal{A}_2(e)(Q(a)) = B$.

PS4: Suppose $e_1 \propto e_2$, $\mathcal{A}_2(e_1)(Q(a)) = F$ and $_2(e_2)(Q(a)) = C$;
then there is an e such that $e_1 \propto e \propto e_2$ and $\mathcal{A}_2(e)(Q(a)) = B$.

PS5: Suppose $e_1 \propto e_2$, $\mathcal{A}_2(e_1)(Q(a)) = B$ and $\mathcal{A}_2(e_2)(Q(a)) = F$;
then there is an e such that $e_1 \propto e \propto e_2$ and $\mathcal{A}_2(e)(Q(a)) = C$.

PS6: Suppose $e_1 \propto e_2$, $\mathcal{A}_2(e_1)(Q(a)) = P$ and $\mathcal{A}_2(e_2)(Q(a)) = B$;
then there is an e such that $e_1 \propto e \propto e_2$ and $\mathcal{A}_2(e)(Q(a)) = C$.

PS7: Suppose $e_1 \propto e_2$, and $\mathcal{A}_2(e_1)(Q(a)) = \mathcal{A}_2(e_2)(Q(a)) = C$;
then either there is an e such that $\mathcal{A}_2(e)(Q(a)) = C$ and which temporally includes both $e_1$ and $e_2$,[15] or there is an e such that $e_1 \propto e \propto e_2$ and $\mathcal{A}_2(e)(Q(a)) = B$.

PS8: Suppose $e_1 \propto e_2$ and $\mathcal{A}_2(e_1)(Q(a)) = \mathcal{A}_2(e)(Q(a)) = B$;
then either there is an e such that $\mathcal{A}_2(e)(Q(a)) = B$ and which temporally includes both $e_1$ and $e_2$,[15] or there is an e such that $e_1 \propto e \propto e_2$ and $\mathcal{A}_2(e)(Q(a)) = C$.

The Principle of Separation asserts the existence of various times of change. As such it directly contradicts the theory we proposed in section 1, but it is in keeping with the central idea of the theory we are developing in this section.

It is natural to impose PS1 - PS8 not only on the event models $\mathfrak{M}$ themselves, but also on their completions. It is readily seen, however, that the fact that $\mathfrak{M}$ itself satisfies these conditions does not warrant that they also hold in its completions. We must therefore stipulate separately that only such completions as agree with PS1 - PS8 are permitted.

For this, and other, similar, reasons it is desirable to introduce, besides the semantic units already discussed, in addition what I shall here call *interpretations*: pairs consisting of an event model $\mathfrak{M}$ and a set $\mathcal{S}$ of completions of $\mathfrak{M}$. It is then possible to define classes of interpretations by imposing restrictions both on $\mathfrak{M}$ and $\mathcal{S}$. Let me state, once more, the

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15 By e *temporally includes* e' I understand the condition (2) on p.150: $(\forall e_1)(e_1 \propto e \rightarrow e_1, \propto e')$ & $(\forall e_1)(e \propto e_1 \rightarrow e' \propto e_1)$ & $(\forall e_1)(e_1 O e' \rightarrow e_1 O e)$.

operators S, U, S', U'. Let the resulting languages be $L_2$ (without B,C,F) and $L'_2$ (with these operators). I have already remarked that the set of valid sentences of such a language is sensitive to the structure of time. Thus we are bound to find different sets of valid sentences relative to model classes $\mathfrak{K}$ ' which derive, in the way just indicated, from different classes $\mathfrak{K}$ of linear orderings. Moreover the conditions PS1 - PS8 are now reflected by the validity of certain sentences. Thus PS1 warrants the validity relative to any class $\mathfrak{K}$' consisting exclusively of interpretations that satisfy it of

(S1) $Q(a)\ \&\ P((FQ)(a)\ \&\ \phi) \rightarrow P((BQ)(a)\ \&\ P((FQ)(a)\ \&\ \phi))$

and

(S2) $(FQ)(a)\ \&\ P(Q(a)\ \&\ \phi) \rightarrow P((CQ)(a)\ \&\ P(Q(a)\ \&\ \phi))$

Similarly PS3-PS6 warrant the validity of, respectively,

(S3) $Q(a)\ \&\ P((CQ)(a)\ \&\ \phi) \rightarrow P((BQ)(a)\ \&\ P((CQ)(a)\ \&\ \phi))$,

(S4) $(CQ)(a)\ \&\ P((FQ)(a)\ \&\ \phi) \rightarrow P((BQ)(a)\ \&\ P((FQ)(a)\ \&\ \phi))$,

(S5) $(FQ)(a)\ \&\ P((BQ)(a)\ \&\ \phi) \rightarrow P((CQ)(a)\ \&\ P((BQ)(a)\ \&\ \phi))$, and

(S6) $(BQ)(a)\ \&\ P(Q(a)\ \&\ \phi) \rightarrow P((CQ)(a)\ \&\ P(Q(a)\ \&\ \phi))$.

PS7 moreover warrants the validity of

(S7) $(CQ)(a)\ \&\ P(\psi\ \&\ P((CQ)(a)\ \&\ \phi)) \rightarrow [P((CQ)(a)\ \&\ \psi\ \&\ P((CQ)(a)\ \&\ \phi))\ v$
$P((BQ)(a)\ \&\ P((CQ)(a)\ \&\ \phi))]$

and PS8 that of

(S8) $(BQ)(a)\ \&\ P(\psi\ \&\ P((BQ)(a)\ \&\ \phi)) \rightarrow [P((BQ)(a)\ \&\ \psi\ \&\ P((BQ)(a)\ \&\ \phi))\ v$
$P((CQ)(a)\ \&\ P((BQ)(a)\ \&\ \phi))]$

For the weaker language $L'_1$, which contains besides B,C,F and the devices of first order logic only the Priorean operators P and F (not to be confused with our predicate operator!) these schemata summarize, I believe, the effect of imposing PS1 - PS8 on interpretations. To be precise, I conjecture that they are *complete* in the following sense: Let $\mathfrak{K}$ be a class of linear orderings and let $\mathfrak{K}$" be the class of interpretations determined by it as indicated in (2). Let $\mathfrak{A}$ be an axiomatization for the language $L_1$ (which lacks B,C,F but contains the Priorean operators), which is complete relative to $\mathfrak{K}$. Then we obtain a complete axiomatization for $L'_1$ relative to $\mathfrak{K}$" by adding to $\mathfrak{A}$ the schemata (11) i - iv) and S1 - S8. S1 - S8 do not suffice to generate, in the presence of appropriate axioms for $L_2$, all sentences of $L'_2$ for whose

validity PS1 - PS8 are responsible. It is not difficult, however, to find axioms which are complete for $L'_2$ in the same sense in which I conjectured S1 - S8 to be complete for $L'_1$. Indeed it follows from the functional completeness of S,U,S',U' that we can express with the help of these operators a three place connective PS, such that $PS(\phi, \psi, \chi)$ is true at t iff either $\phi$ is false at t or else for all t' before t at which $\psi$ holds there is a t" between t' and t so that $\chi$ holds at t". The principle PS1 warrants the validity of each formula of the form PS(Q(a),(FQ)(a),(BQ)(a)). To each of PS2 - PS6 corresponds another set of instances of $PS(\phi,\psi,\chi)$. In very much the same way we can also find formulae corresponding to PS7 and PS8. The resulting 8 schemata will be complete for $L'_2$ in the sense explained.
Before concluding this section I must mention a few other principles which interpretations might be asked to obey.

The first of these is the principle, already mentioned in section 2, that all changes from one state to its opposite are what I termed 'ideally instantaneous'. Let us call this principle the P(rinciple of) I(nstantaneity of) C(hange). We already saw how the condition that e is instantaneous can be expressed, vz. as (1) on p 145. What we need here is a condition which makes explicit that the event in question is indeed a change. We thus get:

PIC $\quad (\exists s)(\mathcal{A}_2(e)(s) = B \vee \mathcal{A}_2(e)(s) = C) \rightarrow \neg(\exists e', e")(eOe' \,\&\, eOe" \,\&\, e' \propto e")$

PIC also has its effect on validity. Thus if all completions of interpretations in $\mathcal{H}$" satisfy PIC then the following formulae of $L'_2$ are all valid relative to $\mathcal{H}$':

$$
\begin{aligned}
(13)\quad & S(\phi,(BQ)(a)) \rightarrow S(\phi,\bot), \\
& S(\phi,(CQ)(a)) \rightarrow S(\phi,\bot), \\
& U(\phi,(BQ)(a)) \rightarrow R(\phi,\bot), \\
& U(\phi,(CQ)(a)) \rightarrow U(\phi,\bot).
\end{aligned}
$$
[15]

Moreover in the presence of PIC PS7 and PS8 can be simplified. Thus PS7 reduces to the condition that any two successive ceasings of Q(a) are separated by at least one becoming of Q(a), and PS8 to the condition that any two becomings are separated by a ceasing. These new versions of PS7 and PS8 warrant the validity of corresponding strengthenings of S7 and S8, so that PIC also has an effect on validity for $L'_1$. I suspect that to get complete axiomatizations for classes $\mathcal{H}$' which are restricted to interpretations whose completions satisfy PIC the only additional axioms required are, in the case of $L'_2$, the formulae (13), and that in the case of $L'_1$ the strengthened

15 By $\bot$ I understand any explicit contradiction of ordinary propositional calculus, e.g. the formula Q(a) & ¬Q(a).

versions of S7 and S8 will suffice.

The Principle of Separation has a counterpart in what I shall call (by lack of a more poignant name) the *Second Principle of Separation*, or $PS_2$. This principle says that between two opposite changes - e.g. a change from not-s to s followed by a change from s to not-s there is an intervening time when the end-state of the first, which is also the initial state of the second change, obtains. Formally:

$PS_2 1$: Suppose $e_1 \propto e_2$, $\mathcal{A}_2(e_1)(Q(a)) = B$ and $\mathcal{A}_2(e_2)(Q(a)) = C$; then there is an e such that $e_1 \propto e \propto e_2$ & $\mathcal{A}_2(a)(Q(a)) = P$.

Suppose $e_1 \propto e_2$, $\mathcal{A}_2(e_1)(Q(a)) = C$ and $\mathcal{A}_2(e_2(Q(a)) = B$; then there is an e such that $e_1 \propto e \propto e_2$ and $\mathcal{A}_2(e)(Q(a)) = F$.

It appears that $PS_2$ is, if possible, even less controversial than PS. For how could an event e be a change from not-s to s if it was not preceded by a time where s failed and followed by a time where s holds? And how could e be followed by a reverse change if a situation in which s obtained had not been reached first? Both these questions invite negative answers. In fact the negative answer to the first embodies yet another uncontroversial principle, to which I shall refer as the *P*(*rinciple of*) *C*(*ompleted*) *C*(*hange*):

PCC1 Suppose $\mathcal{A}_2(e)(Q(a)) = B$; then there is an $e_1$ such that $e_1 \propto e$ and $\mathcal{A}_2(e_1)(Q(a)) = F$ and there is an $e_2$ such that $e \propto e_2$ and $\mathcal{A}_2(e_2)(Q(a)) = P$.

PCC2 Suppose $\mathcal{A}_2(e)(Q(a)) = C$; then there is an $e_1$ such that $e_1 \propto e$ and $\mathcal{A}_2(e_1)(Q(a)) = P$ and there is an $e_2$ such that $e \propto e_2$ and $\mathcal{A}_2(e_2)(Q(a)) = F$.

Note that there is a certain asymmetry between PS and $PS_2$. PS expresses what might be called a 'realist', as opposed to a 'virtual', conception of change. When a time where s fails is followed by a time where s holds then there has been a change. This much is uncontroversial. But the claim that in such cases there always was a time *at* which the change occurred may be regarded as controversial, and this is what PS asserts. $PS_2$ on the other hand seems to do no more than unpack what is implicit in the characterization of an event as a change from one condition to its opposite - and the same goes of course for PCC.

Axioms corresponding to these principles can readily be formulated, both for $L_1'$ and for $L_2'$. But, after all that has already been said on this subject in connection with other principles, it would only be tedious to do yet another exercise of this sort.

Although $PS_2$ and PCC derive from the same consideration they are formally independent from each other (an event model could satisfy either of them without satisfying the other). There is, however, a stronger formal principle which implies both of them and which itself would also seem to follow from the informal consideration that justifies their adoption. I shall refer to this principle - my baptismal powers are deteriorating - as the *Principle of Discrete Change*, or PDC. PDC asserts that a change is *flanked* by its initial and end conditions with nothing intervening between it and the events flanking it. The principle admits of two formulations, a strong and a weak version. I shall give the weak version first:

$PDC_W1$ Suppose $\mathcal{A}_2(e)(Q(a)) = B$; then there is an $e_1$ such that $e_1 \propto e$, $\mathcal{A}_2(e_1)(Q(a)) = F$ and $\neg(\exists e_3)(e_1 \propto e_3 \propto e)$; and there is an $e_2$ such that $e \propto e_2$, $\mathcal{A}_2(e_2)(Q(a)) = P$ and $\neg(\exists e_3)(e \propto e_3 \propto e_2)$.

Suppose $\mathcal{A}_2(e)(Q(a) = C$; then there is an $e_1$ such that $e_1 \propto e$, $\mathcal{A}_2(e_1)(Q(a) = P$ and $\neg(\exists e_3)(e_1 \propto e_3 \propto e)$; and there is an $e_2$ such that $e \propto e_2$, $\mathcal{A}_2(e_2)(Q(a)) = F$ and $\neg(\exists e_3)(e \propto e_3 \propto e_2)$.

This weak version says that there is no event intervening between the change and either of the events which flank it. This does not exclude the possibility, however, that in an instant model derived from an event model in which $PDC_W1$ and $PDC_W2$ hold an instant intervenes between that or those of the change and those at which the initial or end condition obtains. To exclude this eventuality we must strengthen the first half of the conclusion of $PDC_W1$ to:

there is an $e_1$ such that $e_1 \propto e$, $\mathcal{A}_2(e_1)(Q(a)) = F$ and $\neg(\exists e_3, e_4)(e_3 \propto e \ \&\ e_1 \propto e_4 \ \&\ e_3 O e_4)$

The second half of the conclusion, as well as the two halves of the conclusion of $PDC_W2$, need to be strengthened analogously. I shall refer to the resulting principles $PDC_S1$ and $PDC_S2$.

Although PDC may seem to be as well motivated as its consequences $PS_2$ and PCC we shall find in the next section that there are reasons that might induce us to give it up, or at least modify it, but which do not affect the validity of the two weaker principles.

*IV.* Consider once more an event e which is a change from a situation $e_1$ where Q(a) fails to a situation $e_2$ where Q(a) holds. During e BQ is true of a; during $e_2$ it cannot be true of a, for Q is true of a there, and what is the case cannot possibly at the same time be going to be the case. There is thus a transition from a situation where (BQ)(a) holds to one where it fails. It might seem therefore that the very same considerations which led us, in the previous section, to the view that $e_1$ and $e_2$ must be separated by an event such as e, now force us to postulate an event e' to mediate the transition from (BQ)(a) to its oposite.

The very same question can be raised from the vantage point of the predicate Q. Having stipulated that e is an event during which ¬Q(a) is true should not the very same reasoning that led to the assumption of e now also commit us to the existence of an event e' which mediates between it and the following $e_2$ ?

It would be a mistake to suppose that our commitment to such events as e necessarily carries with it the further commitment to such events as e'. One could defend the first commitment while rejecting the second by drawing a distinction between predicates such as Q on the one and predicates such as BQ and CQ on the other hand: the former would express conditions which cannot be fulfilled at one time after having not been fulfilled at some earlier time without an intervening change; the second type of predicate, however, would itself already express that a change w.r.t. Q is happening, so that there is no reason for supposing that an event where such a predicate is satisfied must be separated from one where it is not by yet another event; similarly the contrast between sentences Q(a) and ¬Q(a) is not the sort of contrast that requires for any pair of successive events where these respective sentences are true, an additional event that marks the transition; for, on the interpretation that we gave to ¬ in section 3, ¬Q(a) is compatible with the occurrence if a transition to, or from, a situation where Q(a) holds. Although we might justify our acceptance of the first and rejection of the second commitment, and thus of the theory presented in the preceding section, by an appeal to this distinction, I have given as yet no compelling grounds why we should not make the second commitment as well as the first. In this section I wish to explore, albeit in a preliminary fashion, what is implied by making it.

It is one question whether we must assume that there is always an event which marks the transition from (BQ)(a) to Q(a). It is another question how a another exercise of this sort.

Although $PS_2$ and PCC derive from the same consideration they are formally independent from each other (an event model could satisfy either of them without satisfying the other). There is, however, a stronger formal principle which implies both of them and which itself would also seem to follow from the informal consideration that justifies their adoption. I shall refer to this principle - my baptismal powers are deteriorating - as the *Principle of Discrete Change*, or PDC. PDC asserts that a change is *flanked* by its initial and end conditions with nothing intervening between it and the events flanking it. The principle admits of two formulations, a strong and a weak version. I shall give the weak version first:

$PDC_W1$ Suppose $\mathcal{A}_2(e)(Q(a)) = B$; then there is an $e_1$ such that $e_1 \propto e$, $\mathcal{A}_2(e_1)(Q(a)) = F$ and $\neg(\exists e_3)(e_1 \propto e_3 \propto e)$; and there is an $e_2$ such that $e \propto e_2$, $\mathcal{A}_2(e_2)(Q(a)) = P$ and $\neg(\exists e_3)(e \propto e_3 \propto e_2)$.

Suppose $\mathcal{A}_2(e)(Q(a) = C$; then there is an $e_1$ such that $e_1 \propto e$, $\mathcal{A}_2(e_1)(Q(a) = P$ and $\neg(\exists e_3)(e_1 \propto e_3 \propto e)$; and there is an $e_2$ such that $e \propto e_2$, $\mathcal{A}_2(e_2)(Q(a)) = F$ and $\neg(\exists e_3)(e \propto e_3 \propto e_2)$.

This weak version says that there is no event intervening between the change and either of the events which flank it. This does not exclude the possibility, however, that in an instant model derived from an event model in which $PDC_W1$ and $PDC_W2$ hold an instant intervenes between that or those of the change and those at which the initial or end condition obtains. To exclude this eventuality we must strengthen the first half of the conclusion of $PDC_W1$ to:

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The second half of the conclusion, as well as the two halves of the conclusion of $PDC_W2$, need to be strengthened analogously. I shall refer to the resulting principles $PDC_S1$ and $PDC_S2$.

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The very same question can be raised from the vantage point of the predicate Q. Having stipulated that e is an event during which $\neg$Q(a) is true should not the very same reasoning that led to the assumption of e now also commit us to the existence of an event e' which mediates between it and the following $e_2$ ?

It would be a mistake to suppose that our commitment to such events as e necessarily carries with it the further commitment to such events as e'. One could defend the first commitment while rejecting the second by drawing a distinction between predicates such as Q on the one and predicates such as BQ and CQ on the other hand: the former would express conditions which cannot be fulfilled at one time after having not been fulfilled at some earlier time without an intervening change; the second type of predicate, however, would itself already express that a change w.r.t. Q is happening, so that there is no reason for supposing that an event where such a predicate is satisfied must be separated from one where it is not by yet another event; similarly the contrast between sentences Q(a) and $\neg$Q(a) is not the sort of contrast that requires for any pair of successive events where these respective sentences are true, an additional event that marks the transition; for, on the interpretation that we gave to $\neg$ in section 3, $\neg$Q(a) is compatible with the occurrence if a transition to, or from, a situation where Q(a) holds. Although we might justify our acceptance of the first and rejection of the second commitment, and thus of the theory presented in the preceding section, by an appeal to this distinction, I have given as yet no compelling grounds why we should not make the second commitment as well as the first. In this section I wish to explore, albeit in a preliminary fashion, what is implied by making it.

It is one question whether we must assume that there is always an event which marks the transition from (BQ)(a) to Q(a). It is another question how a change from (BQ)(a) to Q(a) might be expressed. Clearly we cannot use the sentence (BQ)(a) for this purpose for e marks a transition *from* the condition expressed by that sentence to some other condition (expressed by Q(a)). We could introduce an entirely new predicate operator (say $B^2$) – so that $(B^2Q)(a)$ would express the condition in question. But it seems also possible to represent the required predicate as CBQ, using the operator C which is already present in the languages $L_i'$ , and that is the option I shall pursue. Similarly we shall express the condition obtaining at an event which marks the transition from a time where we have (FQ(a) to a time where (BQ)(a) holds as (BBQ)(a); and BCQ and CCQ shall serve to express transitions to and from events described by (CQ)(a). The considerations that earlier led us to stipulate that (BQ)(a) is false when Q(a) or (FQ)(a) are true now yield the similar conclusion that (BBQ)(a) is not true when either Q(a) or (BQ)(a) is true. It appears intuitively clear, moreover, that (BBQ)(a) must be false in all these cases, and the same should hold for (BCQ)(a) and (CCQ)(a). It ought to be equally plausible that the new predicates BBQ, CBQ, BCQ, and CCQ are mutually exclusive. We thus have a set of eight pairwise incompatible predicates.

Having thus extended the application range of the operators B and C we cannot avoid the question whether a similar extention would not be apposite for F also. Previously we distinguished between events where Q(a) fails and events which separate these from events where Q(a) holds, and we agreed to express this difference as that between the truth of (FQ)(a) and that of both ¬Q(a) and ¬(FQ)(a).

In the same way we might now, taking our departure from the predicate BQ instead of Q, distinguish between events such as $e_2$, at which (BQ)(a) definitely fails, and events such as e' which mark the transition either from such a condition to one where (BQ)(a) holds or to such a condition from one where (BQ)(a) holds; and stipulate that at the former (FBQ)(a) holds while at the latter both ¬(BQ)(a) and ¬(FBQ)(a) are true. By this principle (FBQ)(a) will be implied by Q(a) as well as by (FQ)(a), while on the other hand (FBQ)(a) will be false when either (BBQ)(a) or (CBQ)(a) is true.
(CQ)(a) expresses the occurrence of a transition from one condition, expressed by Q(a), to another, expressed by (FQ)(a), both of which have already been recognized as implying (FBQ)(a). I shall take this as a sufficient reason for stipulating that (CQ)(a) itself also implies (FBQ)(a). This is in fact a particular case of a general principle, to which I shall presently have to refer again. I have called it the P(rinciple of) C(ontinuity); the

The exact formulation of it will be given below.

By symmetric considerations one is led to the stipulation that FCQ is implied by Q, FQ, and BQ, and excluded by BCQ and CCQ.

In section 3 I stipulated that (FQ)(a) is, like Q(a), false when either (BQ)(a) or (CQ)(a) is true. The motivation for that stipulation was that these latter sentences express the occurrence of a transition to or from the condition described by (FQ)(a). We must now consider whether (FQ)(a) is true or false when a satisfies any of the new predicates that we have allowed ourselves to form on the basis of Q.

First let us look at the predicate BBQ. This predicate expresses a transition from FQ to BQ, and our reason for stipulating that (FQ)(a) is false when (BQ)(a) is true now leads us to postulate that (FQ)(a) must be false when (BBC(a) is true. Similarly, FQ must be excluded by CCQ.
CBQ and BCQ on the other hand express transitions between the condition expressed by Q and those expressed by BQ and CQ. We agreed earlier that each of these excludes FQ in the weak sense that (FQ)(a) is false when Q(a), (BQ)(a) or (CQ)(a) is true. Yet another application of the Principle of Continuity leads to the conclusion that CBQ and BCQ also exclude FQ.

It would not be natural, now that we have extended the syntax of the languages $L_i'$ this far, to prohibit the application of B, C and F to predicates of the form FQ. In defining the extensions of the resulting predicates I shall be guided by the idea, implicit in all that has been said about F so far, that this operator forms out of any predicate its complementary predicate: If a satisfies $\Pi$ then a fails to satisfy $F\Pi$ , if a satisfies $F\Pi$ then a fails to satisfy $\Pi$, and it is not determined whether a satisfies $\Pi$ iff it is not determined whether a satisfies $F\Pi$ . Thus FFQ is equivalent to Q; and by the same token BFQ is equivalent to CQ and CFQ is equivalent to BQ.

Once we have made the second commitment - that there are event such as e', which marks the transition from (BQ)(a) to Q(a) - and moreover accepted that the conditions which distinguish such events are expressible - e.g. by the sentence (CBQ)(a) - it is hard to see why we should stop here: should there not be supposed to be an event e", for instance, which marks the transition from the condition expressed by (CBQ)(a), which prevails at e', to the incompatible condition Q(a); and could we not express what distinguishes this event from the two between which it mediates as (CCBQ)(a)? Again there are no conclusive reasons why we should take this additional step. Still, it is worth while to reflect on the implications of making this third commitment also, as well as all those which successively arise out of each other in the same way as the second arose from the first and the third from the second.

This infinite sequence of commitments leads to languages $L_i''$ in which the operators B, C and F can be iterated arbitrarily often. That is, $L_i''$ has the same vocabulary as $L_i'$ , but its syntax differs in that it has a wider set of predicates, recursively defined by:

(14) i) each predicate of L is a predicate of $L_i''$ ;

ii) if $\Pi$ is a predicate of $L_i''$ then so are B $\Pi$ , C $\Pi$ and F $\Pi$ .

Let me state, in conjunction with this definition, that of a related notion which will be useful in what is to come:

(15) a) the set S( $\Pi$ ) of predicates of L" *positively generated by* the predicate $\Pi$ is defined recursively by:

i) $\Pi \in S(\Pi)$

ii) if $Q \in S(\Pi)$ then B $\Pi$ and C $\Pi \in S(\Pi)$.

The event models for a language $L''_i$ must also provide information about the extensions of the new predicates which do not occur in $L_i'$ . In view of this it turns out to be convenient to change the format of the functions $\mathcal{A}_1$ and $\mathcal{A}_2$. We make $\mathcal{A}_1$ into a function that maps events onto atomic sentences (thus instead of $\mathcal{A}_1(e_1) = \langle Q(a),B\rangle$ we shall now have: $\mathcal{A}_1(e) = (BQ)(a)$) and $\mathcal{A}_2$ into a function from events to functions from sets of atomic sentences of $L_i''$ to {0,1} - so that instead of $\mathcal{A}_2(e)(Q(a)) = P$ we now have $\mathcal{A}_2(e)(Q(a)) = 1$, while the cases in which a belongs to the extension of BQ, CQ or FQ are conveyed thereby that $\mathcal{A}_2(e)$ assigns the value 1 to, respectively, (BQ)(a), (CQ)(a) or (FQ)(a).

Besides the consistency requirements already noted in section 3 (see p. 150 ) we must now state constraints which insure matching between the assignments to atomic sentences that involve the same individuals and structurally related predicates. Some of these constraints have already been stated informally, and it is straightforward to translate these into conditions on models for $L_i''$ . Thus we get, among others,

(16) i) if $\mathcal{A}_2(e)$ assigns 1 to one of $\Pi$(a), (F$\Pi$)(a), (B$\Pi$)(a) and (C$\Pi$)(a) then it assigns 0 to the remaining three.

ii) if $\mathcal{A}_2(e)$ assigns 1 to $\Pi$(a), (C $\Pi$)(a) or (F $\Pi$)(a) then it assigns 1 to (FB $\Pi$)(a).

iii) if $\mathcal{A}_2(e)$ assigns 1 to one of $\Pi$(a), (B $\Pi$)(a) and (C $\Pi$)(a) then it assigns 1 to (FC $\Pi$)(a).

iv) $\mathcal{A}_2(e)(FF\Pi)(a) = {}^1_0$ iff $\mathcal{A}_2(e)(\Pi(a)) = {}^1_0$, $\mathcal{A}_2(e)(BF\Pi)(a)) = {}^1_0$ iff $\mathcal{A}_2(e)((C\Pi)(a)) = {}^1_0$ and $\mathcal{A}_2(e)((CF\Pi)(a)) = {}^1_0$ iff $\mathcal{A}_2(e)((B\Pi)(a)) = {}^1_0$.

The other connections between the extensions of structurally related predicates which I stated earlier can similarly be converted into conditions on event models. There still remain a number of such connections, however, which our discussion has so far left untouched. It appears that all of these can be settled by appeal to two principles. The first of these is the Incompatibility Principle, in the form in which we used it in section 3, according to which a predicate $\Pi$ which expresses the occurrence of a transition between two conditions expressed by respectively the predicates R and $\Sigma$, excludes each of these. The second is the Principle of Continuity, which we shall now state in its intended generality:

PC Suppose that the predicate R expresses the occurrence of a transition between conditions expressed by $\Sigma$ and T.

a) Suppose that $\Pi$ is a predicate which is implied by both $\Sigma$ and T. Then $\Pi$ is implied by R.

b) Suppose $\Pi$ is excluded by both $\Sigma$ and T. Then $\Pi$ is excluded by R.

c) Suppose $\Pi$ implies both $F\Sigma$ and FT. Then $\Pi$ implies FR.

It is easy to verify that if R and $\Sigma$ are distinct predicates which are positively generated by the same predicate $\Pi$ then they are mutually exclusive. Moreover, if R is positively generated by $C\Pi$ then $\Pi$ as well as all predicates positively generated by $B\Pi$ imply FR, and if R is positively generated by $B\Pi$ then $\Pi$ as well as all predicates positively generated by $C\Pi$ imply FR. Clearly the condition which we could impose upon completions of models for $L_i'$ - that in the derived instant model at each instant at least one of Q, BQ, CQ, and FQ must be true - must now be dropped. We can, however, replace it by a similar condition, to the effect that

(17) for each instant t and each primitive predicate Q at least one sentence $\Pi(a)$ is true at t, where $\Pi$ is either FQ, or else a predicate that is positively generated by Q.

All these conditions are also easily translated into restrictions on the function $\mathcal{A}_2$, but there seems to be no point in carrying this out explicitly. We have already committed ourselves informally to a generalization of the Principle of Separation, to apply to arbitrary predicates of $L_i''$. In view of the way I have chosen to treat the operator F where it applies to composite predicates this generalized principle is correctly expressed by the conditions PS'1 - PS'8, which we obtain through replacing in PS1 - PS8 the letter Q, which ranges over predicates of L, by the letter Π whose intended range is the set of all predicates of the $L_1''$.

It is clear that once we impose PS'1 - PS'8 we cannot also impose the Principle of Discrete Change. For any event characterized by, say, (BΠ)(a) will be separated from any other that is characterized by Π(a) or by (FΠ)(a) by infinitely many other events. PS'1 - PS'8 are compatible, however, with a weaker condition than PDC, to the effect, roughly, that a change is flanked by two events, characterizable as the presence of the initial and the end condition of the change, respectively, between which there are just the events which PS'1 - PS'8 require. Unfortunately, although the intuitions behind this condition are quite straightforward, it is nonetheless a little difficult to state.

Let $\mathfrak{M}$ be an event model, $e_1$ and $e_2$ members of E such that $e_1 \propto e_2$, $\mathcal{A}_2(e_1)(F\Pi)(a)) = 1$ and $\mathcal{A}_2(e_2)(\Pi(a)) = 1$. We say that a subset E' is *generated by* Π *and* $e_1$ *and* $e_2$ iff

> i) $e_1, e_2 \in E'$; ii) if $e, e' \in E'$, $e \propto e'$ and for some predicate R generated by Π $\mathcal{A}_2(e)((F\,R)(a)) = 1$ and $\mathcal{A}_2(e)(R(a)) = 1$ then there is an $e'' \in E'$ such that $e \propto e'' \propto e'$ and $\mathcal{A}_2(e'')((B\,R)(a)) = 1$; iii) if $e, e' \in E'$, $e \propto e'$ and for some predicate R generated by Π $\mathcal{A}_2(e)(R(a)) = 1$ and $\mathcal{A}_2(e')((F\,R)(a)) = 1$ then there is an $e'' \in E'$ such that $e \propto e'' \propto e'$ and $\mathcal{A}_2(e'')((C\,R)(a)) = 1$; iv) if $E'' \subseteq E'$ and E" satisfies i) - iii) then E" = E'.

In a similar way we define what it is for E' to be generated by Π and a from $e_1$ and $e_2$ for the case where $\mathcal{A}_2(e_1)(\Pi(a)) = 1$ and $\mathcal{A}_2(e_2)(F\Pi)(a)) = 1$. We can now formulate the condition referred to above as follows:

(18) i) Suppose $\mathcal{A}_2(e)(B\Pi)(a)) = 1$; then there are $e_1$ and $e_2$ such that a) $e_1 \propto e \propto e_2$; b) $\mathcal{A}_2(e_1)((F\Pi)(a)) = 1$; c) $\mathcal{A}_2(e_2)(\Pi(a)) = 1$; d) there is a unique event set E' generated by Π and a from $e_1$ and $e_2$; and e) for each set E" of pairwise overlapping events such that for each $e' \in E''$ either $e_1 \propto e'$ or $e' \propto e_2$ there is an $e' \in E'$ such that e' overlaps each of the events in E".

ii) Suppose that $\mathcal{A}_2(e)(C\Pi)(a)) = q$. Then there are $e_1$ and $e_2$ such that $e_1 \propto e \propto e_2$, $\mathcal{A}_2(e_1)(\Pi(a)) = q$, $\mathcal{A}_2(e_2)((F\Pi)(a)) = 1$, and the remaining conditions are as i) a and i) c.

The meaning of (18) should be clear: the events $e_1$ and $e_2$ which "flank" a change are so to speak *only* separated from e by those events which PS'1 - PS'8 demand - this in the precise sense that each of the predicates generated by $\Pi$ is represented by at most one event between $e_1$ and $e_2$, and that these representatives moreover fill the space between $e_1$ and $e_2$ in the sense that any instant of the instant structure derived from the model in question which lies between $e_1$ and $e_2$ [16] will contain one such representative.

Clearly it is a consequence of (18) that each change lies in a dense interval,i.e. let e be a change, $e_1$ and $e_2$ the events flanking it whose existence is ensured by (18), and $t_1$ and $t_2$ two elements of the corresponding instant structure containing $e_1$ and $e_2$ respectively. Then the interval $(t_1,t_2)$ of the instant structure will be dense.

Consider moreover the following condition:

(19) Let E' and E" be two sets of pairwise overlapping events, containing events e' and e" such that $e' \propto e''$. Then either there is a change (i.e. an event which verifies at least one sentence of the form $(B\Pi)(a)$ or the form $(C\Pi)(a)$) which overlaps all the events in E', or there is such an event overlapping all events in E", or else there is a change e and members e' and e" of E' and E" respectively such that $e' \propto e \propto e''$.

(19) implies that if $t_1$ and $t_2$ are distinct instants then either at least one of them is an instant of change - in the sense that at least one change is occurring at it - or else they are separated by a change. It thus embodies the idea that only change can produce temporal succession.

If the model satisfies (19) as well as (18) then the derived instant structure is dense.

The languages $L_i''$ suggest many questions of the sort we considered in section 3 in connection with their counterparts $L_i'$. I shall not try to formulate, let alone answer, these here, however. In fact, before occupying himself seriously with the metamathematics of these systems, one might wish for more evidence that they are conceptually adequate. There have been a number of places in the development of the syntax and semantics of the $L_i''$ at which we might have taken out a different option from the one I chose, and some of these should be considered with some care, if only to strengthen our conviction that the choices which I did make were the right ones. This task I shall also leave largely unaccomplished. In the next section I will address myself to what I think is the most serious difficulty with the present

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16 t *lies between* $e_1$ and $e_2$ iff $(\exists e_3 \varepsilon t)(e_1 \propto e_3)$ and $(\exists e_4 \varepsilon t)(e_4 \propto e_2)$

languages. I wish to conclude this section by drawing attention to a peculiarity by which the reader may already have been struck, and perhaps mystified.

This peculiarity is related to the particular way in which I have dealt with the operator F. As I have set up the semantics FBQ is implied by Q while on the other hand FQ is excluded by BQ. F thus fails to obey one of the typical properties of negation, vz. a form of the Law of Contraposition, according to which, if $\Pi$ implies the negation of R then R implies the negation of $\Pi$. The failure of this principle is indicative of an asymmetry between certain predicates, e.g. between Q and BQ, which is not only central to the ideas that led to the formulation of the $L_i'$ but which has at least been partly retained by the $L_i''$ : Whereas BQ is treated as indicating change from FQ to Q - which entails that it excludes FQ as well as Q - Q is not treated as marking a transition to or from a situation where BQ is satisfied; rather, BQ is introduced as a predicate which can never be satisfied when Q is: Q(a) is taken as conclusive evidence that a fails to satisfy BQ, in the strong sense expressed by (FBQ)(a). A similar asymmetry is found between, e.g., BQ and BBQ: BQ implies FBBQ, but FBQ is incompatible with BBQ; and so on.

In the preceding paragraph I use the phrase 'B$\phi$ is *introduced* as a predicate ...'. This locution suggests a way of looking at the languages $L_i''$ which the presentation I gave did not make fully explicit, vz. as the limits of sequences of ever stronger languages: $L_i' \subset L_i^2 \subset \ldots \subset L_i^n \subset \ldots$ , where $L_i^n$ contains only those predicates of $L_i''$ which consist of a primitive predicate preceded by at most n operators. The predicates which contain exactly n operators are those which express true changes in $L_i^n$. In $L_i^{n+1}$ these same predicates come to function as expressions for conditions which cannot turn into their contraries without an intervening change; these intervening changes are then expressed with the help of the new predicates of $L_i^{n+1}$.

Thus conceived the transition from $L_i^n$ to $L_i^{n+1}$ might be expected to involve not only a change in status of these predicates but also a modification of the way in which they interact with negation. Thus we might expect that in, say, $L_i^2$ the predicate BQ should be taken to exclude Q in a stronger sense than that in which it excludes that predicate in $L_i'$ and in which for instance BBQ excludes BQ in $L_i^2$ . We might contemplate the possibility of using F to express this stronger sense in which BQ excludes Q in $L_i^2$. This policy would lead in the limit represented by $L_i''$ to the situation that whenever a predicate $\Pi$ excludes a predicate R in the weak sense (i.e. for all a if $\Pi$(a) then $\neg$R(a)) it also excludes R in the strong sense (i.e. $\Pi$ implies FR).

Thus F has, where applicable, the same force as ¬, and would thereby become redundant. The resulting languages are simpler than the $L_i''$. They are distinguished from the underlying languages $L_i$ only by the greater diversity of their predicates, and the logical relations which hold between these.

The event models for $L_i''$ differ from those for $L_i'$ in that the former must, for any two events $e_1 \propto e_2$ characterized by, respectively, (FQ)(a) and Q(a), have infinitely many intervening events, while the latter are required to have only one event that intervenes. By the same token a model for $L_i^2$ should have at least three intervening events, characterized by (BBQ)(a), (BQ)(a) and (CBQ)(a); and similarly in a model for $L_i^n$ there ought to be at least $2^n-1$ such events. Moreover, just as PDC is a natural condition on models for $L_i'$ and its counterpart (18) a natural requirement to be imposed on models for $L_i''$, so we may ask of the models for $L_i^n$ that each event characterized by a sentence of the form (BQ)(a) lies between two events characterized by (FQ)(a) and Q(a), between which there are precisely the $2^n-1$ events which the preceding condition demands. Let us refer to this condition as $PDC^n$(thus $PDC^1$ is the condition PDC itself).

Note that if $\mathfrak{M}$ is an event model for $L_i^n$ which satisfies $PDC^n$ than there is an essentially unique[17] minimal extension $\mathfrak{M}'$ of $\mathfrak{M}$ which is a model for $L_i^{n+1}$ and which satisfies $PDC^{n+1}$.

Up to this point we have considered, in connection with a given event model $\mathfrak{M}$, only instant structures which derive via Wiener's construction from completions of $\mathfrak{M}$. This carries the suggestion that the structural features of time, as determined by the model $\mathfrak{M}$, which in turn reflects a particular conceptual frame, are precisely those which all these instant structures have in common. In section 2 I already remarked, when I mentioned the issue whether time is order complete, that this fails to account for the possibility of attributing to time properties that some, or even all, of the instant structures lack. To do justice to that possibility within the general framework developed so far we must consider other ways in which a given event model $\mathfrak{M}$ can determine instant models.

The connection between the languages $L_i'$ and $L_i''$ may be exploited to provide one such alternative (albeit not one which will provide us with instant structures that are order complete). For let $\mathfrak{M}$ be an event model for $L_i'$

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17 By 'essentially unique' I mean that any two such models are isomorphic, i.e. there is a 1-1 map from the event set of the first to that of the second which preserves $\propto$, O and the functions $\mathcal{A}_1$ and $\mathcal{A}_2$.

which satisfies PDC. Then there is a (n essentially) unique sequence $\mathfrak{M}' = \mathfrak{M}, \mathfrak{M}^2, \ldots, \mathfrak{M}^n, \ldots$ of models $\mathfrak{M}^n$ for $L_i^n$, each $\mathfrak{M}^{n+1}$ determined in the obvious way by its predecessor $\mathfrak{M}^n$. The limit of this sequence (limit in a sense evident enough to obviate the need of a formal definition) is a model $\mathfrak{M}''$ for $L_i''$ which satisfies (18) and is complete in the sense of satisfying (17).

In this way we can associate with a given model $\mathfrak{M}$ for $L_i'$ a set of event models $\mathfrak{M}''$ for $L_i''$ as the set of limits of sequences generated by completions of $\mathfrak{M}'$.[18]

As we introduced them in section 3 completions of event models for $L_i'$ represent hypothetical situations in which all indeterminacies have been resolved. The relation between $L_i'$ and $L_i''$ suggests another type of hypothetical situation in which not only the predicates of $L_i'$ have become precise but in which moreover new predicates, those of $L_i''$, have been added, and thereby the means of identifying events enlarged.

The models that represent these situations must contain also the events identifiable only with the help of the extended vocabulary. In as much as the expressive capacities of $L_i''$ are understood as being already implicit in $L_i'$ it would seem natural to concentrate on such models rather than on the completions with which we worked up to now. Moreover, it seems reasonable to assume that the extra vocabulary of $L_i''$ is inextricably connected with the existence assumptions which led us to introduce that vocabulary in the first place. Thus the models which are to replace the earlier completions may be assumed to satisfy (18), and in fact to be the unique minimal extentions, in the sense explained above, of these completions. In this way we come to the following alternative notion of an interpretation for $L_i'$ - to avoid confusion I shall refer to such interpretations as interpretations$_2$. Each *interpretation*$_2$ is determined by an interpretation $\langle \mathfrak{M}, \mathcal{S} \rangle$ for $L_i'$ . The interpretation$_2$ which $\langle \mathfrak{M}, \mathcal{S} \rangle$ determines is the pair $\langle \mathfrak{M}, \mathcal{S} \rangle$ where $\mathcal{S}$ is the set of all models for $L_i''$ which are minimal extensions, in the sense explained above, of members of $\mathcal{S}$ . Clearly each class $\mathcal{K}$ of interpretations for $L_i'$ determines a corresponding class $\mathcal{K}'$ of interpretation$_2$'s. Whoever is persuaded that the iterated construction that led to the languages $L_i''$ and their models is an intrinsic part of the ideas already embodied in $L_i'$ should study metamathematical notions

18 Alternatively we could associate with $\mathfrak{M}$ itself, in an analogous fashion, one or more models $\mathfrak{M}''$ for $L_i''$ and then consider the completions of these. As far as I can see, however, the effect of this way of proceeding is the same, while it is more difficult to state.

such as validity relative to such classes $\mathcal{K}'$ rather than to the classes of interpretations from which they derive.

In general where $\langle \mathfrak{M}, \mathcal{S}' \rangle$ is an interpretation$_2$ for $L_i'$ the members of $\mathcal{S}'$ will contain many more events than $\mathfrak{M}$ itself. Indeed, if $\langle \mathfrak{M}, \mathcal{S}' \rangle$ is derived from the interpretation $\langle \mathfrak{M}, \mathcal{S} \rangle$ and the members of $\mathcal{S}$ satisfy PDC then the instant structures derived from the members of $\mathcal{S}'$ are dense, while those derived from the members of $\mathcal{S}$ could even be finite.[19]

There are various alternatives to the semantics we have just developed. Thus we might consider interpretations $\langle \mathfrak{M}, \mathcal{S} \rangle$ for $L_i'$ where the members of $\mathcal{S}$ are reductions to $L_i'$ of models for some language $L_i^n$, with $n > 1$. I doubt that such interpretations will be of more than a purely technical interest, however. On the other hand we might study interpretations $\langle \mathfrak{M}, \mathcal{S}' \rangle$ for a language $L_i^n$ where $\mathfrak{M}$ is a model for $L_i^n$ and the members of $\mathcal{S}'$ are reductions to $L_i^n$ of models for $L_i''$. This kind of interpretation promises to have some conceptual significance. A natural language such as English provides a great variety of predicates, some simple and some complex, for expressing change, and some of these express changes in conditions expressed by other predicates. It is a feature typical of natural languages, and one which is often ignored in attempts to analyse aspects of natural discourse through modelling by formal systems, that devices used to form complex expressions out of simpler

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19 According to the formal definition (10) of an interpretation for $L_i'$ the members of $\mathcal{S}$ may contain more events than $\mathfrak{M}$ itself; in particular, where $\langle \mathfrak{M}, \mathcal{S} \rangle$ is the interpretation$_2$ derived from $\langle \mathfrak{M}, \mathcal{S} \rangle$ then according to (10) $\langle \mathfrak{M}, \mathcal{S}' \rangle$ is itself an interpretation for $L_i'$. There is a difficulty here to which I might have drawn attention earlier, but which we cannot ignore any longer now. The informal explanation that I gave of a completion might be thought to imply that if $\mathfrak{M}'$ is a completion of the event model $\mathfrak{M}$ then the event set E' is identical with E. This need not be so, however. For (an observation made by David Lewis) it is conceivable that because of the vagueness in some predicate Q it is undetermined whether the object a actually moved out of the extention of that predicate at one time, and back into it at a subsequent time, or whether it remained inside the extension throughout. This means that on some resolutions of the vagueness of the predicate there will be two changes, from and to Q respectively, while in others there will be none. Thus the event set of the model $\mathfrak{M}$ representing such a situation cannot be identical with the event set of each of its completions.

It would seem natural to maintain that in such a case the changes are not part of the event model $\mathfrak{M}$ itself, so that among its completions some will contain more events, but none fewer. Although it seems intuitively clear from this account that as a rule the models for $L_i''$ which extend completions of $\mathfrak{M}$ should not themselves qualify as completions of $\mathfrak{M}$, I do not quite know how to sharpen the formal characterization of what a completion is in such a way as to exclude them. I must leave this issue here unresolved.

ones cannot be iterated indefinitely. Certainly some, and possibly all of the devices available in English to transform a given predicate into another which expresses change with respect to the condition expressed by the first have this feature. It seems quite possible therefore that for English, as well as many other natural languages, there is an upper bound to the degree of the predicates it contains - where the *degree* of a predicate is characterized as follows:

A predicate which does not express a change with respect to a condition expressible by any other predicate of the language has degree 0; if a predicate expresses change with respect to a condition expressed by a predicate of degree n then it is of *degree* n+1. If a given language has such a finite upper bound n then that language, or at least the fragment whose constructions other than those used to express change can be represented in $L_i$, can be regarded as corresponding to some part of $L_i^n$. It is thus the semantics for that language $L_i^n$, rather than that for $L_i^!$ , that would be relevant to the study of change as expressible in that natural language.

Whether natural languages admit of such upper bounds, and what these are, are empirical questions, to which I intend to address myself in Part II.

*V.* I should have liked to think that the way in which we arrived at the conclusion that instant models are densely ordered reveals the roots of the prevailing conviction that density is an intrinsic feature of the structure of time. But I am afraid that such thinking would be altogether too wishful.

There is a difficulty connected with the languages $L_i^!$ to which I must now, belatedly, draw attention: if i) there are changes intervening between times where a satisfies Q and times where it fails to satisfy Q; ii) there are means of characterizing these changes; and iii) the resulting characterization is incompatible with both the first and the second of these two conditions; then the first two conditions were not really contradictiories, but only incompatible: the very idea of a change between *contradictory* conditions which is describable as distinct from either of these appears to be incoherent.

I believe we can escape this conclusion by distinguishing between such predicates as Q and FQ on the one hand and such predicates as BQ and CQ on the other; Q and FQ, although both incompatible with the predicates BQ and CQ, would nevertheless remain contradictories, in as much as there are no predicates of the type to which Q and FQ themselves belong which are incompatible with both of them - that they should be incompatible with certain predicates expressing change with respect to the very conditions they express themselves is not to be taken to disprove that they are contradictory predicates. Note that this amounts to a reassessment of the concept of contradictoriness (the need for which I already announced in the footnote to p.131). I shall not go into a defense of this particular suggestion for revision here, but just make the following observation: The *only* way to maintain that Q and FQ are contradictories is to classify BQ and CQ as predicates of a different sort from that to which Q and FQ themselves belong, and to argue that predicates of this new sort are not relevant to the question whether predicates of the other type are contradictories or only incompatible.

Once we introduce such a subclassification of predicates, however, it becomes clear that we must distinguish between what I termed in the preceding section the 'first' and 'second' hypothesis: the hypothesis that there is a time of transition between a time where a satisfies Q and a time where a fails to satisfy Q, is distinct from the hypothesis that there is a time of transition between an event characterized as (BQ)(a) and an event characterized by the 'contradictory' condition (FBQ)(a). And precisely because the two categories of predicates are supposed to play fundamentally distinct roles we cannot assume that the second hypothesis is entailed by the first. Nor is there any entailment between the second hypothesis and the third, according to which there are times of change between e.g. events characterized as (CBQ)(a) and times where the 'contradictory' condition obtains. For to maintain that BQ and FBQ are contadictories we must classify predicates such as CBQ as belonging to yet a third category. Indeed, as I already indicated in section 4, all the hypotheses which thus arise are independent of each other; and the infinite progression which leads from $L_i'$ to $L_i''$ could, for all I can see, be interrupted at any point.

The need for such a progression - and therewith the conclusion that time is dense, which in the argument of the preceding section, depends on it - would be a good deal more compelling if there were grounds for assuming that

> (20) each change can be characterized - or is temporally included in an event that can be characterized - with the help of predicates that are recognized from the outset as belonging to precisely the same category as those used to characterize the conditions between which that change is a change.

In case (20) holds the very same hypothesis will keep asking, again and again, for interpolating changes, which, in the limit, must lead to the event models which we have come to associate with the languages $L_i''$.

Note that in order that the instant structure derived from an event model be densely ordered it is sufficient that there be *some* group of predicates for which i) (20) is true; and ii) changes characterizable with the help of predicates of this group are always going on (i.e. if $e_1$ and $e_2$ are any two successive events then either one of them is a change of this sort or else they are separated by such a change.)

I believe there is such a group of predicates, vz. those used to express motion. Motion is change of position; and each individual motion of a particular body a is a change from, or to, a condition where a occupies some particular position p, to, or from a condition where a is not in that position. It is a common assumption, however, that: Whenever a is *not* in position p then it is in some position *other* than p. We assume, that is, that there is a class of predicates, which I shall represent for the sake of the present discussion as $(\lambda x)L(x,p)$, $(\lambda x)L(x,p'),\ldots$, such that whenever an object a fails to satisfy one of them it must satisfy one of the others which is incompatible with the first. It is an equally common assumption that whenever $t_1$ and $t_2$ are successive times during which a is in, respectively, positions $p_1$ and $p_2$, there must be an intermediate event e which constitutes the transition from the first position to the second; while this event is going on a is neither in position $p_1$ nor in position $p_2$, and thus must, by the first assumption, be in a different position. Let e' be the event of a's being in some one such intermediate position $p_3$. e' must be temporally included in the event e. The very same hypothesis that led us to the assumption that there must be such an event as e now leads to the conclusion that there must be an event separating e' from the time $t_1$, and also one which separates e' from $t_2$; and so on.

Note, however, that this time our departure from the restricted notion of change between contradictories is irrevocable. The condition that a is in position $p_1$ is *not* the contradictory of the condition that a is in position $p_2$, for both of these are incompatible with the condition that a is in position $p_3$, and this last condition is of precisely the same kind as the former two. A proper analysis of the concept of motion therefore requires a more general concept of change than we have studied in this first part of the paper - the concept of an event which is a change between conditions which are incompatible but which need not be contradictories (in any sense of the word which distinguishes it from incompatibility tout court). The theory of change of this more general type will be one of the main topics of part II.

I shall conclude this part with a few short remarks, to be seen merely as prognoses of what will be discussed in the second installment of this paper.

1) Clearly the changes between conditions that are not contradictories are not in general instantaneous - in fact the short discussion of motion already bore this out, vz. where I observed that a change in a from position $p_1$ to position $p_2$ might temporally include (and of course it *must properly* include) the event of a's being in position $p_3$. The circumstance that such changes are often protracted causes certain metamathematical difficulties, which do not arise in connection with instantaneous changes. It has been partly out of a desire to avoid these complications that I deferred the treatment of the more general notion of change for what might appear, in the light of the discussion with which I began this section, an inexcusably long time.

2) I have failed to deal with the question of change with respect to conditions expressible with the help of complex formulae, built from atomic predicates with the help of truthfunctional connectives, quantifiers, tense operators, or possibly other devices which this paper has not even mentioned. It is in particular in this context that protracted changes are particularly troublesome and lead (I think inevitably) to a partial valued semantics - the sort of semantics, that is, the mathematical theory of which is still very poorly developed.

3) Even after we have accepted changes between conditions which are not contradictories, and thus protracted changes, as primitives, it remains an important question how they relate to the ideally instantaneous changes first discussed in section 2. At the end of section 1 I suggested that protracted changes might be analyzed as sequences of instantaneous changes. Whether, and how, that suggestion could be made good is a question which I must also defer to part II. Here I only wish to draw attention to a much more modest, but related, claim, vz. the assumption that

> (21) Each change e from a given condition p to some incompatible condition temporally includes a change from p to the condition that is the contradictory of p.

In one sense (21) is incontrovertible. Surely when there is a transition from p to some other condition incompatible with p then there is also a transition from p to its contradictory. But it does not automatically follow from this that the event which constitutes the first transition must be accompanied by (or, casu quo, be identical with) an *event* which is the change from p to its contradictory. Indeed, our discussion of motion suggested that this implication might be denied in connection with motion predicates: that all changes are changes between satisfaction of one such precicate and satisfaction of another which,though incompatible with the first, is not its contradictory, while the transitions from satisfaction of such a predicate to failure to satisfy it are in some, or even all, cases 'virtual' - in the sense of there being no time at which the transition occurs. (Whether this is indeed one of the assumptions implicit in our conception of motion is a question to which I shall address myself in part II.). I should say in this connection that it may well be that different predicates require different analyses of the nature of instantaneous and protracted changes with respect to the conditions that these predicates express, and of how these two sorts of changes are connected. E.g. what we wish to say about motion may well differ substantially from what we would regard as the appropriate theory of change of colour, or pitch.

This brings me to the last major issue which must eventually be discussed but about which too I have been almost entirely silent: the means of expressing change that are available in natural languages, in particular English. This is a complex problem, and the little that I may have achieved in this first part of the paper can at best serve as a formal setting in which various aspects of that problem might be more fruitfully discussed than would be possible without it.[20]

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20 Part of the research for this paper was done while the author was supported by DFG Projekt 'Die Beschreibung der Zeitlogik von Zeitformen und Verbalperiphrasen im Französischen, Portugiesischen und Spanischen', led by Chr. Rohrer, University of Stuttgart.

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