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Johan van BENTHEM
University of Amsterdam & Stanford University

Alice ter MEULEN
Indiana University, Bloomington, Indiana

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CHAPTER 3

Representing Discourse in Context*

Jan van Eijck

CWI, PO Box 94079, 1090 GB Amsterdam, The Netherlands
E-mail: jve@cwi.nl

Hans Kamp

Universität Stuttgart, Institut für Maschinelle Sprachverarbeitung, Azenbergstrasse 12, D-70174 Stuttgart, Germany
E-mail: hans@ims.uni-stuttgart.de

Commentator: Kees Vermeulen

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1. Introduction

The key idea behind the theory of the semantics of coherent multi-sentence discourse and text that is presented in this chapter – Discourse Representation Theory, or DRT for short – is that each new sentence S of a discourse is interpreted in the context provided by the sentences preceding it. The result of this interpretation is that the context is updated with the contribution made by S; often an important part of this process is that anaphoric elements of S are hooked up to elements that are present in the context. An implication of this conception of text interpretation is that one and the same structure serves simultaneously as content and as context – as content of the sentences that have been interpreted already and as context for the sentence that is to be interpreted next. This double duty imposes special constraints on logical form, which are absent when, as in most older conceptions of semantics and pragmatics, the contents and contexts are kept separate.

The initial problem that motivated the present theory is the interpretation of nominal and temporal anaphora in discourse. The key idea in the way of thinking about the semantics of discourse in context exemplified in (Heim, 1982) and (Kamp, 1981) is that each new sentence or phrase is interpreted as an addition to, or 'update' of, the context in which it is used and that this update often involves connections between elements from the sentence or phrase with elements from the context.

In the approach of Kamp (1981), which we will follow more closely here than the largely equivalent approach of Heim (1982), this idea is implemented in the form of interpretation rules – each associated with a particular lexical item or syntactic construction. When applied to a given sentence S, these rules identify the semantic contributions which S makes to the context C in which S is used and add these to C. In this way C is transformed into a new context, which carries the information contributed by S as well as the information that was part of the context already. The result can then serve as context for the interpretation of the sentence following S (in the given discourse or text), which leads to yet another context, and so on until the entire discourse or text has been interpreted.

An important aspect of this kind of updating of contexts is the introduction of elements – so-called reference markers or discourse referents – that can serve as antecedents to anaphoric expressions in subsequent discourse. These reference markers play a key part in the the context structures posited by DRT, the so-called Discourse Representation Structures or DRSs.

With its emphasis on representing and interpreting discourse in context, discourse representation theory has been instrumental in the emergence of a *dynamic perspective* on natural language semantics, where the center of the stage, occupied so long by the concept of truth with respect to appropriate models, has been replaced by context change conditions, with truth conditions defined in terms of those. Thus, under the influence of discourse representation theory, many traditional Montague grammarians have made the switch from static to dynamic semantics (see the Chapter on *Dynamics* in this Handbook). This shift has considerably enriched the enterprise of formal semantics, by bringing areas formerly belonging to informal pragmatics within its compass.

In the next section we will first look at some examples of DRSs and at the considerations which have led to their specific form. After that we will look more closely at the relationship between DRSs and the syntactic structure of sentences, discourses or texts from which they can be derived. This will lead us naturally to the much debated question whether the theory presented here is compositional. The compositionality issue will force us to look carefully at the operations by means of which DRSs can be put together from minimal building blocks. Next we will show, by developing a toy example, what a compositional discourse semantics for a fragment of natural language may look like. This is followed by sample treatments of quantification, tense and aspect. The chapter ends with some pointers to the literature on further extensions of the approach and to connections with related approaches.

2. The problem of anaphoric linking in context

The semantic relationship between personal pronouns and their antecedents was long perceived as being of two kinds: a pronoun either functions as an individual constant coreferential with its antecedent or it acts as a variable bound by its antecedent. However, in the examples (1)–(4) below, neither of these two possibilities seems to provide a correct account of how pronoun and antecedent are related.

(1) A man^1 entered. He$_1$ smiled.

(2) Every man who meets a nice woman^1 smiles at her$_1$.

(3) If a man^1 enters, he$_1$ smiles.

(4) Hob believes a witch^1 blighted his mare. Nob believes she$_1$ killed his sow.

In these examples we have used subscripts and superscripts to coindex anaphoric pronouns and their intended antecedents.

The first option – of pronoun and antecedent being coreferential – does not work for the simple reason that the antecedent does not refer (as there is no one particular thing that can be counted as the referent!); so a fortiori antecedent and pronoun cannot *co*refer (that is, refer to the same thing). The second option, the bound variable analysis, runs into problems because the pronoun seems to be outside the scope of its antecedent. For instance, in (1) the antecedent of the pronoun is an indefinite noun phrase occurring in the preceding sentence. In the approaches which see pronouns as either coreferring terms or bound variables, indefinite NPs are viewed as existential quantifiers whose scope does not extend beyond the sentence in which they occur. In such an approach there is no hope of the pronoun getting properly bound. Examples (2)–(4) present similar difficulties. Example (2) is arguably ambiguous in that *a nice woman* may be construed either as having wide or as having narrow scope with respect to *every man*. If *a nice woman* is construed as having narrow scope, i.e. as having its scope restricted to the relative clause, then the pronoun won't be bound; the phrase can bind the pronoun if it is given wide scope, as in that case its scope is the entire sentence, but this leads to an interpretation which, though perhaps marginally possible, is clearly not the preferred reading of (2). We find much the same problem with (3): in order that the indefinite *a man* bind the pronoun *he*, it must be construed as having scope over the conditional as a whole, and not just over the if-clause; but again, this yields a reading that is marginal at best, while the preferred reading is not available.

Sentences with the patterns of (2) and (3) have reached the modern semantic literature through Geach (1980), who traces them back to the Middle Ages and beyond. Geach's discussion revolves around examples with donkeys, so these sentences became known in the literature as *donkey sentences*. Also due to Geach are sentences like (4), which pose a binding problem across a sentential boundary, complicated by the fact that antecedent and anaphoric element occur in the scopes of different attitude predications, with distinct subjects.

Problems like the ones we encountered with (1)– (4) arise not just with pronouns. There are several other types of expressions with anaphoric uses that present essentially the same difficulties to the traditional ways of viewing the relationship between natural language and logic. First, there are other anaphoric noun phrases besides pronouns, viz. definite descriptions and demonstratives; and these also occur in the contexts where the problems we have just noted arise. Moreover, as was remarked already more than twenty years ago in (Partee, 1973), there are striking similarities in the behaviour of anaphoric pronouns and tenses, and it turns out that the interpretation of tense involves the same sort of anaphoric dependencies which (1)–(4) exhibit. More precisely, the past tense is often to be understood as referring to some particular time in the past (rather than meaning 'sometime in the past') and more often than not this particular time is to be recovered from the context in which the given past tense sentence is used.

(5) John entered the room. He switched on the light.

(6) Whenever John entered the room, he switched on the light.

In (5) the switching time is understood as temporally related to the time at which John entered the room (presumably the time of switching was directly after the time of entering) and a full interpretation of (5) needs to make this explicit. A quantificational sentence such as (6) suggests the same relationship between switching times and entering times; and insofar as the tense of the main clause is to be interpreted as anaphoric to that of the whenever-clause, this anaphoric connection raises the same questions as those of (2) and (3).

3. Basic ideas of discourse representation

The central concepts of DRT are best explained with reference to simple examples such as (1) in the previous section. The logical content of (1) appears to be that there was some man who entered and (then) smiled. That is, the content of (1) is what in standard predicate logic would be expressed by an existential quantification over material coming in part from the first and in another part from the second sentence of (1), roughly as in (7).

(7) $\exists x(\text{man } (x) \wedge \text{entered } (x) \wedge \text{smiled } (x))$.

As observed in the last section, according to DRT the interpretation of (1) results from a process in which an interpretation is obtained for the first sentence, which then serves as context for the interpretation of the second sentence. The interpretation of the second sentence transforms this context into a new context structure, the content of which is essentially that of (7).

The problem with (1) is that the first sentence has an existential interpretation and thus must in some way involve an existential quantifier, and that the contribution which the second sentence makes to the interpretation of (1) must be within the scope of that quantifier. Given the basic tenets of DRT, this means that (i) the first sentence of (1) must get assigned a representation, i.e. a DRS, K_1 which captures the existential interpretation of that sentence; and (ii) this DRS K_1 must be capable of acting as context for the interpretation of the second sentence in such a way that this second interpretation process transforms it into a DRS K_2 representing the truth conditions identified by (7). (i) entails that the reference marker introduced by the indefinite NP *a man* – let it be x – must get an existential interpretation within K_1; and (ii) entails that it is nevertheless available subsequently as antecedent for the pronoun *he*. Finally, after x has been so exploited in the interpretation of the second sentence, it must then receive once more an existential interpretation within the resulting DRS K_2.

Heim (1982) uses the metaphor of a filing cabinet for this process. The established representation structure K_1 is a set of file cards, and additions to the discourse effect a new structure K_2, which is the result of changing the file in the light of the new information. Here is how DRT deals with these desiderata. The DRS K_1 is as given in (8).

(8)

x
man x entered x

This can also be rendered in canonical set-theoretical notation, as in (9).

(9) $(\{\mathrm{x}\}, \{\mathrm{man\ x}, \mathrm{entered\ x}\})$.

Precisely how this DRS is derived from the syntactic structure of the first sentence of (1), and how DRS construction from sentences and texts works generally is discussed in Section 9. For now, suffice it to note that the reference marker x gets introduced when the NP *a man* is interpreted and that this interpretation also yields the two conditions *man(x)* and *entered(x)*, expressing that any admissible value a for x must be a man and that this man was one who entered.

A DRS like (8) can be viewed as a kind of 'model' of the situation which the represented discourse describes. The modeled situation contains at least one individual a, corresponding to the reference marker x, which satisfies the two conditions contained in (8), i.e. a is a man and a is someone who entered.

When a DRS is used as context in the interpretation of some sentence S, its reference markers may serve as antecedents for anaphoric NPs occurring in S. In the case of our example we have the following. (8), serving as context for the second sentence of (1), makes x available as antecedent for the pronoun *he*. That is, the interpretation of *he* links the reference marker it introduces, y say, to the marker x for the intended antecedent, something we express by means of the equational condition $y \doteq x$. In addition, the interpretation step yields, as in the case of the indefinite *a man*, a condition expressing the clausal predication which involves *he* as argument. Through the application of this principle (8) gets expanded to the DRS (10), which represents the content of all of (1).

(10)

x y
man x enter x $y \doteq x$ smiled y

DRS (10) models situations in which there is at least one individual that is a man, that entered and that smiled. It is easy to see that these are precisely the situations which satisfy the predicate formula (7). (This claim will be made formal by the model theory for DRSs, to be presented in Section 4.)

As illustrated by the above examples (8) and (10), a DRS generally consists of two parts, (i) a set of reference markers, the universe of the DRS, and (ii) a set of conditions, its condition set. There are some other general points which our example illustrates:

(i) The reference markers in the universe of a DRS all get an existential interpretation;

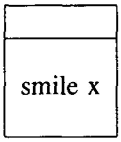
(ii) All reference markers in the universe of a context DRS are available as anaphoric antecedents to pronouns and other anaphoric expressions that are interpreted within this context;

(iii) The interpretation of a sentence S in the context provided by a DRS K results in a new DRS K', which captures not only the content represented by K but also the content of S, as interpreted with respect to K.

It should be clear that DRSs such as (8) and (10) can only represent information that has the logical form of an existentially quantified conjunction of atomic predications. But there is much information that is not of this form. This is so, in particular, for the information expressed by (3). So the DRS for (3) will have to make use of representational devices different from those that we have used up to this point.

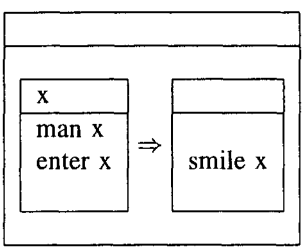
The DRT conception of conditional information is this. The antecedent of a conditional describes a situation, and the conditional asserts that this situation must also satisfy the information specified in its consequent. When conditionals are seen from this perspective, it is not surprising that the interpretation of their consequents may use the interpretations of their antecedents as contexts much in the way the interpretation of a sentence S may build upon the interpretation assigned to the sentences preceding it in the discourse to which it belongs; for the consequent extends the situation description provided by the antecedent in essentially the same way in which S extends the situation described by its predecessors.

In the case of (3) this means that the DRS (8), which represents its antecedent (see the discussion of (1) above), can be exploited in the interpretation of the consequent, just as (8), as interpretation of the first sentence of (1), supported the interpretation of the second sentence of (1). To make this work out, we need a suitable representation for the consequent. This turns out to be (11).

(11)


To obtain a representation of (3), (8) and (11) must be combined in a way which reveals the conditional connection between them. We represent this combination by a double arrow in between the two DRSs. The result $K \Rightarrow K'$, where K and K' are the two DRSs to be combined, is a DRS condition (a complex condition as opposed to the simple DRS conditions we have encountered so far). The DRS for a conditional sentence such as (3) will consist just of such a condition and nothing else.

Intuitively the meaning of a condition $K \Rightarrow K'$ is that a situation satisfying K also satisfies K'. This is indeed the semantics we adopt for such conditions (for details see Section 4). Applying this to the case of (3) we get the representation (12).

(12)


Conditions of the form $K \Rightarrow K'$ illustrate an important feature of DRT: The logical role played by a reference marker depends on the DRS-universe to which it belongs. Markers belonging to the universe of the main DRS get an existential interpretation – this is, we saw, a consequence of the principle that a DRS is true if it is possible to find individuals corresponding to the reference markers in the DRS universe which satisfy its conditions. This principle, however, applies only to the reference markers in the main DRS universe. The logic of reference markers in subordinate universes, such as for instance x in (12), is determined by the principles governing the complex DRS conditions to which they belong. Thus the semantics of conditions of the form $K \Rightarrow K'$ implies that for all individuals corresponding to reference markers in the universe of K which satisfy the conditions of K it is the case that K' is satisfiable as well. Thus the $\Rightarrow$-condition of (12) has the meaning that for every individual corresponding to the marker x – that is, for every man that enters – the right hand side DRS of (12) is satisfied, i.e. that individual smiles. Reference markers in the left hand side universe of an $\Rightarrow$- condition thus get a universal, not an existential interpretation.

It is worth noting explicitly the ingredients to this solution of the semantic dilemma posed by conditionals like (3). Crucial to the solution are:

(i) the combination of the principles of DRS construction, which assign to conditional sentences such as (3) representations such as (12), and

(ii) the semantics for $\Rightarrow$-conditions that has just been described.

Like any other DRS, (12) is a pair consisting of a set of reference markers and a set of conditions. But in (12) the first of these sets is empty. In particular, the reference marker x which does occur in (12) belongs not to the universe of the 'main' DRS of (12) but to that of a subordinate DRS, which itself is a constituent of some DRS condition occurring in (12). One important difference between reference markers in such subordinate positions and those belonging to the universe of the main DRS is that only the latter are accessible as antecedents for anaphoric pronouns in subsequent sentences. In general, in order that a reference marker can serve as antecedent to a subsequent pronoun, it must be accessible from the position that the pronoun occupies. Compare for instance the discourses (13) and (14).

(13) A man came in. He smiled. He was holding a flower in his right hand.

(14) If a man comes in, he smiles. ?He is holding a flower in his right hand.

While in (13) the second *he* is as unproblematic as the first *he*, in (14) the second *he* is hard or impossible to process. This difference is reflected by the fact that in the DRS for the first two sentences of (13) the reference marker for *a man* belongs to the universe of the main DRS and so is accessible to the pronoun of the last sentence, whereas in (14) this is not so.

The rules for processing sentences in the context of a representation structure impose formal constraints on availability of discourse referents for anaphoric linking. The set of available markers consists of the markers of the current structure, plus the markers of structures that can be reached from the current one by a series of steps in the directions *left*, (i.e. from the consequent of a pair $K \Rightarrow K'$ to the antecedent), and *up*, (i.e. from a structure to an encompassing structure).

For universally quantified sentences such as (2) DRT offers an analysis that closely resembles its treatment of conditionals. According to this analysis a universally quantifying NP imposes a conditional connection between its own descriptive content and the information expressed by the predication in which it participates as argument phrase; and this connection is interpreted in the same way as the $\Rightarrow$-conditions that the theory uses to represent conditional sentences. In particular, (2) gets an analysis in which any individual satisfying the descriptive content *man who meets a nice woman*, i.e. any individual corresponding to the reference marker x in the DRS (15), satisfies the DRS representing the main predication of (2). According to this way of looking at quantification, the descriptive content of the quantifying phrase can be taken as presupposed for purposes of interpreting the predication in which the phrase partakes, just as the antecedent of a conditional can be taken as given when interpreting its consequent. Thus, just as we saw for the consequent of the conditional (3), the construction of the DRS for the main predication of (2) may make use of information encoded in the 'descriptive content' DRS (15). The result is the DRS in (16).

(15)

x y
man x woman y nice y meet (x,y)

(16)

u
$u \doteq y$ smiles-at (x,u)

To get a representation of (2), DRSs (15) and (16) have to be combined into a single DRS condition. It is clear that $\Rightarrow$ has the desired effect. The result is (17).

(17)

x y		u
man x woman y nice y meet (x,y)	$\Rightarrow$	$u \doteq y$ smiles-at (x,u)

The constraints on marker accessibility are used to account for the awkwardness of anaphoric links as in (18).

(18) *If every man[1] meets a nice woman[2], he_1 smiles at her_2.

The difference between pronominal anaphora and the variable binding we find in classical logic is also nicely illustrated by anaphora involving the word *other*. Consider, e.g., (19).

(19) A man walked in. Another man followed him.

Here *another man* is anaphoric to *a man*, but the sense is that the two men should be different, not that they are the same. In other words, while any phrase of the form *another CN* must, just as an anaphorically used pronoun, find an antecedent in its context of interpretation, the semantic significance of the link is just the opposite here. The DRS for (19) is (20).

(20)

x y z
man x walk-in x $y \neq x$ man y $z \doteq x$ follow (y,z)

Note that the representation of *other*-anaphora always needs two reference markers, one introduced by the anaphoric NP itself and one for the antecedent; there is no question here of replacing the former marker by the latter (that is: eliminating the y at the top of (20) and the inequality $y \neq x$ and replacing the other occurrences of y by x), as that would force the two men to be the same, rather than different. In this regard *other*-anaphora differs from pronoun anaphora, for which the substitution treatment yields representations that are equivalent to the ones we have been constructing above.

One reason for preferring the treatment of pronoun anaphora we have adopted is that it brings out the similarity as well as the difference between pronouns and phrases with *other*: In both cases interpretation involves the choice of a suitable antecedent. But the 'links' between the chosen antecedent and the marker for the anaphoric NP are different in nature: they express equality in one case, inequality in the other.

We have said something about the interpretation of three kinds of NPs: indefinite descriptions, anaphoric pronouns and quantified NPs, and we have introduced *linking* as a central theme in DRT. More about quantification in Section 10. We will now briefly turn to definite descriptions. One of the most obvious facts about them, but a fact systematically ignored or played down in the classical theories of denoting phrases (Frege, 1892; Russell, 1905; Strawson, 1950), is that, like pronouns, definite descriptions often act as anaphoric expressions.

Indeed, there seems to be a kind of interchangeability in the use of pronouns and descriptions, with a description taking the place of a pronoun in positions where the latter would create an unwanted ambiguity; thus, in discourses like (21) the use of a definite description in the second sentence serves to disambiguate the intended anaphoric link.

(21) A man and a boy came in. The man/he(?) smiled.

Anaphoric definite descriptions are, like pronouns, linked to existing discourse referents, and thus, like pronouns, they impose certain conditions on the context in which they are used: the context must contain at least one discourse referent that can serve as an antecedent. In this sense both pronouns and anaphoric definite descriptions may be said to carry a certain presupposition: only when the context satisfies this presupposition is it possible to interpret the pronoun, or to interpret the description anaphorically. The descriptive content then serves as information to guide the anaphora resolution process. This will permit anaphora resolution in cases like (21).

Matters are not always this simple, however. Definite descriptions have uses that can hardly be described as anaphoric. For instance, in (22), the description *the street* is certainly not anaphoric in the strict sense of the word, for there is no antecedent part of the given discourse which has introduced an element that the description can be linked up with.

(22) A man was walking down the street. He was smiling.

It is argued in (Heim, 1982) that the use of a definite description is a means for the speaker to convey that he takes the referent of the description to be in some sense familiar. The hearer who is already acquainted with the street that is intended as the referent of *the* *street* by the speaker of (22) may be expected to interpret the description as referring to this street; in such cases speaker and hearer are said to *share a common ground* (see, e.g., Stalnaker, 1974) which includes the street in question, and it is this which enables the hearer to interpret the speaker's utterance as he meant it. Such common grounds can also be represented in the form of DRSs. Thus, the common ground just referred to will contain, at a minimum, a component of the form (23), where we assume that the marker u in (23) is *anchored* to a suitable object (the street that speaker and hearer have in mind).

(23)

u
street u

On the assumption of such a 'common ground DRS' (including a suitable anchor) it becomes possible to view the NP *the street* of (22) as anaphoric. Interpretation of (22) will then be relative to the context DRS (23) and the interpretation of its definite description will yield, by the same principle that governs the interpretation of *the man* in (21), a DRS like (24).

(24)

u x v y
street u man x $v \doteq u$ street v was-walking-down (x,v) $y \doteq x$ was-smiling y

This way of dealing with definite descriptions such as *the street* in (24) may seem to restore uniformity to the analysis of definites. An important difference between definite descriptions and pronouns remains, however. Definite descriptions can be linked much more easily than pronouns to objects that are implicit in the common ground, but have not been explicitly introduced by earlier parts of the same discourse.

To assimilate the use of definite descriptions as unique identifiers (the use that Frege and Russell focus on to the exclusion of all others) to the present anaphoric analysis one must allow for accommodation. When the context available to the hearer does not contain a representation of the referent of a definite description, he may accommodate this context so that it now does contain such a representation, and then proceed as if the representation had been there all along. However, under what conditions precisely accommodation is possible is still a largely unsolved problem.

Interesting cases where the anaphoric account and the unique identification account of definite description have to be combined are the so-called 'bridging descriptions', as in (25) and (26).

(25) (Yesterday) an M.P. was killed. The murderer got away.

(26) Usually when an M.P. is killed, the murderer gets away.

In (25) *the murderer* is naturally interpreted as referring to the murderer of the M.P. mentioned in the preceding sentence. In other words, the context provides a referent x, and the definite description is interpreted as *the unique individual who murdered x.* This account also works for (26), where x varies over murdered M.P.s, and the definite description ranges over the set of unique murderers for all those x.

We conclude with a brief remark on proper names. As has been emphasized in the philosophical literature (see in particular (Kripke, 1972)) a proper name has no descriptive content, or at any rate its descriptive content plays no essential part in the way it refers. One consequence of this is that a name cannot have more than one referential value (a point which should not be confused with the evident fact that many names – *Fred, Fido, John Smith, Fayetteville* – are many ways ambiguous). This means that a name cannot have the sort of anaphoric use which we found with *the murderer* in (25) and (26), and that the antecedent to which the reference marker for a name will have to be linked will always be a marker in the main universe of the context DRS. Logically speaking, therefore, a proper name will always have 'maximally wide scope'. One might think about this process in several ways. One might assume, as in the construction rule for proper names in (Kamp, 1981), that the processing of a proper name always leads to the introduction of a marker in the top DRS, even if the name gets processed in a subordinate DRS somewhere way down. Or one might assume an external element in the semantics of proper names, namely the presence of external anchors: reference markers that are already in place in the top box of a DRS. Any proper name, then, comes equipped with its fixed anaphoric index for linking the name to its anchor. This is the approach we will follow in Section 9.

4. Discourse representation structures

It is now time to turn to formal details. Let A be a set of constants, and U a set of reference markers or discourse referents (variables, in fact). We also assume that a set of predicate letters with their arities is given. In the following definition, c ranges over A, v over the set U, and P over the set of predicates.

DEFINITION 4.1 (*DRSs; preliminary definition*).

terms $t ::= v \mid c$.

conditions $C ::= \top \mid Pt_1 \cdots t_k \mid v \doteq t \mid v \neq t \mid \neg D$.

DRSs $D ::= (\{v_1, \ldots, v_n\}, \{C_1, \ldots, C_m\})$.

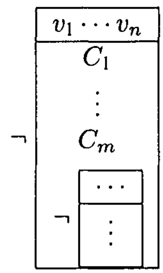
Note that this definition of the representation language is provisional; it will be modified in Section 6. We introduce the convention that $D_1 \Rightarrow D_2$ is shorthand for $\neg(\{v_1, \ldots, v_n\}, \{C_1, \ldots, C_m, \neg D_2\})$, where $D_1 = (\{v_1, \ldots, v_n\}, \{C_1, \ldots, C_m\})$.

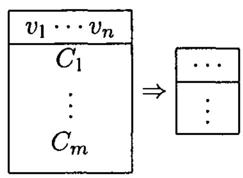
As in the previous sections DRSs will sometimes be presented in the box notation:

DRSs $D ::=$

$v_1 \cdots v_n$
C_1 $\vdots$ C_m

The abbreviation $D_1 \Rightarrow D_2$ is rendered in box format by the agreement to write (27) as (28).

(27) 

(28) 

Conditions can be *atoms*, *links*, or *complex conditions*. Complex conditions are negations or implications. As the implications are abbreviations for special negations, we can assume that all complex conditions are negations.

An atom is the symbol $\top$ or a predicate name applied to a number of terms (constants or discourse referents), a link is an expression $v \doteq t$ or $v \neq t$, where v is a marker, and t is either a constant or a marker. The clause for complex conditions uses recursion: a complex condition is a condition of the form $\neg D$, where D is a discourse representation structure.

We will first give a static truth definition for discourse representation structures. Later on, when discussing the problem of compositionality for DRSs, we turn to a context change formulation of those same conditions. Call a first order model $\mathcal{M} = \langle M, I\rangle$ (we assume the domain M is non-empty) an *appropriate* model for DRS D if I maps the n-place predicate names in the atomic conditions of D to n-place relations on M, the individual constants occurring in the link conditions of D to members of M, and (here is the recursive part of the definition) $\mathcal{M}$ is also appropriate for the DRSs in the complex conditions of D.

Let $\mathcal{M} = \langle M, I\rangle$ be an appropriate model for DRS D. An assignment s for $\mathcal{M} = \langle M, I\rangle$ is a mapping of the set of reference markers U to elements of M. The term valuation determined by $\mathcal{M}$ and s is the function $V_{\mathcal{M},s}$ defined by $V_{\mathcal{M},s}(t) := I(t)$ if $t \in A$ and $V_{\mathcal{M},s}(t) := s(t)$ if $t \in U$. In the following definition we use $s[X]s'$ for: s' agrees with s except possibly on the values of the members of X.

DEFINITION 4.2 (*Assignments verifying a DRS*). An assignment s verifies $D = (\{v_1, \ldots, v_n\}, \{C_1, \ldots, C_m\})$ in $\mathcal{M}$ if there is an assignment s' with $s[\{v_1, \ldots, v_n\}]s'$ which satisfies every member of $\{C_1, \ldots, C_m\}$ in $\mathcal{M}$.

DEFINITION 4.3 (*Assignments satisfying a condition*).

(i) s always satisfies $\top$ in $\mathcal{M}$.

(ii) s satisfies $P(t_1, \dots, t_n)$ in $\mathcal{M}$ iff $\langle V_{\mathcal{M},s}(t_1), \dots, V_{\mathcal{M},s}(t_n)\rangle \in I(P)$.
(iii) s satisfies $v \doteq t$ in $\mathcal{M}$ iff $s(v) = V_{\mathcal{M},s}(t)$.
(iv) s satisfies $v \neq t$ in $\mathcal{M}$ iff $s(v) \neq V_{\mathcal{M},s}(t)$.
(v) s satisfies $\neg D$ in $\mathcal{M}$ iff s does not verify D in $\mathcal{M}$.

DEFINITION 4.4. Structure D is true in $\mathcal{M}$ if there is an assignment which verifies D in $\mathcal{M}$.

Note that it follows from Definition 4.4 that $(\{x\}, \{Pxy\})$ is true in $\mathcal{M}$ iff $(\{x, y\}, \{Pxy\})$ is true in $\mathcal{M}$. In other words: free variables are existentially quantified.

We leave it to the reader to check that the definition of verifying assignments yields the following requirement for conditions of the form $D_1 \Rightarrow D_2$:

- s satisfies $D_1 \Rightarrow D_2$ in $\mathcal{M}$, where $D_1 = (X, \{C_1, \dots, C_k\})$, iff every assignment s' with $s[X]s'$ which satisfies $C_1, \dots, C_k$ in $\mathcal{M}$ verifies D_2 in $\mathcal{M}$.

These definitions are easily modified to take anchors (partial assignments of values to fixed referents) into account. This is done by focusing on assignments extending a given anchor.

It is not difficult to see that the expressive power of basic DRT is the same as that of first order logic. In fact, there is an easy recipe for translating representation structures to formulae of predicate logic. Assuming that discourse referents can do duty as predicate logical variables, the atomic and link conditions of a representation structure are atomic formulae of predicate logic. The translation function $^\circ$ which maps representation structures to formulae of predicate logic is defined as follows:

DEFINITION 4.5 (*Translation from DRT to FOL*).

- For DRSs: if $D = (\{v_1, \dots, v_n\}, \{C_1, \dots, C_m\})$ then
  $D^\circ := \exists v_1 \cdots \exists v_n (C_1^\circ \wedge \cdots \wedge C_m^\circ)$.
- For atomic conditions (i.e. atoms or links): $C^\circ := C$.
- For negations: $(\neg D)^\circ := \neg D^\circ$.

It follows from this that the translation instruction for implications becomes (assume $D_1 = (\{v_1, \dots, v_n\}, \{C_1, \dots, C_m\})$)

- $(D_1 \Rightarrow D_2)^\circ := \forall v_1 \cdots \forall v_n ((C_1^\circ \wedge \cdots \wedge C_m^\circ) \rightarrow D_2^\circ)$.

The following is now easy to show:

PROPOSITION 4.6. *s verifies D in $\mathcal{M}$ iff $\mathcal{M}, s \models D^\circ$, where $\models$ is Tarski's definition of satisfaction for first order predicate logic.*

It is also not difficult to give a meaning preserving translation from first order predicate logic to basic DRT. In the following definition, $\phi^\bullet$ is the DRS corresponding to the predicate logical formula ϕ, and $\phi_1^\bullet$ and $\phi_2^\bullet$ are its first and second components.

DEFINITION 4.7 (*Translation from FOL to DRT*).

- For atomic formulas: $C^\bullet := (\emptyset, C)$.
- For conjunctions: $(\phi \wedge \psi)^\bullet := (\emptyset, \{\phi^\bullet, \psi^\bullet\})$.
- For negations: $(\neg\phi)^\bullet := (\emptyset, \neg\phi^\bullet)$.
- For quantifications: $(\exists v \phi)^\bullet := (\phi_1^\bullet \cup \{v\}, \phi_2^\bullet)$.

PROPOSITION 4.8. *$\mathcal{M}, s \models \phi$ iff s verifies $\phi^\bullet$ in $\mathcal{M}$, where $\models$ is Tarski's definition of satisfaction for first order predicate logic.*

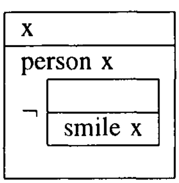
The difference between first order logic and basic DRT has nothing to do with expressive power but resides entirely in the different way in which DRT handles context. The importance of this new perspective on context and context change is illustrated by the following examples with their DRS representations.

(29) Someone did not smile. He was angry.

(30) Not everyone smiled. *He was angry.

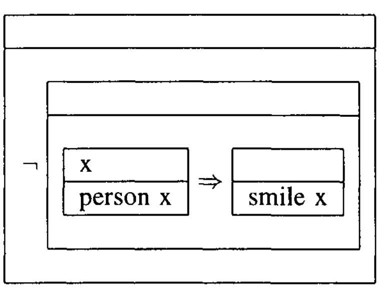
A suitable DRS representation (ignoring tense) for the first sentence of (29) is the following.

(31)



Here we see that the pronoun *he* in the next sentence of (29) can be resolved by linking it to the marker x occurring in the top box. The anaphoric possibilities of (30) are different, witness its DRS representation (32).

(32)



In this case there is no suitable marker available as an antecedent for *he* in the next sentence of (30).

What we see here is that DRSs with the same truth conditions, such as (31) and (32), may nevertheless be semantically different in an extended sense. The context change potentials of (31) and (32) are different, as the former creates a context for subsequent anaphoric links whereas the latter does not. This is as it should be, of course, as the pronoun in the second sentence of (29) can pick up the reference marker in the first sentence, but the pronoun in the second sentence of (30) cannot. The comparison of (31) and (32) illustrates that meaning in the narrow sense of truth conditions does not exhaust the concept of meaning for DRSs. The extended sense of meaning in which (31) and (32) are different can be informally phrased as follows: (31) creates a new context that can furnish an antecedent for a pronoun is subsequent discourse, (32) does not. This is because (31) *changes* the context, whereas (32) does not.

5. The static and dynamic meaning of representation structures

DRT has often been criticized for failing to be 'compositional'. It is important to see what this criticism could mean and to distinguish between two possible ways it could be taken. According to the first of these DRT fails to provide a direct compositional semantics for the natural language fragments to which it is applied. Given the form in which DRT was originally presented, this charge is justifiable, or at least it was so in the past. We will address it in Section 9. In its second interpretation the criticism pertains to the formalism of DRT itself. This objection is groundless. As Definitions 4.2 and 4.3 more or less directly imply, the formal language of Definition 4.1 is as compositional as standard predicate logic. We can make the point more explicit by rephrasing Definitions 4.2 and 4.3 as a definition of the semantic values $[\![\ ]\!]_{\mathcal{M}}$ that is assigned to each of the terms, conditions and DRSs of the DRT language by an appropriate model $\mathcal{M}$. As values for DRSs in $\mathcal{M}$ we use pairs $\langle X, F\rangle$ consisting of a finite set of reference markers $X \subseteq U$ and a set of functions $F \subseteq M^U$, and as meanings for conditions we use sets of assignments.

DEFINITION 5.1 (*Semantics of DRSs*).

$$[\![(\{v_1,\ldots,v_n\},\{C_1,\ldots,C_m\})]\!]_{\mathcal{M}} := (\{v_1,\ldots,v_n\},[\![C_1]\!]_{\mathcal{M}} \cap\cdots\cap [\![C_m]\!]_{\mathcal{M}}).$$

DEFINITION 5.2 (*Semantics of conditions*).

(i) $[\![P(t_1,\ldots,t_n)]\!]_{\mathcal{M}} := \{s \in M^U \mid \langle V_{\mathcal{M},s}(t_1),\ldots,V_{\mathcal{M},s}(t_n)\rangle \in I(P)\}$.
(ii) $[\![v \doteq t]\!]_{\mathcal{M}} := \{s \in M^U \mid s(v) = V_{\mathcal{M},s}(t)\}$.
(iii) $[\![v \neq t]\!]_{\mathcal{M}} := \{s \in M^U \mid s(v) \neq V_{\mathcal{M},s}(t)\}$.
(iv) $[\![\neg D]\!]_{\mathcal{M}} := \{s \in M^U \mid$ for no $s' \in M^U : s[X]s'$ and $s' \in F\}$,
where $(X, F) = [\![D]\!]_{\mathcal{M}}$.

To see the connection with the earlier definition of verification, 4.2, note that the following proposition holds:

PROPOSITION 5.3.

- *s verifies D in $\mathcal{M}$ iff $[\![D]\!]_{\mathcal{M}} = \langle X, F\rangle$ and there is an $s' \in M^U$ with $s[X]s'$ and $s' \in F$.*
- *D is true in $\mathcal{M}$ iff $[\![D]\!]_{\mathcal{M}} = \langle X, F\rangle$ and $F \neq \emptyset$.*

If one asks what are the DRS components of a DRS $(\{v_1,\ldots,v_n\},\{C_1,\ldots,C_m\})$, then the answer has to be: there aren't any. For those who do not like this answer, it turns out to be possible to view DRSs as built from atomic building blocks which are also DRSs. This was first pointed out by Zeevat(1989). The DRS language is now given in a slightly different way:

DEFINITION 5.4 (*Building DRSs from atomic DRSs*).
(i) If v is a reference marker, $(\{v\}, \emptyset)$ is a DRS.
(ii) If $(\emptyset, \{\top\})$ is a DRS.
(iii) If P is an n-ary predicate and $t_1, \ldots, t_n$ are terms, then $(\emptyset, \{P(t_1, \ldots, t_n)\})$ is a DRS.
(iv) If v is a reference marker and t is a term, then $(\emptyset, \{v \doteq t\})$ is a DRS.
(v) If v is a reference marker and t is a term, then $(\emptyset, \{v \neq t\})$ is a DRS.
(vi) If D is a DRS, then $(\emptyset, \neg D)$ is a DRS.
(vii) If $D = (X, C)$ and $D' = (X', C')$ are DRSs, then $(X \cup X', C \cup C')$ is a DRS.
(viii) Nothing else is a DRS.

It is clear that this defines the same DRS language. Let us use $-$ for the construction step that forms negated DRSs (that is, we use $-D$ for $(\emptyset, \neg D)$) and $\oplus$ for the operation of merging the universes and the constraint sets of two DRSs (that is, if $D = (X, C)$ and $D' = (X', C')$, then $D \oplus D' := (X \cup X', C \cup C')$).

Under this DRS definition, DRSs have become structurally ambiguous. DRS $(\{x\}, \{Px, Qx\})$, for example, has several possible construction histories:

- $(\{x\}, \emptyset) \oplus ((\emptyset, \{Px\}) \oplus (\emptyset, \{Qx\}))$,
- $(\{x\}, \emptyset) \oplus ((\emptyset, \{Qx\}) \oplus (\emptyset, \{Px\}))$,
- $((\{x\}, \emptyset) \oplus (\emptyset, \{Px\})) \oplus (\emptyset, \{Qx\})$,
- and so on.

The DRS semantics to be given next ensures that these structural ambiguities are harmless: the semantic operation corresponding to $\oplus$ is commutative and associative.

The following two semantic operations correspond to the syntactic operations $\oplus$, $-$ on DRSs (note that we overload the notation by calling the semantic operations by the same names as their syntactic counterparts):

$$\langle X, F\rangle \oplus \langle Y, G\rangle := \langle X \cup Y, F \cap G\rangle,$$
$$-\langle X, F\rangle := \langle \emptyset, \{g \in M^U \mid \neg\exists f \in F \text{ with } g[X]f\}\rangle.$$

The DRS semantics now looks like this:

DEFINITION 5.5.
(i) $[\![(\{v\}, \emptyset)]\!]_{\mathcal{M}} := (\{v\}, M^U)$.
(ii) $[\![(\emptyset, \{\top\})]\!]_{\mathcal{M}} := (\emptyset, M^U)$.
(iii) $[\![(\emptyset, \{Pt_1, \ldots, t_n\})]\!]_{\mathcal{M}} := (\emptyset, \{f \in M^U \mid \langle V_{\mathcal{M},f}(t_1), \ldots, V_{\mathcal{M},f}(t_n)\rangle \in I(P)\})$.
(iv) $[\![(\emptyset, \{v \doteq t\})]\!]_{\mathcal{M}} := (\emptyset, \{f \in M^U \mid f(v) = V_{\mathcal{M},f}(t)\})$.
(v) $[\![(\emptyset, \{v \neq t\})]\!]_{\mathcal{M}} := (\emptyset, \{f \in M^U \mid f(v) \neq V_{\mathcal{M},f}(t)\})$.
(vi) $[\![-D]\!]_{\mathcal{M}} := -[\![D]\!]_{\mathcal{M}}$.
(vii) $[\![D \oplus D']\!]_{\mathcal{M}} := [\![D]\!]_{\mathcal{M}} \oplus [\![D']\!]_{\mathcal{M}}$.

Clearly, this provides an elegant and compositional model- theoretic semantics for DRSs. Moreover, it is easily verified that Definition 5.5 is equivalent to Definitions 5.1 and 5.2

in the sense that if $[\![D]\!]_{\mathcal{M}} = \langle X, F\rangle$, then for any assignment s, $s \in F$ iff s verifies D in $\mathcal{M}$.

The semantics considered so far defines the *truth conditions* of DRSs. But as we noted at the end of Section 4, there is more to the meaning of a DRS than truth conditions alone. For DRSs which define the same truth conditions may still differ in their context change potentials.

To capture differences in context change potential, and not just in truth conditions, we need a different kind of semantics, which makes use of a more finely differentiated (and thus, necessarily, of a more complex) notion of semantic value. There are several ways in which this can be achieved. The one which we follow in the next definition defines the semantic value of a DRS as a relation between assignments – between *input assignments*, which verify the context to which the DRS is being evaluated, and *output assignments*, which reflect the way in which the DRS modifies this context. A semantics which characterizes the meaning of an expression in terms of its context change potential is nowadays usually referred to as *dynamic semantics*, while a semantics like that of the Definitions 4.2 and 4.3 or Definitions 5.1 and 5.2, whose central concern is with conditions of truth, is called *static*. The first explicit formulation of a dynamic semantics in this sense can be found in (Barwise, 1987). An elegant formulation is given in (Groenendijk and Stokhof, 1991).

Although they are quite different from a conceptual point of view, the dynamic and the static semantics for formalisms like those of DRT are nonetheless closely connected. Thus, if we denote the dynamic value of DRS D in model $\mathcal{M}$, – i.e. the relation between assignments of $\mathcal{M}$ which D determines – as ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$, with s the input assignment and s' the output assignment, we have:

– If $D = (X, C)$ then: ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$ iff $s[X]s'$ and s' verifies D in $\mathcal{M}$.

We can also characterize this relation directly, by a definition that is compositional in a similar spirit as Definition 5.5 in that it characterizes the dynamic value of a complex DRS in terms of the dynamic values of its constituents. It will be convenient to base this definition on a slightly different syntactic characterization of the DRS formalism than we have used hitherto, one in which the symmetric merge of Definition 5.5 is replaced by an asymmetric merge $\oslash$ defined as follows:

– If $D = (X, C)$ and $D' = (Y, C')$ then $D \oslash D' := (X, C \cup C')$ is a DRS.

It is clear that all DRSs can be built from atomic DRSs using $-$ and $\oslash$ (but note that $\oslash$ disregards the universe of its second argument).

The dynamic semantics is given as follows. We use ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$ for s, s' is an input/output state pair for D in model $\mathcal{M}$, and $s[v]s'$ for: s and s' differ at most in the value for v.

DEFINITION 5.6.

(i) ${}_s[\![(\{v\}, \emptyset)]\!]^{\mathcal{M}}_{s'}$ iff $s[v]s'$.
(ii) ${}_s[\![(\emptyset, \{\top\})]\!]^{\mathcal{M}}_{s'}$ iff $s = s'$.
(iii) ${}_s[\![(\emptyset, \{Pt_1, \dots, t_n\})]\!]^{\mathcal{M}}_{s'}$ iff $s = s'$ and $\langle V_{\mathcal{M},s}(t_1), \dots, V_{\mathcal{M},s}(t_n)\rangle \in I(P)$.
(iv) ${}_s[\![(\emptyset, \{v \doteq t\})]\!]^{\mathcal{M}}_{s'}$ iff $s = s'$ and $s(v) = V_{\mathcal{M},s}(t)$.
(v) ${}_s[\![(\emptyset, \{v \neq t\})]\!]^{\mathcal{M}}_{s''}$ iff $s = s'$ and $s(v) \neq V_{\mathcal{M},s}(t)$.

(vi) ${}_s[\![-D]\!]^{\mathcal{M}}_{s'}$ iff $s = s'$ and for no s'' it is the case that ${}_s[\![D]\!]^{\mathcal{M}}_{s''}$.
(vii) ${}_s[\![D \oslash D']\!]^{\mathcal{M}}_{s'}$ iff ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$ and ${}_{s'}[\![D']\!]^{\mathcal{M}}_{s'}$.

The static and the dynamic semantics of DRSs are equivalent, for we have the following proposition:

PROPOSITION 5.7. $[\![D]\!]_{\mathcal{M}} = \langle X, F\rangle$, $s[X]s'$, $s' \in F$ *iff* ${}_s[\![D]\!]^{\mathcal{M}}_{s'}$.

Still, the relation between static and dynamic semantics that we have given here leaves something to be desired. The composition operations for static semantics and dynamic semantics are different. The basic reason for this is that the dynamic semantics has a notion of sequentiality built in, a notion of processing in a given order. Therefore the commutative merge operation $\oplus$ does not quite fit the dynamic semantics: $\oplus$ is commutative, and sequential merging of DRSs intuitively is not. The operation $\oslash$ is not commutative, but it is unsatisfactory because it discards the dynamic effect of the second DRS (which is treated as if it had an empty universe).

To give a true account of the context change potential of DRSs one has to be able to answer the question how the context change potential of a DRS D_1 and that of a DRS D_2 which follows it determine the context change potential of their composition. This leads directly to the question how DRSs can be built from constituent DRSs by an operation of sequential merging.

6. Sequential composition of representation structures

Taking unions of universes and constraint sets is a natural commutative merge operation on DRSs, but it is not quite the operation on DRS meanings one would expect, given the dynamic perspective on DRS semantics. Intuitively, the process of gluing an existing DRS representing the previous discourse to a DRS representation for the next piece of natural language text is a process of sequential composition, a process which one would expect not to be commutative.

How should DRS meanings be composed sequentially? Before we address this question, it is convenient to switch to a slightly modified language for DRSs. It turns out that if one introduces a sequencing operator ; the distinction between DRSs and conditions can be dropped. This move yields the following language that we will call the language of proto-DRSs or pDRSs.

pDRSs $D ::= v \mid \top \mid Pt_1 \cdots t_n \mid v \doteq t \mid \neg D \mid (D_1; D_2)$.

In this language, a reference marker taken by itself is an atomic pDRS, and pDRSs are composed by means of ;. Thus, introductions of markers and conditions can be freely mixed. Although we drop the distinction between markers and conditions and that between conditions and pDRSs, a pDRS of the form v will still be called a marker, and one of the form $\top$, $Pt_1 \cdots t_n$, $v \doteq t$ or $\neg D$ a condition. Thus, a pDRS is a reference marker or an atomic condition or a negation or a ;-composition of pDRSs.

From now on, we will consider $v \not\doteq t$ as an abbreviation of $\neg v \doteq t$, and $D_1 \Rightarrow D_2$ as an abbreviation of $\neg(D_1; \neg D_2)$. It will turn out that the process of merging pDRSs with ';' is associative, so we will often drop parentheses where it does no harm, and write $D_1; D_2; D_3$ for both $((D_1; D_2); D_3)$ and $(D_1; (D_2; D_3))$.

It is possible to give a commutative semantics for pDRSs, by using the semantic operation $-$ to interpret $\neg$, and $\oplus$ to interpret ;.

DEFINITION 6.1 (*Commutative Semantics of pDRSs*).
(i) $[\![v]\!]_{\mathcal{M}} := \langle \{v\}, M^U \rangle$.
(ii) $[\![\top]\!]_{\mathcal{M}} := \langle \emptyset, M^U \rangle$.
(iii) $[\![Pt_1, \ldots, t_n]\!]_{\mathcal{M}} := \langle \emptyset, \{f \in M^U \mid \langle V_{\mathcal{M},f}(t_1), \ldots, V_{\mathcal{M},f}(t_n) \rangle \in I(P)\} \rangle$.
(iv) $[\![v \doteq t]\!]_{\mathcal{M}} := \langle \emptyset, \{f \in M^U \mid f(v) = V_{\mathcal{M},f}(t)\} \rangle$.
(v) $[\![\neg D]\!]_{\mathcal{M}} := -[\![D]\!]_{\mathcal{M}}$.
(vi) $[\![D; D']\!]_{\mathcal{M}} := [\![D]\!]_{\mathcal{M}} \oplus [\![D']\!]_{\mathcal{M}}$.

This interpretation of ; makes merging of pDRSs into a commutative operation. To see the effect of this, look for instance at examples (33) and (34).

(33) A man entered.

(34) A boy smiled.

How should pDRSs for these examples be merged? The commutative merge that we just defined gives the result (35).

(35) $$\begin{array}{|l|}\hline \text{x} \\ \hline \text{man x} \\ \text{enter x} \\ \hline \end{array} \; ; \; \begin{array}{|l|}\hline \text{x} \\ \hline \text{boy x} \\ \text{smile x} \\ \hline \end{array} = \begin{array}{|l|}\hline \text{x} \\ \hline \text{man x} \\ \text{enter x} \\ \text{boy x} \\ \text{smile x} \\ \hline \end{array}$$

In the pDRT semantics the two discourse referents for *a man* and a *a boy* will be fused, for according to the operation $\oplus$ the fact that a marker is mentioned more than once is irrelevant. This shows that (35) cannot be the right translation of the sequential composition of (33) and (34).

A different approach to merging pDRSs is suggested by the fact that in a dynamic perspective merging in left to right order has a very natural relational meaning:

– ${}_s[\![D_1; D_2]\!]^{\mathcal{M}}_{s'}$ iff there is an assignment s'' with ${}_s[\![D_1]\!]^{\mathcal{M}}_{s''}$ and ${}_{s''}[\![D_2]\!]^{\mathcal{M}}_{s'}$.

This semantic clause complies with the intuition that the first pDRS is interpreted in an initial context s yielding a new context s'', and this new context serves as the initial context for the interpretation of the second pDRS.

Once we are here a natural way to extend the dynamic approach to the full language suggests itself, as was noted by Groenendijk and Stokhof (1991). Their observation is basically this. If we interpret the DRS conditions in terms of pairs of assignments, the dynamic semantic values of DRS conditions can be given in the same form as the dynamic values of DRSs.

At first sight, DRS conditions do not look like context changers. If (s, s') is a context pair for a condition, then always $s = s'$, representing the fact that the condition does not change anything. But who cares? If we allow degenerate context changers, we can drop the distinction between conditions and DRSs altogether. What is more, even the distinction between marker introductions and conditions is not essential, for the introduction of a marker u can also be interpreted in terms of context pairs, and the introduction of a list of markers can be obtained by merging the introductions of the components.

These considerations yield the following relational semantics for the pDRS format (this is in fact the semantic format of the dynamic version of first order predicate logic defined in (Groenendijk and Stokhof, 1991)):

DEFINITION 6.2 (*Relational Semantics of pDRSs*).
(i) ${}_s[\![v]\!]_{s'}^{\mathcal{M}}$ iff $s[v]s'$.
(ii) ${}_s[\![\top]\!]_{s'}^{\mathcal{M}}$ iff $s = s'$.
(iii) ${}_s[\![Pt_1,\dots,t_n]\!]_{s'}^{\mathcal{M}}$ iff $s = s'$ and $\langle V_{\mathcal{M},s}(t_1),\dots,V_{\mathcal{M},s}(t_n)\rangle \in I(P)$.
(iv) ${}_s[\![v \doteq t]\!]_{s'}^{\mathcal{M}}$ iff $s = s'$ and $s(v) = V_{\mathcal{M},s}(t)$.
(v) ${}_s[\![\neg D]\!]_{s'}^{\mathcal{M}}$ iff $s = s'$ and for no s'' it is the case that ${}_s[\![D]\!]_{s''}^{\mathcal{M}}$.
(vi) ${}_s[\![D;D']\!]_{s'}^{\mathcal{M}}$ iff there is an s'' with ${}_s[\![D]\!]_{s''}^{\mathcal{M}}$ and ${}_{s''}[\![D']\!]_{s'}^{\mathcal{M}}$.

Truth is defined in terms of this, as follows.

DEFINITION 6.3 (*Truth in relational semantics for pDRSs*). D is true in $\mathcal{M}$, given s, notation $\mathcal{M}, s \models D$, iff there is an s' with ${}_s[\![D]\!]_{s'}^{\mathcal{M}}$.

Note that the difference with the previous semantics (Definition 6.1) resides in the interpretation of ; and has nothing to do with with the static/dynamic opposition. To see that, observe that the relational semantics Definition 6.2 can also be given a static formulation. For that, the only change one has to make to Definition 6.1 is in the clause for $D_1; D_2$, by interpreting ; as the operation $\circ$ defined as follows:

$$\langle X, F\rangle \circ \langle X', F'\rangle := \langle X \cup X', \{f' \in F' \mid \exists f \in F\ f[X']f'\}\rangle.$$

Given this change to Definition 6.1, we have the following proposition:

PROPOSITION 6.4. *$\mathcal{M}, s \models D$ iff $[\![D]\!] = \langle X, F\rangle$ and $\exists f \in F$ with $s[X]f$.*

So we see that 6.2 can be given an equivalent static formulation. Conversely, it is not hard to give a relational clause for $\oplus$:

$$fR \oplus Sg \iff f[R^\bullet \cup S^\bullet]g \ \&\ g \in \text{rng}\,(R) \cap \text{rng}\,(S),$$

where $R^\bullet = \{v \in U \mid (f,g) \in R \ \&\ f(v) \neq g(v)\}$ (and similarly for $S^\bullet$).

According to the relational semantics of Definition 6.2, (36) and (37) have the same meanings.

(36) x; y; man x; woman y; love (x,y).

(37) x; man x; y; woman y; love (x,y).

This means that we can use the same box representation (38) for both:

(38)

x y
man x woman y love (x,y)

Unfortunately, other examples show that the box notation does not really fit the relational semantics for the pDRSs given in Definition 6.2. The use of collecting discourse referents in universes, as it is done in the box format, is that this allows one to see the anaphoric possibilities of a representation at a glance: the discourse referents in the top box are the markers available for subsequent anaphoric linking.

However, when the composition operation ; is interpreted as in Definition 6.2 (or, alternatively, as the operation $\circ$), the pDRS notation becomes capable of expressing distinctions that cannot be captured in the box notation we have been using. Note, for instance that the pDRSs in (39) and (40) are not equivalent with regard to the semantics of Definition 6.2, although they are equivalent with regard to that given by (the unmodified) Definitions 5.1 and 5.2.

(39) x; man x; dog y; y; woman y; love (x,y).

(40) x; y; man x; dog y; woman y; love (x,y).

To take this difference into account the box representation for (39) would have to be something like (41).

(41)

x	y
man x dog y	woman y love (x,y)

The vertical dividing line in (41) separates the occurrences of y that receive their interpretation from the previously given context from those that are linked to the new introduction.

Thus we see that the relational semantics for pDRSs provides a natural notion of sequential merging, which allows sharing of introduced markers between two DRSs. However, it distinguishes between different introductions of the same marker. This introduces a problem of *destructive assignment*: after a new introduction of a marker v that was already present, its previous value is lost. This feature of Definition 6.2 is the root cause of the mismatch between box representation and sequential presentation that we just noted. It is also the source of the non-equivalence of the commutative and the relational composition semantics for the pDRS format.

For a fruitful discussion of the problem of sequential merge, it is necessary to be clear about the nature of the different kinds of marker occurrences in a pDRS. In the following discussion we compare the role of reference markers with that of variables in classical logic and in programming languages. Classical logic has two kinds of variable occurrences: bound and free. In the dynamic logic that underlies DRT there are three kinds of variable or marker occurrences (see Visser, 1994b).

(i) marker occurrences that get their reference fixed by the larger context,
(ii) marker occurrences that get introduced in the current context,

(iii) markers occurrences that get introduced in a subordinate context.

We will call the first kind *fixed* marker occurrences, the second kind *introduced* marker occurrences, and the third kind *classically bound* marker occurrences. The first kind corresponds roughly to the free variable occurrences of classical logic, and the third kind to the bound variable occurrences of classical logic (hence the name). The second kind is altogether different: these are the markers that embody the context change potential of a given pDRS.

As the distinction between these three kinds of marker occurrences is given by 'dynamic' considerations, it is not surprising that there is a close connection with the various roles that variables can play in imperative programming. Here are the correspondences:

(i) Fixed markers correspond to variables in read memory.

(ii) Introduced markers correspond to variables in write memory.

(iii) Bound markers correspond to scratch memory (memory used for intermediate computations that are not part of the output of the program under consideration).

Due to the semantic motivation for this tripartite distinction, the formal definition will depend on the semantics for ; that we adopt. We will give the definition based on the relational semantics.

The set of discourse referents which have a fixed occurrence in a pDRS is given by a function $\mathit{fix} : \mathit{pDRSs} \to \mathcal{P}U$. The set of discourse referents which are introduced in a pDRS is given by a function $\mathit{intro} : \mathit{pDRSs} \to \mathcal{P}U$, and the set of discourse referents which have a classically bound occurrence in a pDRS is given by a function $\mathit{cbnd} : \mathit{pDRSs} \to \mathcal{P}U$. To define these functions, we first define a function *var* on the atomic conditions of a DRS.

$$\mathit{var}(Pt_1 \cdots t_n) := \{t_i \mid 1 \leqslant i \leqslant n,\ t_i \in U\},$$

$$\mathit{var}(v \doteq t) := \begin{cases} \{v, t\} & \text{if } t \in U, \\ \{v\} & \text{otherwise.} \end{cases}$$

DEFINITION 6.5 (*fix, intro, cbnd*).

- $\mathit{fix}(v) := \emptyset$, $\mathit{intro}(v) := \{v\}$, $\mathit{cbnd}(v) := \emptyset$.
- $\mathit{fix}(\top) := \emptyset$, $\mathit{intro}(\top) := \emptyset$, $\mathit{cbnd}(\top) := \emptyset$.
- $\mathit{fix}(Pt_1 \cdots t_n) := \mathit{var}(Pt_1 \cdots t_n)$, $\mathit{intro}(Pt_1 \cdots t_n) := \emptyset$, $\mathit{cbnd}(Pt_1 \cdots t_n) := \emptyset$.
- $\mathit{fix}(v \doteq t) := \mathit{var}(v \doteq t)$, $\mathit{intro}(v \doteq t) := \emptyset$, $\mathit{cbnd}(v \doteq t) := \emptyset$.
- $\mathit{fix}(\neg D) := \mathit{fix}(D)$, $\mathit{intro}(\neg D) := \emptyset$, $\mathit{cbnd}(\neg D) := \mathit{intro}(D) \cup \mathit{cbnd}(D)$.
- $\mathit{fix}(D_1; D_2) := \mathit{fix}(D_1) \cup (\mathit{fix}(D_2) - \mathit{intro}(D_1))$,
  $\mathit{intro}(D_1; D_2) := \mathit{intro}(D_1) \cup \mathit{intro}(D_2)$,
  $\mathit{cbnd}(D_1; D_2) := \mathit{cbnd}(D_1) \cup \mathit{cbnd}(D_2)$.

We will occasionally use $\mathit{activ}(D)$ for the set of markers $\mathit{fix}(D) \cup \mathit{intro}(D)$.

The set of conditions of a pDRS is given by the function $\mathit{cond} : \mathit{pDRSs} \to \mathcal{P}(\mathit{pDRSs})$, which collects the conditions of D together in a set:

DEFINITION 6.6 (*cond*).

(i) $\mathit{cond}(v) := \emptyset$.

(ii) $cond(\top) := \{\top\}$.
(iii) $cond(Pt_1 \cdots t_n) := \{Pt_1 \cdots t_n\}$.
(iv) $cond(v \doteq t) := \{v \doteq t\}$.
(v) $cond(\neg D) := \{\neg D\}$.
(vi) $cond(D_1; D_2) := cond(D_1) \cup cond(D_2)$.

Note that there are pDRSs D with $intro(D) \cap fix(D) \neq \emptyset$. An example is given in (42).

(42) $Px; x; Qx.$

Also, there are pDRSs D where a marker is introduced more than once. An example is given in (43).

(43) $x; Px; x; Qx.$

We will call a pDRS *proper* (or a DRS) if these situations do not occur. Thus, the set of DRSs is defined as follows:

DEFINITION 6.7 (*DRSs*).

- If v is a marker, then v is a DRS.
- $\top$ is a DRS.
- If $t_1, \ldots, t_n$ are terms and P is an n-place predicate letter, then $Pt_1 \cdots t_n$ is a DRS.
- If v is a marker and t is a term, then $v \doteq t$ is a DRS.
- If D is a DRS, then $\neg D$ is a DRS.
- If D_1, D_2 are DRSs, and $(fix(D_1) \cup intro(D_1)) \cap intro(D_2) = \emptyset$, then $D_1; D_2$ is a DRS.
- Nothing else is a DRS.

Note that examples (42) and (43) are not DRSs. Indeed, we have:

PROPOSITION 6.8. *For every DRS D, $intro(D) \cap fix(D) = \emptyset$.*

Proposition 6.8 entails that DRSs of the form $D; v$ are equivalent to $v; D$. This means that any DRS D can be written in box format (44) without change of meaning. Indeed, we can view the box format for DRSs as an abstract version of the underlying real syntax.

(44)

$intro(D)$
$cond(D)$

Note that if a DRS D has $intro(D) \neq \emptyset$ and $cond(D) \neq \emptyset$, then D must be of the form $D_1; D_2$, where $(fix(D_1) \cup intro(D_1)) \cap intro(D_2) = \emptyset$. We say that D is a simple merge of D_1 and D_2.

According to the DRS definition, DRSs are either of one of the forms in (45) or they are simple merges of two DRSs (but note that taking simple merges is a partial operation).

(45)

v

$\top$

$Pt_1 \cdots t_n$

$v \doteq t$

$\neg D$

For DRSs, the truth conditions according to the commutative semantics coincide with those according to the relational semantics:

PROPOSITION 6.9. *For all models $\mathcal{M}$, all DRSs D:*

$$\textit{if } [\![D]\!]_{\mathcal{M}} = \langle X, F\rangle \textit{ then } {}_s[\![D]\!]^{\mathcal{M}}_{s'} \textit{ iff } s[X]s' \textit{ and } s' \in F.$$

7. Strategies for merging representation structures

To get a clear perspective on the problem of merging DRSs, note that the issue does not even occur in an approach where a natural language discourse is processed by means of a DRS construction algorithm that proceeds by 'deconstructing' natural language sentences in the context of a given DRS, as in (Kamp, 1981) or (Kamp and Reyle, 1993).

The problem emerges as soon as one modifies this architecture by switching to a set-up where representations for individual sentences are constructed first, and next these have to be merged in left to right order. Suppose we want to construct a DRS for the sequential composition of S_1 and S_2 on the basis of a DRS D_1 for S_1 and a DRS D_2 for S_2. Now it might happen that $D_1; D_2$ is not a DRS, because $(\mathit{fix}(D_1) \cup \mathit{intro}(D_1)) \cap \mathit{intro}(D_2) \neq \emptyset$. Our idea is to resolve this situation by applying a renaming strategy. In the example sentences given so far the problem has been avoided by a prudent choice of indices, but example (46) would pose such a conflict.

(46) A man^1 entered. A boy^1 smiled.

The initial representation for the sequential composition of D_1 and D_2 can be given by $D_1 \bullet D_2$. The problem of sequential merge now takes the form of finding strategies for reducing DRS-like expressions with occurrences of $\bullet$ to DRSs.

Before we list of a number of options for 'merge reduction', we define a class of reducible DRSs or RDRSs (assume D ranges over DRSs):

RDRSs $R ::= D \mid \neg R \mid (R_1 \bullet R_2)$.

Thus, RDRSs are compositions out of DRSs by means of $\neg$ and $\bullet$. It is useful to extend the definitions of *intro*, *fix* and *cbnd* to RDRSs:

DEFINITION 7.1 (*fix, intro, cbnd for RDRSs*).

- $\mathit{fix}(\neg R) := \mathit{fix}(R)$, $\mathit{intro}(\neg R) := \emptyset$, $\mathit{cbnd}(\neg R) := \mathit{intro}(R) \cup \mathit{cbnd}(R)$.
- $\mathit{fix}(R_1 \bullet R_2) := \mathit{fix}(R_1) \cup (\mathit{fix}(R_2) - \mathit{intro}(R_1))$,
  $\mathit{intro}(R_1 \bullet R_2) := \mathit{intro}(R_1) \cup \mathit{intro}(R_2)$,
  $\mathit{cbnd}(R_1 \bullet R_2) := \mathit{cbnd}(R_1) \cup \mathit{cbnd}(R_2)$.

We use $\bullet$ for sequential merge. The various options for how to merge DRSs all have a semantic and a syntactic side, for they must handle two questions:

(i) What is the semantics of $\bullet$?

(ii) How can RDRSs be reduced to DRSs?
In order to talk about these reductions in a sensible way, we must take negative context into account. Here is a definition of negative contexts (D ranges over DRSs, R over RDRSs).

Negative Contexts $N ::= \neg\Box \mid \neg N \mid (N;D) \mid (D;N) \mid (N \bullet R) \mid (R \bullet N)$.

Condition on $(N;D)$: $\mathit{activ}(N) \cap \mathit{intro}(D) = \emptyset$. Condition on $(D;N)$: $\mathit{activ}(D) \cap \mathit{intro}(N) = \emptyset$, where $\mathit{activ}(N)$ and $\mathit{intro}(N)$ are calculated on the basis of $\mathit{intro}(\Box) := \mathit{fix}(\Box) := \mathit{cbnd}(\Box) := \emptyset$.

What the definition says is that a negative context is an RDRS with one constituent RDRS immediately within the scope of a negation replaced by $\Box$. If N is a negative context, then $N[R]$ is the result of substituting RDRS R for $\Box$ in N. The definition of negative contexts allows us to single out an arbitrary negated sub-RDRS R of a given RDRS by writing that RDRS in the form $N[R]$.

Contexts $C ::= \Box \mid N$.

A context is either a $\Box$ or a negative context. If C is a context, then $C[R]$ is the result of substituting RDRS R for $\Box$ in N. Thus, if we want to say that a reduction rule applies to an RDRS R that may (but need not) occur immediately within the scope of a negation sign within a larger RDRS, we say that the rule applies to $C[R]$. If we specify a reduction rule

$$R \Longrightarrow R',$$

this is meant to be understood as licensing all reductions of the form:

$$C[R] \longrightarrow C[R'].$$

This format ensures that the rule can both apply at the top level and at a level bounded by a negation sign inside a larger RDRS.

We will now discuss several options for merge reduction: symmetric merge, prudent merge, destructive merge, deterministic merge with substitution, and indeterministic merge with substitution.

Symmetric merge. Interpret $\bullet$ as $\oplus$ and ; as $\circ$. The reduction rules that go with this are:

$$(R \bullet v) \Longrightarrow (v;R),$$
$$(R \bullet \top) \Longrightarrow (R;\top),$$
$$(R \bullet Pt_1,\ldots,t_n) \Longrightarrow (R;Pt_1,\ldots,t_n),$$
$$(R \bullet \neg R') \Longrightarrow (R;\neg R'),$$
$$((R \bullet v) \bullet R') \Longrightarrow ((v;R) \bullet R'),$$
$$((R \bullet \top) \bullet R') \Longrightarrow ((R;\top) \bullet R'),$$

$$((R \bullet Pt_1,\dots,t_n) \bullet R') \Longrightarrow ((R; Pt_1,\dots,t_n) \bullet R'),$$
$$((R \bullet \neg R_1) \bullet R_2) \Longrightarrow ((R; \neg R') \bullet R_2),$$
$$(R \bullet (R_1; R_2)) \Longrightarrow ((R \bullet R_1) \bullet R_2),$$
$$(R \bullet (R_1 \bullet R_2)) \Longrightarrow ((R \bullet R_1) \bullet R_2).$$

Partial merge. Interpret $\bullet$ as a partial operation (see, e.g., Muskens, 1996) while retaining $\circ$ as the interpretation of ; (as we will do throughout the remainder of this section). To give the semantics, we have to take context into account. Assume that the semantics of a DRS D is given as a triple $\langle X, Y, F\rangle$, where $X = \mathit{fix}(D)$, $Y = \mathit{intro}(D)$ and F is a set of assignments, then the following partial operation gives the semantics of partial merge:

$$\langle X, Y, F\rangle \odot \langle X', Y', F'\rangle := \begin{cases} \langle X \cup X', Y \cup Y', F \cap F'\rangle \\ \quad \text{if } (X \cup Y) \cap Y' = \emptyset, \\ \uparrow \quad \text{otherwise.} \end{cases}$$

The reduction rules that go with this: same as above, except for the following change in the rules that handle marker introductions:

$$(R \bullet v) \Longrightarrow (R; v) \text{ if } v \notin \mathit{fix}(R) \cup \mathit{intro}(R),$$
$$(R \bullet v) \Longrightarrow \text{ERROR} \text{ if } v \in \mathit{fix}(R) \cup \mathit{intro}(R),$$
$$((R \bullet v) \bullet R') \Longrightarrow ((R; v) \bullet R') \text{ if } v \notin \mathit{fix}(R) \cup \mathit{intro}(R),$$
$$((R \bullet v) \bullet R') \Longrightarrow \text{ERROR} \text{ if } v \in \mathit{fix}(R) \cup \mathit{intro}(R).$$

Prudent merge. To give the semantics of prudent merging for $\bullet$ (see Visser, 1994b), one again has to take context fully into account.

$$\langle X, Y, F\rangle \odot \langle X', Y', F'\rangle := \langle X \cup (X' - Y), Y \cup (Y' - X), F \cap F'\rangle.$$

Reduction rules that go with this: same as above, except for the following change in the rules that handle marker introduction:

$$(R \bullet v) \Longrightarrow (R; v) \text{ if } v \notin \mathit{fix}(R) \cup \mathit{intro}(R),$$
$$(R \bullet v) \Longrightarrow R \text{ if } v \in \mathit{fix}(R) \cup \mathit{intro}(R),$$
$$((R \bullet v) \bullet R') \Longrightarrow (R; v) \bullet R') \text{ if } v \notin \mathit{fix}(R) \cup \mathit{intro}(R),$$
$$((R \bullet v) \bullet R') \Longrightarrow R \bullet R' \text{ if } v \in \mathit{fix}(R) \cup \mathit{intro}(R).$$

Destructive merge. Interpret $\bullet$ as $\circ$ (relational composition), and allow destructive assignment. The reduction rule that goes with this is very simple: replace all occurrences of $\bullet$ in one go by ;, and interpret ; as $\circ$. But of course, this reduction does not yield DRSs but only proto-DRSs.

For the next two perspectives on merging DRSs, we need to develop a bit of technique for handling substitution, or, more precisely, marker renamings.

DEFINITION 7.2. A marker renaming is a function $\theta : U \rightarrow U$, such that its domain $\text{Dom}(\theta) := \{v \in U \mid v \neq \theta(v)\}$ is finite. If θ is a renaming with $\text{Dom}(\theta) = \{v_1, \ldots, v_n\}$, then $\text{Rng}(\theta) := \{\theta(v_1), \ldots, \theta(v_n)\}$. A renaming θ avoids a set $X \subseteq U :\Leftrightarrow \text{Rng}(\theta) \cap X = \emptyset$. If θ is a renaming, then $\theta - v :=$ the renaming σ that is like θ but for the fact that $\sigma(v) = v$. If $X \subseteq U$ then $\theta X := \{\theta(x) \mid x \in X\}$. A marker renaming θ is injective on $X :\Leftrightarrow |X| = |\theta X|$.

We will refer to a renaming θ with domain $\{v_1, \ldots, v_n\}$ as $[\theta(v_1)/v_1, \ldots, \theta(v_n)/v_n]$. Thus, $[x/y]$ is the renaming θ with $\theta(u) = x$ if $u = y$ and $\theta(u) = u$ otherwise. This renaming is of course injective on $\{x\}$, but not on $\{x, y\}$. $[x/y, x/z]$ is a renaming which is not injective on $\{y, z\}$. $[x/y, x/z] - z = [x/y]$.

A renaming of a subset of *intro*(D) intuitively has as its semantic effect that the write memory of D gets shifted. Renaming in a dynamic system like DRT works quite differently from variable substitution in classical logic, because of the three kinds of marker occurrences that have to be taken into account: *fix*, *intro* and *cbnd*. In particular, a renaming of *intro*(D) has to satisfy the following requirements:

(i) it should be injective on *intro*(D),
(ii) it should avoid *fix*(D),
(iii) it should leave *cbnd*(D) untouched.

The first two of these requirements can be imposed globally. Requirement (iii) should be part of the definition of the effects of renamings on (R)DRSs: we will handle it by distinguishing between outer and inner renaming. For an outer renaming of RDRS R with θ we employ θR, for an inner renaming $\overline{\theta} R$. Inner renaming is renaming within a context where marker introductions act as classical binders, i.e. within the scope of an occurrence of $\neg$. For example, if $\theta = [v/x, w/y]$, then:

$$\theta(x; \neg(y; Rxy)) = v; \neg(y; Rvy).$$

A renaming θ induces functions from terms to terms as follows:

$$\theta(t) := \begin{cases} \theta(v) & \text{if } t = v \text{ with } v \in U, \\ t & \text{if } t \in C. \end{cases}$$

A renaming $\theta - v$ induces functions from terms to terms as follows:

$$\theta - v(t) := \begin{cases} \theta(w) & \text{if } t = w \neq v \text{ with } w \in U, \\ v & \text{if } t = v, \\ t & \text{if } t \in C. \end{cases}$$

The induced renaming functions from (R)DRSs to (R)DRSs are given by:

$$
\begin{aligned}
\theta v &:= \theta(v), \\
\theta\top &:= \top, \\
\overline{\theta}\top &:= \top, \\
\theta(Pt_1 \cdots t_n) &:= P\theta t_1 \cdots \theta t_n, \\
\overline{\theta}(Pt_1 \cdots t_n) &:= P\theta t_1 \cdots \theta t_n, \\
\theta(v \doteq t) &:= \theta v \doteq \theta t, \\
\overline{\theta}(v \doteq t) &:= \theta v \doteq \theta t, \\
\theta(\neg R) &:= \neg\overline{\theta}R, \\
\overline{\theta}(\neg R) &:= \neg\overline{\theta}R, \\
\theta(v; R) &:= \theta v; \theta R, \\
\overline{\theta}(v; R) &:= v; \overline{\theta - v}R, \\
\theta(C; R) &:= \theta C; \theta R, \quad C \in \{Pt_1 \cdots t_n, v \doteq t, \neg R'\}, \\
\overline{\theta}(C; R) &:= \overline{\theta}C; \overline{\theta}R, \quad C \in \{Pt_1 \cdots t_n, v \doteq t, \neg R'\}, \\
\theta((R_1; R_2); R_3) &:= \theta(R_1; (R_2; R_3)), \\
\overline{\theta}((R_1; R_2); R_3) &:= \overline{\theta}(R_1; (R_2; R_3)),
\end{aligned}
$$

plus rules for • exactly like those for ;.

For the semantics, let us again assume that a meaning for DRS D is a triple $\langle X, Y, F\rangle$, where $X = \mathit{fix}(D)$, $Y = \mathit{intro}(D)$, and F is the set of assignments satisfying $\mathit{cond}(D)$.

DEFINITION 7.3. θ is a proper renaming for DRS D :⇔

(i) $\mathrm{Dom}(\theta) \subseteq \mathit{intro}(D)$,

(ii) θ is injective on $\mathit{intro}(D)$,

(iii) $\mathrm{Rng}(\theta) \cap \mathit{fix}(D) = \emptyset$.

DEFINITION 7.4. If $F \subseteq M^U$, $\theta F := \{g \in M^U \mid g \circ \theta \in F\}$.

For example, if $F = \{f \in M^U \mid f(x) \in I(P)\}$, and $\theta = [y/x]$, then:

$$[y/x]F = \{g \in M^U \mid g \circ [y/x](x) \in I(P)\} = \{g \in M^U \mid g(y) \in I(P)\}.$$

PROPOSITION 7.5. *If θ is a proper renaming for D and $|D|^{\mathcal{M}} = \langle X, Y, F\rangle$ then $|\theta D|^{\mathcal{M}} = \langle X, \theta Y, \theta F\rangle$.*

The upshot if this proposition is that a proper renaming only changes the write memory of a DRS.

Deterministic merge with substitution. The sequence semantics for dynamic predicate logic defined in (Vermeulen, 1993) can be used as a semantics for a language of unreduced DRSs:

$$R ::= \text{PUSH } v \mid \top \mid Pt_1 \cdots t_n \mid v \doteq t \mid \neg R \mid (R_1 \bullet R_2),$$

where v ranges over a set U of markers without indices. The meaning of a variable introduction v in sequence semantics is: push a new value for v on a stack of v values. Clearly, this prevents the destructive use of memory that we saw in connection with Definition 6.2. Suggestive notation for this: PUSH v.

We can reduce expressions of this language to a language of proper DRSs where the markers are taken from the set of indexed markers $U' := \{u_i \mid u \in U,\ i > 0\}$. The corresponding merge reduction rules for this use fully determined renamings, as follows.

First we do a global renaming, by replacing every occurrence of $v \in U$, except those immediately preceded by a PUSH, by $v_1 \in U'$. Next, assume that we are in a situation $D \bullet \text{PUSH } v \bullet R$, where D is a DRS (no occurrences of PUSH in D, no occurrences of $\bullet$ in D). Then there are two cases to consider.

It may be that v_j does not occur in $\mathit{fix}(D) \cup \mathit{intro}(D)$, for any index j. In that case, rewrite as follows:

$$(D \bullet \text{PUSH } v) \bullet R \Longrightarrow (D; v_1); R.$$

It may also be that v_j does occur in $\mathit{fix}(D) \cup \mathit{intro}(D)$, for some index j. In that case, let i be $\sup(\{j \in \mathbb{N} \mid v_j \in \mathit{fix}(D) \cup \mathit{intro}(D)\})$, and rewrite as follows:

$$(D \bullet \text{PUSH } v) \bullet R \Longrightarrow (D; v_{i+1}); [v_{i+1}/v_i]R.$$

The idea behind these instructions is that if v_j does not occur in D, then v_1 can safely be introduced, and it will actively bind the occurrences of v_1 which occur in open position on the right. If v_j does occur in D, then the present push should affect the v-variables with the highest index in open position on the right. This is precisely what the renaming $[v_{i+1}/v_i]$ effects.

Indeterministic merge with substitution. Indeterministic merge does involve a family $\odot_\theta$ of merge operations, where θ is a renaming that is constrained by the two DRSs D_1 and D_2 to be merged, in the sense that θ is proper for D_2 and θ avoids the set $\mathit{intro}(D_1) \cup \mathit{fix}(D_1)$. If the interpretations of D_1 and D_2 are given by $\langle X_1, Y_1, F_1\rangle$ and $\langle X_2, Y_2, F_2\rangle$, respectively, then the interpretation of $D_1 \bullet_\theta D_2$ is given by:

$$\langle X_1 \cup X_2, Y_1 \cup \theta Y_2, F_1 \cap \theta F_2\rangle.$$

If θ is constrained in the way stated above this is a proper DRS denotation.

The rules for indeterministic merge reduction use renamings, as follows (we use $\mathit{activ}(R)$ for $\mathit{intro}(R) \cup \mathit{fix}(R)$):

$$(R \bullet v) \Longrightarrow \begin{cases} (R; v) & \text{if } v \notin \mathit{activ}(R), \\ (R; w) & \text{if } v \in \mathit{activ}(R), \\ & \quad w \notin \mathit{activ}(R), \end{cases}$$

$$(R \bullet \top) \Longrightarrow (R; \top),$$

$$(R \bullet Pt_1, \ldots, t_n) \Longrightarrow (R; Pt_1, \ldots, t_n),$$

$$(R \bullet \neg R') \Longrightarrow (R; \neg R'),$$

$$((R \bullet v) \bullet R') \Longrightarrow \begin{cases} ((R; v); R') & \text{if } v \notin \mathit{activ}(R), \\ ((R; w); [w/v]R' \\ \quad \text{if } v \in \mathit{activ}(R),\ w \notin \mathit{activ}(R) \cup \mathit{activ}(R'), \end{cases}$$

$$((R \bullet \top) \bullet R') \Longrightarrow ((R; \top) \bullet R'),$$

$$((R \bullet Pt_1, \dots, t_n) \bullet R') \Longrightarrow ((R; Pt_1, \dots, t_n) \bullet R'),$$

$$((R \bullet \neg R_1) \bullet R_2) \Longrightarrow ((R; \neg R_1) \bullet R_2),$$

$$(R \bullet (R_1; R_2)) \Longrightarrow ((R \bullet R_1) \bullet R_2),$$

$$(R \bullet (R_1 \bullet R_2)) \Longrightarrow ((R \bullet R_1) \bullet R_2).$$

Note that under the indeterministic merge regime, $\bullet$ does not get an independent semantics, so one cannot talk about 'the' meaning of $D \bullet D'$ anymore, only about its meaning modulo renaming of $\mathit{intro}(D')$. One can still prove that different reductions of R to normal form (i.e. to proper DRSs) are always write variants of one another, i.e. $R \twoheadrightarrow D$ and $R \twoheadrightarrow D'$ together entail that there is some proper renaming θ of D with $\theta D = D'$.

A set of RDRSs together with a set of merge reduction rules like the example sets given above is a so-called abstract reduction system (Klop, 1992), and the theory of abstract reduction systems can fruitfully be applied to their study. What all merge reduction rule sets above, with the exception of destructive merge, have in common is that they start out from reducible DRSs and produce proper DRSs as normal forms. They all take into account that the merge operation $\bullet$ should not destroy anaphoric links. Merge with substitution has as an additional feature that it preserves anaphoric sockets, and that is what we will use in the sequel. For practical reasons we opt for the indeterministic version, to avoid possible confusion due to the appearance of a new kind of indices (indicating stack depth).

The picture we end up with in indeterministic merge reduction is given in Figure 1. Each RDRS or DRS has a set of anaphoric plugs and a set of anaphoric sockets. The plugs anchor the representation structure to previous discourse or to contextually given antecedents. In both reduced and unreduced RDRSs, these plugs have fixed names, given by $\mathit{fix}(R)$. The sockets are the anchoring ground for the next bit of discourse. In unreduced RDRSs, the sockets do not have fixed names yet, and they may not yet represent the

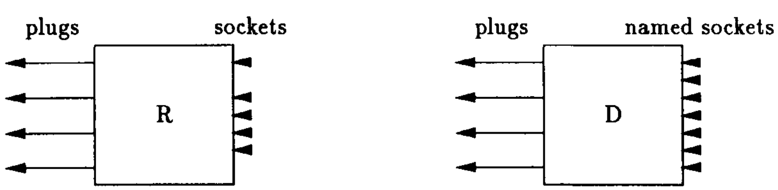
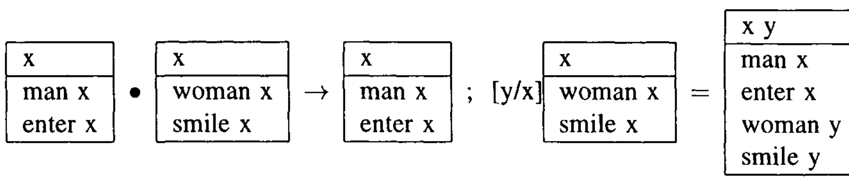


Fig. 1. Unreduced and reduced DRS, with plugs and sockets.

full set of anaphoric possibilities of the represented discourse. During the process of merge reduction, the internal wiring of the representation structure gets re-shuffled and some members of *intro*(R) may end up with a new name, to make room for extra sockets. If D is a fully reduced DRS, however, the sockets have fixed names, given by *intro*$(D) \cup$ *fix*(D), and this set of markers represents the full set of anaphoric possibilities for subsequent discourse.

Here is a concrete example of how disjoint merging according to the indeterministic merge regime works:

(47)



In DRT with indeterministic merge, introduced markers are always new, so no information is ever destroyed, and merging of representations preserves all anaphoric possibilities of the parts that are merged.

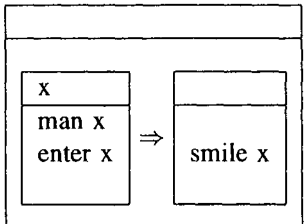
We now know what the basic building blocks of DRT are, namely structures as given in (45), and what is the glue that puts them together, namely the disjoint merge operation involving marker renaming. This concludes the discussion of compositionality for DRSs. Quite a few philosophical and technical questions concerning the natural notion of information ordering in DRT remain. See (Visser, 1994a) for illumination on these matters.

8. Disjoint merge and memory management

Reference markers are similar to variables, but differ from them in that they are not bound by logical operators in the usual sense. In fact, reference markers behave more like variables in programming languages than like variables in ordinary first order logic (Section 6 above).

Anaphoric links are created by linking new reference markers to available ones. How does one discard references? By de-allocating storage space on popping out of a 'subroutine'. The representation, in box format, for (3) is given in (48).

(48)



The semantic treatment of this uses a subroutine for checking if every way of making a reference to a man who enters (where the reference is established via marker x) makes the property given by the consequent of the clause succeed. Next the storage space for x

is de-allocated, which explains why an anaphoric link to *a man* in subsequent discourse is ruled out, or at least infelicitous (see example (49)).

(49) If a man[1] enters, he_1 smiles. *He_1 is happy.

Thus we see that anaphoric linking is not subsumed under variable binding, or at least not under variable binding perceived in a standard fashion, as in first order logic. The process is much more akin to variable binding in programming, where storage space is created and discarded dynamically, and where links to a variable remain possible until the space occupied by the variable gets de- allocated to be used for something else, so that further anaphoric links remain possible as long as the variable space for the antecedent remains accessible.

Reference markers, as we have seen, are allocated pieces of storage space for (representations of) things in the world. We can picture the building of a representation structure as an interactive process, where we give instructions to make memory reservations and to provide names for the allocated chunks of memory, as in (50).

(50) new(Var).

The system responds by allocating a chunk of memory of the correct size and by returning a name as value of *Var*, say $u385$, indicating that a piece of storage space is allocated and henceforth known under the name $u385$, where 385 presumably is the offset from the beginning of the piece of memory where the representation under construction is stored. Once storage space has been allocated to a discourse referent, it is useful to know the scope of the allocation. In DRT the scope of the introduction of a discourse referent is closed off by the closest $\neg$ operator (or the closest $\Rightarrow$ operator, in case $\Rightarrow$ is taken as a primitive) that has that introduction in its scope.

Of course, this interactive picture is an *inside* picture of what happens during the representation building process. We must also be able to look at the situation *from the outside*, and answer the question what happens if we assume that we have built and stored two representation structures D_1, D_2 in the memory of a computer, one after the other. Next, we want to store them in memory simultaneously, i.e. to merge them, where the merging has to preserve sequential order. This will in general involve changing the names of those variables declared in the second representation that would otherwise overwrite the area of memory already used by the first representation.

What if some very suspicious semanticist still has qualms about disjoint merge because of the indeterminism of the operation? We then would have to explain to him (or her) that the indeterminism is entirely natural, as it reflects the fact that the renaming operation is nothing but the familiar operation of copying variable values to a different (unused) part of memory before combining two memory states (Figure 2). Disjoint merge is indeterministic simply because any way of copying part of memory to a safe new location will do. This suggests that indeterminism is a strength rather than a weakness of the disjoint merge.

The story of a reasonable definition of merge is a story of memory management. Assuming we have an unlimited supply of memory available, we may picture the data part of memory where the active markers of representation structure D reside as an