The Logic of Historical Necessity *)

I.

Consider the following game - I will call it GOF. GOF is played by two players, A and D . It goes as follows. On his first and only move A makes an assertion, of his own choice, about the immediate future - to be precise about the period from one to two minutes after his utterance. D has to reply by saying whether this assertion is correct or not. He has to reply within one minute.

Suppose you are asked to play this game and you may decide whether to play as A or as D . It is pretty clear which side you should choose. For if A is not a complete idiot D doesn't have a chance in this game. All A has to do is to pick a sentence which he is in a position to make true or false. For example he can stick to the sentence 'Between *one* and *two* minutes from now I will say 'Booh''. 'If D then says 'No' A waits until one minute has passed at which time he says 'Booh'; if D says 'Yes' A keeps his mouth shut. Not a very amusing game, - certainly not for D ; and, unless the stakes are significant, not really for A either.

The only reason I mention it is in order to contrast it with another game, GOP , which is superficially similar to GOF , yet works out quite differently in practice. GOP is also played between two players A and D and also involves only two moves. The difference with GOF is that now A is to make an assertion about the period between *one* and *two* minutes before his utterance.

*) for footnote see p. 1a .

*) The main formal result of this paper, which appears as Theorems 1 and 2 in section IV, was obtained in the spring of 1975. The paper was completed in the summer of 1978, when the author was supported by DFG project "Die Beschreibung mithilfe der Zeitlogik von Zeitformen und Verbalformen im Französischen, Portugiesischen und Spanischen".

Among the many with whom I have communicated about the contents of this essay over the years I would like to mention in particular ~~[illegible]~~ Professor John Burgess of Princeton University who made valuable suggestions as to how the formal results reported here might be generalized, and mr. R. Spencer-Smith, of Bedford College London, who ~~pointed out~~ drew attention to a number of silly, and one or two more serious errors in an almost final draft.

Let me add the familiar triviality that, as the kindness to point out some mistakes does not entail the obligation to point out all, ~~thexxxundoubtedlyxxxremaining~~ those which - undoubtedly - remain, only reflect upon myself.

Here it is not quite so obvious which side you should prefer. You might argue: If I am D then, even if I know nothing that is relevant to the truth value of the sentence which A has picked, I still have a 50% chance of winning, by mere guessing. And with a bit of luck A may now and then choose a sentence the truth value of which I can guess with a somewhat better chance of being right.

Actually the situation may not be quite so simple. For A could try to influence D insidiously by choosing sentences of which he himself, but of which D does not, knows the truth value, and to alternate true and false sentences in such a manner that D gets lured into a higher percentage of wrong guesses than he would be likely to make if both his own moves and those of A were entirely independent of the previous plays of the game. In fact you might find it rather fun to play this game - lots of people like poker.

Even if the advantage is on A's side in this game, however, it won't be anything like the advantage he has in GOF. Indeed, suppose both GOF and GOP are played a large number of times, by large numbers of people. Then we may expect that the rate of plays won by A will be very high in the case of GOF, (say, over 90%) while for GOP it will be somewhere in the neighbourhood of 50%. This asymmetry between GOF and GOP deserves, like any other empirical phenomenon, to be explained.

The explanation which will occur immediately to anybody, and which I have already as much as given in describing GOF

and GOP, is that in GOF A can choose sentences which he still has the power to "make" true or false after D has made his countermove.* Of course there are innumerable asymmetries of this sort which are connected with the apparent circumstance that we have a certain control over the future but none over the past; and you may well be puzzled why I should have bothered to give an illustration, and at that a rather contrived one, of something so utterly familiar. My reason was that the difference between GOF and GOP, as I have described these games, lies solely in the types of sentences A may choose in each of them ; this difference moreover could be stated in purely grammatical terms. We could for example stipulate that the sentences A may choose in GOF are always to be of the form 'Between one and two minutes from now α will β' where α is a noun phrase and β is a simple infinitive phrase (i.e. a verb phrase consisting of a verb in the infinitive followed by an appropriate sequence of noun phrases and adverbials) - and analogously for the class of sentences available to A in GOP.

The difference in the winning rates for A in GOF and GOP might then be accounted for by remarking that a proposition asserted through the utterance of a sentence of the second

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* This is not the only possible type of explanation of this and similar asymmetries. It does not seem a priori impossible that somebody could come up with a very sophisticated theory which treats humans as immensely complicated automata, succeeds in giving an empirically meaningful characterization of the states that such an automaton is in when playing GOF or GOP respectively and to derive from this theory among other things the differences that we want to explain. Given our present knowledge of causal mechanisms such a theory would be absolutely fantastic. But logically impossible it is not, at least no absolutely conclusive proof of its impossibility has so far been given.

type is invariably 'settled' - whether it is to be true or false is no longer within anybody's power. On the other hand any proposition one asserts by uttering a sentence of the first type need not be settled at the time of assertion, and among those who are still in a position to influence its eventual truth or falsehood the utterer himself may occupy a central position.*

Thus while all propositions asserted through uttering a sentence of the second type are settled, this is apparently not so for some, though only some, propositions which can be made through uttering a sentence of the first type. As I would rather not talk about propositions in this paper** I shall rephrase this difference as a difference between the sentences themselves. I shall call a sentence S (<u>historically</u>) <u>necessary</u>, or <u>determinately true</u>, <u>at</u> a given time t if the truth of what S says is immune to the influence of chance events as well as to that of agents, at all times after t. I shall represent determinateness by means of a 1-place sentential operator D: Ds is to mean 's is determinately true'. Thus if S is historically necessary at t, Ds is

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* This is for all we know not so for every sentence of the first class. If for example I say at a given time t_o "Between one and two minutes from now the distance between the sun and the earth will be $X \pm Y$ km" where X is the distance between the sun and the earth at t_o, and Y is large enough then what I say will surely be settled. In particular, if $Y > 36{,}000{,}000$ km then falsification of my assertion would require that the sun and the earth travel apart during the next two minutes at a relative velocity exceeding the speed of light. We have no reason to believe this to be possible.

** To explain my reasons here for wishing to avoid the need for propositions would carry us unnecessarily far from the issues that are essential to this paper.

true at t; while Ds is <u>false</u> at t if s is not determinately true at t. Certain sentences , such as e.g. all sentences about the past, have the property that if the sentence s itself is true at t then Ds must also be true at t. But from what I have said so far it appears that for certain other sentences, viz sentences about the future, this implication is in general not valid.

We should not be too hasty, however, to conclude that there do indeed exist sentences s such that s itself can be true at t while Ds is not. In fact there are two quite distinct reasons for challenging this conclusion. In the first place it might be questioned whether there really is such a thing as control over the future. It is not a priori impossible that everything that ever happens was in fact at all earlier times determined to happen at the appointed time, and that in all those cases where we have the impression that something would not have happened had it not been for our own intervention, this impression would be justified at best in the weak sense that the causal mechanism that produces the event in question involves us in a way in which we are not involved in the causation of events that we would regard as happening independently of ourselves.

Once we consider the possibility that this may be all there is to the difference between the things we "bring about" and the things we do not "bring about" there would seem to be little difficulty with the additional hypothesis that those events which appear to us as pure chance events are also the products of fully deterministic causal mechanisms, which however are too complicated to be recognized for what they are.

—I do not know if there is any argument that shows the untenability of these possibilities conclusively.*

For what I intend to do in this paper those questions do not need to be settled. For although the results I shall present do strictly speaking collapse into simpler and already familiar ones if chance and control over the future are illusory, there nevertheless remains an interpretation on which they retain their significance. This is a claim which I cannot fully explain at this point before having given these results themselves. Even so, let me try for some elucidation already here. The investigations on which I shall report below are all concerned with what might be called the logic of the concept "it is (already) determined that". My interest in this subject comes primarily from the conviction that only if this concept is properly understood can we hope for better insight into the logical structure of a cluster of concepts, such as voluntary - as opposed to involuntary - action, responsibility, coercion, the difference between right and wrong, etc., ~~which~~ occupy a central position ~~in~~ ethics and the theory of action. I think there is no question but that our understanding of each of these concepts is premised on the assumption that among those things that will happen in

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* It would appear that quantum physics provides us with an argument for the existence of chance events. But even that argument requires, at least ~~in many of~~ the forms in which it is at present available, assumptions which make it unclear whether we can regard it as definitive.

the future some are still within our power while others are not, and that this understanding presupposes in particular that the formal representations of this distinction on which I will base my analysis of historical necessity do not reduce to the much simpler structures which ~~correspond to the~~ reflect a condition of complete determinism.

The theory I shall propose, and the results which issue from it, will therefore, even if our world is in fact deterministic, still be valid as part of a conceptual analysis of historical necessity, and cognate notions. If at some future time we shall become really convinced that determinism is true the theory I am going to present will become obsolete. But in that case the same fate will also befall that whole fabric of concepts into which the distinction between events within and events ~~outside~~ beyond our power enters as an essential ingredient. For ~~This does nothing to diminish the fact that~~, as they function, all these concepts presuppose the nondeterministic structure on which the semantic analyses of this paper are based.*

The second reason for questioning the possibility that S is true while D S fails is of a very different sort. Aristotle, in his famous discussion of the proposition 'There will be a sea battle tomorrow',** to my knowledge the first explicit discussion of the relation between truth and determinateness,

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* I expect that for psychological reasons it is simply impossible to seriously believe in physical determinism, to believe it with a conviction that not only colours your thoughts during the intermittent spells of philosophical meditation, but also permeates your perception of the world and your place in it at all times, including all times of action - if any such times would be left; presumably a person truly convinced of determinism would not be capable of any action at all.

** De Interpretatione, book IX.

argues that a claim about the future which is not yet determined can be neither true nor false. His argument, which I will not analyse here, appears to be fallacious ; but that of course is not to say that its conclusion is wrong. I think, however, that there are considerations which, without being absolutely conclusive, nevertheless point towards its rejection.

Suppose I say to you on Monday: 'The Government will lose its motion of confidence tomorrow'. You tell me that I must be out of my mind to hold such an opinion, but I persist. On Tuesday things happen as I predicted. On Wednesday I run into you again. While still at some distance I shout triumphantly: 'You see, it was quite true what I told you about the motion'. Your reaction to this is to remonstrate that although things turned out as I predicted, what I said <u>wasn't</u> true; for when I uttered my sentence on Monday the future was still <u>open</u> in this regard - many M.P.s hadn't made up their minds yet and even those who had could have changed their minds overnight, etc. etc. I think a third party present during this exchange would agree you are unreasonable. Once it has been conceded that things have turned out as I said, it seems not just pedantic, but in fact <u>wrong</u> to argue that what I said wasn't true. Indeed it is perfectly natural to say with hindsight that what I said <u>was</u> (and not <u>is</u>) true.

Of course it would be hazardous to draw so general a conclusion about the concept of truth from a mere convention about the use of tense. And with regard to the peevish objection you made on Wednesday it <u>could</u> be maintained that that was simply irrelevant, without being strictly speaking false. Even so, I feel that the illustration brings out an

important aspect of the significance of claims about the future. What we are as a rule primarily concerned about is whether what such a claim says is going to happen will indeed come to pass. Whether the thing does come to pass or not is something which we usually find out in the end, even if this may essentially involve sitting around and waiting until it either does or does not happen. To decide, in those cases where the thing does eventually comes true, whether or not it was already bound to happen at the time the prediction was made is almost always beyond our capacities. Thus if the truth of the claim (at the time of its making) required not only that the predicted event subsequently occurs, but also that this subsequent occurrence was already determined at the time of the claim itself, the demarcation between truth and non-truth would involve a factor which

(i) is virtually irrelevant to the practical purposes of discourse about the future, and

(ii) is almost invariably beyond detection.

Of course this does not establish in any way that it is undesirable to introduce for theoretical reasons a concept of truth which is demarcated along these lines. This however we have in fact already done by adopting the operator D which allows us to formalize the concept 'is determinately true'. But rather than making this the only semantic concept I prefer to analyse it in conjunction with a truth concept which applies to a statement about the future solely in virtue of how things will in fact unfold.

The purpose of this paper is to investigate the logic of this operator D. Now what is it to investigate the 'logic of' a particular notion? There are various answers to such a question. For there is no such thing, or at least nothing of great interest, as the 'logic' of the concept in isolation. The logic of a concept shows itself usually most tellingly in its interaction with others. Which other concepts we select for joint consideration with the concept that primarily interests us is largely a matter of choice. The choice is guided in the first place by our intuition which interactions will reveal the most significant features of the concept, and in the second place by how much we already know about the logic that governs the cluster of the remaining concepts, in the absence of the one we want to study. It is this second consideration which has led to the almost universal policy of studying new sentential operators - whether deontic, modal, probabilistic or temporal - in conjunction with the truth functions, of whose logic the ordinary propositional calculus offers an exhaustive analysis.

To study D just in conjunction with the truth functions would not teach us very much, however. We have already seen that D is significant only where it combines with sentences about the future. I have not given any explicit analysis of what it is for a sentence to be 'about the future', but in our discussion of GOF and GOP we already noted that in some cases the three-part distinction between sentences that are 'about the future', 'about the present' and 'about the past' can be drawn in terms of the tense of the main verb. This suggests that we should include in our formal analysis certain devices which correspond to the three tenses that mark this division. There is a further reason for including representatives of the tenses: What is determinate changes

with time - or, to be more precise, more and more things become determinate as time goes on. Thus it may be correct to say tomorrow morning, 'It is determined that there will be a sea battle today', while yet it would be wrong to say today 'It is determined that there will be a sea battle tomorrow'. More generally, 'It is determined that it will be the case that q' always implies, but is not necessarily implied by, 'It will be determined that q'.

II.

One of the simplest ways to incorporate formal counterparts of the tenses is to adopt the by now familiar 'tense-operators' P ("it was the case that") and F ("it will be the case that") first introduced, and extensively studied, by Prior. The immediate intuition behind the operators P and F is that they form past and future tense sentences respectively out of sentences in the present tense. Thus if q stands for 'It is raining' then Pq stands for 'It was raining' and Fq for 'It will be raining'. In Prior's (P,F)-calculus the operators may be iterated, i.e. we can also form PPq, PFq, FPq, PFPq, etc. The tense systems of ordinary languages do not seem to contain anything to match these possibilities of iteration - at least this is so for the tense systems of the languages with which I am familiar, in particular for English - and the truth conditions of formulae involving iterations of the operators are obtained through an obvious (but nevertheless problematic) extrapolation of the principles which relate the truth conditions of 'It was raining' and 'It will be raining' to those of 'It is raining':

(1) 'It was raining' is true at time t if there is any time t' earlier than t such that 'It is raining' is true at t', and

(2) 'It will be raining' is true at t if there is a time t' later than t such that 'It is raining' is true at t'.

Generalizing from (1) and (2) we get

(1') $P\varphi$ is true at t iff there is a t' earlier than t such that φ is true at t'; and

(2') $F\varphi$ is true at t iff there is a t' later than t such that $F\varphi$ is true at t';

and these clauses are of course also meaningful if φ itself already contains one or more tense operators.

¶ The iterability of P and F is in my opinion the most important - although by no means the only - reason why Prior's system is a rather poor approximation to the mechanisms of temporal reference which natural languages such as English realize via their tenses. This is a drawback at least from a linguistic, and possibly also from a more general conceptual perspective, and some may therefore be sceptical about any formal analysis that takes this calculus as its point of departure. I personally feel that the essentials of the interaction between D and devices of temporal reference are much clarified through studying the combination of D with Prior's P and F. However, to show that not all of the results crucially depend on this particular choice of temporal device I shall later on also discuss a system in which D

is combined with a very different mode of temporal reference.

It will be useful to briefly state the syntax and semantics of Prior's system.

<u>Definition 1</u>

A) Let L be the language whose symbols are

i) the sentence letters $q_1, q_2, q_3, \ldots$

ii) the sentential connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

iii) parentheses: (,).

The <u>formulae</u> of L are defined recursively by:

fi) q_i is a <u>formula</u>;

fii) if φ, ψ are formulae then $\neg\varphi$, $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$, $(\varphi \leftrightarrow \psi)$ are <u>formulae</u>.

B) L(P,F) is the result of adding to L the two 1-place connectives P and F.

So the <u>formulae</u> of L(P,F) are those given by clauses fi) and fii) above together with

fiii) if φ is a formula then $P\varphi$ and $F\varphi$ are <u>formulae</u>.

C) L(P,F,D) is the language obtained by adding to L(P,F) the 1-place connective D.

I will use '$H\varphi$' as an abbreviation for '$\neg P \neg \varphi$', '$G\varphi$' as an abbreviation for '$\neg F \neg \varphi$', '$\Box\varphi$' as an abbreviation for '$-D-\varphi$', and 'T' as an abbreviation for '$q_1 \vee \neg q_1$'.

The standard semantics for L is well-known. A <u>model</u> for L is simply a function M which assigns a truth value (o or 1) to each of the letters q_i. M determines the truth values of all complex formulae via the familiar clauses. The models for L(P,F) are structures of the form $\langle \mathcal{T}, M \rangle$, where

a) $\mathcal{T}$ is a structure $\langle T, < \rangle$ consisting of

i) a non-empty set T (intuitively of moments of time)

ii) $<$, a linear ordering of T.

b) M is a function which assigns a truth value to each letter q_i at each time $t \in T$ - so $M(q_i,t) \in \{0,1\}$ whenever $t \in T$.

We write $[\varphi]_{\mathcal{M},t}$ for 'the truth value of φ in $\mathcal{M}$ at t'.

The truth values are determined by the following clauses:

(t-) $$\neg[\varphi]_{\mathcal{M}t} = \begin{cases} 1 & \text{if } [\varphi]_{\mathcal{M},t} = 0 \\ 0 & \text{otherwise} \end{cases}$$

(t∧) $$[(\varphi \wedge \psi)]_{\mathcal{M},t} = \begin{cases} 1 & \text{if } [\varphi]_{\mathcal{M},t} = 1 \text{ and } [\psi]_{\mathcal{M}t} = 1 \\ 0 & \text{otherwise} \end{cases}$$

(tv) $$[(\varphi \vee \psi)]_{\mathcal{M},t} = \begin{cases} 1 & \text{if } [\varphi]_{\mathcal{M},t} = 1 \text{ or } [\psi]_{\mathcal{M},t} = 1 \\ 0 & \text{otherwise} \end{cases}$$

(t→) $$[(\varphi \rightarrow \psi)]_{\mathcal{M},t} = \begin{cases} 1 & \text{if } [\varphi]_{\mathcal{M},t} = 0 \text{ or } [\psi]_{\mathcal{M},t} = 1 \\ 0 & \text{otherwise} \end{cases}$$

(t↔) $$[(\varphi \leftrightarrow \psi)]_{\mathcal{M},t} = \begin{cases} 1 & \text{if } [\varphi]_{\mathcal{M},t} = [\psi]_{\mathcal{M},t} \\ 0 & \text{otherwise} \end{cases}$$

(tP) $$[P\varphi]_{\mathcal{M},t} = \begin{cases} 1 & \text{if } [\varphi]_{\mathcal{M},t'} = 1 \text{ for some } t' < t \\ 0 & \text{otherwise} \end{cases}$$

(tF) $$[F\varphi]_{\mathcal{M},t} = \begin{matrix} 1 & \text{if } [\varphi]_{\mathcal{M},t'} = 1 \text{ for some } t' > t \\ 0 & \text{otherwise} \end{matrix}$$

<u>Comments</u>

1) Some authors also consider models $\mathcal{M}$ whose time structures are only partially ordered. Within the context of this investigation there seems no reason to consider such time structures in connection with L(P,F). However we will have to introduce something very much like partially ordered time in connection with L(P,F,D).

2) The notions of <u>validity</u> and <u>logical consequence</u> are somewhat more complex for L(P,F) than they are for L. In the first place we may consider the concept of validity <u>with respect to</u> a given time structure $\mathcal{T}$: φ is valid <u>with respect to</u> $\mathcal{T}$ if for each model $\mathcal{M}$ such that $\mathcal{M} = \langle \mathcal{T}, A \rangle$ for some A, and each $t \in T$ $[\varphi]_{\mathcal{M},t} = 1$.
Similarly φ is a <u>consequence of</u> the set Γ of formulae <u>with respect</u> to $\mathcal{T}$ if for each model $\mathcal{M}$ of the form $\langle \mathcal{T}, A \rangle$ and each $t \in T$, if for all $\psi \in \Gamma$, $[\psi]_{\mathcal{M},t} = 1$ then $[\varphi]_{\mathcal{M},t} = 1$. It is well known that the class of formulae valid with respect to $\mathcal{T}$ depends on the structure of $\mathcal{T}$. For example the formula $Pq_1 \rightarrow PPq_1$ will belong to that class if and only if $\mathcal{T}$ is densely ordered, $P(q_1 \vee \neg q_1)$ belongs to it iff $\mathcal{T}$ has no first point, $(HFq_1 \rightarrow q_1 \vee Fq_1) \wedge (GPq_1 \rightarrow q_1 \vee Pq_1)$ iff $\mathcal{T}$

is discretely ordered; and $(Pq_1 \wedge PH\neg q_1 \to P(H\neg q_1 \wedge \neg F H\neg q_1))$ is valid iff $\mathcal{T}$ is order complete (i.e. closed under Dedekind cuts).

If one holds the view that the question whether time has any of these further properties (density, discreteness, etc.) is an empirical question that depends on the contingent structure of the physical world - then he would want a concept of validity which is independent of any of these properties. This naturally leads to the concepts of validity and logical consequence with respect to a class $\mathcal{K}$ of time structures: φ is <u>valid with respect to</u> $\mathcal{K}$ iff φ is valid with respect to each $\mathcal{T} \in \mathcal{K}$; and similarly for <u>logical consequence with respect to</u> $\mathcal{K}$. Thus for example those who consider no properties ~~other than linearity~~ to be logical features of time which are not entailed by its being a linear order will be interested in validity and consequence with respect to the class $\mathcal{L}in$ of <u>all</u> (linearly ordered) time structures.

Much is known about complete axiomatizations of a variety of such validity concepts - for different time structures and classes of time structures. But we will not go into this here. The class of time structures that will play an important role in the sequel of this paper is the class of all densely ordered structures ~~without a last point~~. I will refer to this class as 'Den.'

III.

How should we characterize the models for the language L(P,F,D)? Since this language contains L(P,F) as a sublanguage a model for L(P,F,D) must provide at least a natural interpretation for the sentences of this sublanguage, i.e. it must incorporate a representation of the complete history of some particular world, from the beginning of time till the end, - let us for the sake of discussion, think of that world as the actual world. If, however, at any stage t of

this history the future is indeed open, then besides the way in which the history in fact continues after t there will be other alternative continuations, in which certain propositions (which are not yet determined at t) receive a truth value opposite to that which they have in the actual future. Moreover, it is reasonable to assume that in any such alternative course of events w the future could again be open at times t'' subsequent to t, so that we must also countenance alternative continuations of w after t' which differ from the way w itself develops after that time. Such considerations lead naturally to the following conception. The actual world, let us call it w_o, determines, in the manner of the models for L(P,F), the truth values of all sentences of L(P,F) at each time t. Besides the future of w_o after t there are alternative futures. It might be temping to think of these alternative futures as 'truncated' histories which run till the end of time but start only at t. This, however, would introduce an asymmetry between these alternative futures and the actual future that would seem to be without foundation. There is nothing that distinguishes the actual future from these alternatives other than the merely accidental circumstance that it ~~the actual future~~ represents the way in which things will, as a matter of fact, turn out. To avoid any such asymmetry I shall treat these alternative futures after t as temporally complete worlds in their own right, each representing, just like w_o, a history that runs from the very beginning of time till its end. Of course, in as much as such a world w represents an 'alternative future for w_o after t' ~~at all times preceding~~ w must coincide with w_o at all times preceding (and including) t. If moreover the future of w is open at the subsequent time t', then the alternative continuations of w after t' will similarly be represented as temporally complete worlds, which coincide with w up to t' (and thus with w_o up to t). We thus come to the view that a model for $L(P,F,D)$ is to specify the development through time of a bundle of worlds. Any two worlds w and w' in the bundle may coincide up to a certain point in time, which means intuitively that, up to that time, w and w' really are one and the same world. Clearly if w and w' coincide up to t they must also coincide up to any time t' antecedent to t. This last relationship, i.e. that of coinciding at least up to a certain time, we shall symbolize as: $\approx$. Thus $w \approx_{t'} w'$ is to mean that w and w' coincide at least up to t' (They may, but do not have to, coincide for longer).

From what has already been said about the operator D it should be clear how the relation $\approx$ enters into the truth definition for $L(P,F,D)$: $D\varphi$ is to be true in w at t iff φ is true at t in all w' such that $w \approx_t w'$.

Notice that this implies that in case the truth value of φ at t is entirely determined by what has happened up to that time then the truth of φ will entail that of $D\varphi$. But if the truth value of φ at t depends on what happens after t - something which is particularly likely to arise when φ has itself the form $F\psi$ - this need not be so.

The question which sentences depend for their truth value at t only on what has been the case at times not later than t is one which cannot be decided on the strength of their tense structure alone. English has, we have already noted, sentences which although they are in the past or present tense are nevertheless forward looking - recall the word 'correct', as it occurs in such sentences as 'Jones's prediction about tomorrow's weather is not correct'.

There do not seem to be many lexical items in English, which cause the sentences that contain them to depend for their truth values on what is going to happen, even if the sentence does not contain the future tense. Moreover it may well be the case that all sentences which are forward looking because they contain such expressions are capable of paraphrases in which the forward looking aspect is made explicit through the use of the future tense. 'Jone's prediction about tomorrow's weather is not correct', for example, could be paraphrased as "There is a property P such that Jones has predicted that tomorrow's weather will have P , but as a matter of fact tomorrow's weather will not have P . "

If we assume that this can always be done and that the truth conditions for sentences containing such expressions may therefore be specified via those for their paraphrases, it becomes possible to insist that each atomic sentence q_i of $L(P,F,D)$, in as much as it is meant to represent a sentence of the simplest possible form, i.e. a sentence in the present tense, has a truth value in w at t which is independent of what happens in w after t. This implies that if $w' \approx_t w$ q_i must have the same truth value in w' at t as it has at t in w . However, as I do not know if forward looking lexical items are always eliminable in this way I have decided to consider also models in which this condition on the truth values of atomic sentences ~~does~~ is not ~~have to be~~ necessarily fulfilled. Models in which the condition holds without exception will be called well-founded.

Since the intended meaning of "$w \approx_t w'$" is that up to t w and w' are really 'the same world' the reader may wish to question the appropriateness of the description I am proposing here. Is it not misleading to speak as if w and w' were fully distinct worlds - even at times where they coincide - and would it therefore not be better to base our models on structures whose elements are not complete histories but rather what might be called 'worlds-up-to-time t', i.e. histories which run from the beginning of time up to, and including t, and which would specify (instead of the relation $\approx$ ~~such a structure would specify~~, for any times t and t' where t' is later than t, which worlds w' up-to- t' extend which worlds w up-to- t, - extend in the sense that w is an initial segment of w' ?

I mention this alternative primarily in order to point out that for the formalization of that sense of the future tense that I advocated earlier it will not do. For what could we say about the truth value of $F\varphi$ at t in w, where w is a "world-up-to t"? I have proposed that such a sentence is to be taken as true at t in w if, as a matter of fact, what φ says turns out to be the case at some later time. But the structures we have just been considering leave us no room for saying this.

The condition cannot be expressed because on the here relevant conception of the future tense future tense sentences - or at least some of them - are essentially forward looking. The truth of such a sentence in w at t is a property of t which transcends the total of that which is, one might say, 'given' at t, whereas the structures based on worlds up-to-t would obliterate any such 'transcendental' predicates. *) To state the truth ~~conditions for sentences of the~~

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*) This last observation makes perhaps more evident than it has been so far the apparent oddity - absurdity, some might want to say - of the position I have defended. Indeed it is undoubtedly this 'oddity' which has led some people to adopt the Aristotelian view which, earlier on, I opposed. As a supplement to that earlier discussion (cf. section 3) let me add here one further point. The desire to avoid properties whose applicability at a time t transcends what is given at t can by itself not be a sufficient reason to adopt the Aristotelian view. For even if we adopt that position with regard to the notion of

truth we must still face the problem of transcendentality in connection with certain other concepts. For example, it would seem that the question whether a prediction made at t is correct also transcends what is 'given' at t. A diehard might perhaps insist that it is a mistake to hold that at the time it is made a prediction is either correct or incorrect. But it seems to me that in the case of correctness this even more clearly at odds with the way we use the concept than it is in the case of truth.
If on the other hand one wishes to defend the Aristotelian view without making a similar claim concerning correctness he must argue that there is something about the concept of truth specifically that conflicts with the transcendental character that my proposal imposes upon it.

conditions for sentences of the form $F\varphi$ the way I want to we must, at each time, have at our disposal not only the past and present at that time of the world in question, but also its (actual) future.

Even when we start from a structure whose worlds are worlds-up-to a time, and in which some of these worlds are extensions of others, we can construct from it a structure in which the worlds all run through the whole of time. These latter worlds can be defined as maximal chains of worlds up-to- t under the relation of extension. A similar construction is possible if we start from yet another type of structure. The elements of such a structure are to be not times and worlds-up-to-a time, but rather, what might be called 'worlds-at-a time', or '(instantaneous) states of affairs', or 'world-states'. These world states are assumed to be partially ordered by a relation $<$. "$s_1 < s_2$" is to mean that s_1 is a state that temporally precedes the state s_2. - The relation should be backwards linear, but not necessarily forwards linear, in as much as at any time the past is taken to be fully determined while the future is not. Here again we may define the worlds-through-time of my first proposal, viz. as maximal chains of world-states under the relation $<$. This is the procedure followed in particular by Thomason in [6], who calls such chains '(complete) histories'. Thomason formalizes a non-transcendental sense of the future tense, and thus needs his histories only as auxiliaries in the recursive definition of the truth value of a sentence φ at a world-state s. However, as long as we are prepared to recognize his auxiliary concept 'φ is true in t with respect to the history h' as a genuine notion in its own right, the partically ordered structures which underlie his models can serve for the analysis of this paper as well.

As it turns out not every structure based on times and worlds-through-time can be obtained by identifying the worlds as maximal chains of some partial ordering. To explain this it is better, however, to first give the

formal definition of such structures, and of the truth conditions of L(P,F,D)- sentences in models based on them.

Before we can do that, however, there are two more issues to which we must devote some informal comments.

In the first place it should be asked whether for any pair of worlds w and w' it must be assumed that there is some time t (it could be very far in the past!) up to which they coincide; or should we accept that there may be worlds which share no initial segment whatever? As far as I can see it would be unwarranted to exclude this possibility. However, for the analysis of L(P,F,D) the issue is without importance. For, as the reader may verify from the formal definitions below, these worlds which have no initial segment in common with a given world w are irrelevant to the truth value in w pertinent of any L(P,F,D)-sentence φ at any time t.

The second, and more important, issue concerns the uniformity of time. Suppose that w and w' coincide up to t but diverge after that time, then, since really they are the same world until t time in these two worlds must also be the same up to that moment. But after t w and w' may develop in ways so utterly distinct that their respective time flows become incommensurable. If we believe that certain structural properties of time, e.g. whether or not it is dense, are determined by the structure of natural events, it is even conceivable that after t the time in w differs from that of w' in respect of such structural properties.)

(It seems unreasonable to exclude this possibility. I have therefore made allowance for it in the formal definition below: with each world w is associated its own time $\mathcal{T}_w$, with the obvious restriction that if w and w' coincide up to t then the segments of $\mathcal{T}_w$ and $\mathcal{T}_{w'}$ which end at t are identical.

It is nevertheless reasonable to also consider the conceptual possibility that when ever w and w' have a common past then the laws which govern those natural phenomena that fully define time, and which these worlds must at one time ~~must~~ have shared, will warrant that in either of the now divergent worlds those phenomena persist - and in particular that both w and w' continue to manifest these periodical physical processes which in a wide (but the only reasonable!) sense of the word, may be called 'clocks' .
According to this view any two worlds that share any part of their pasts at all have the same time throughout. If moreover we assume that any two worlds have a common initial segment, then all worlds will have the same time. Structures in which this is so will be called perfect.

After these preliminary remarks the following formal definitions will be self-explanatory.

Definition 2

A model for $L(P,F,D)$ is quadruple $\langle W, \mathcal{T}, \approx, A\rangle$, where

i) W is a non-empty set (of "worlds");

ii) $\mathcal{T}$ is a function from W to linear time structures; we write $\mathcal{T}_w$ for $\mathcal{T}(w)$ and will assume $\mathcal{T}_w = \langle T_w, <_w\rangle$; we let $T = \bigcup_{w \in W} T_w$.

iii) $\approx$ is a 3-place relation $\subseteq W^2 \otimes T$; we write $w \approx_t w'$ instead of $\approx (w,w',t)$; for fixed t $\approx_t$ is an equivalence relation. Moreover, if $w \approx_t w'$, then $t \in T_w \cap T_{w'}$ and the initial segments of $\mathcal{T}_w$ and $\mathcal{T}_{w'}$ which end with t coincide. Finally, if $w \approx_t w'$ and $t' <_w t$ then $w \approx_{t'} w'$.

iv) A assigns to each q_i , each $w \in W$ and each $t \in T_w$ a truth value. A model $\mathfrak{M} = \langle W, \mathcal{T}, \approx, A\rangle$ is well-founded if for all

w,w',t,q_i, if $w \approx_t w'$ then $A(q_i,w,t) = A(q_i,w',t)$ *)

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*) The concept of a model given in Definition 2, and the truth definition for $L(P,F,D)$ in terms of it the relevant clauses of which are to be found on p. 25 below, are virtually identical with those formulated by Montague in [9] (cf. p.112). The only differences with Montague's proposal are
i) he defines the relation $\approx$ in terms of the values which the model assigns to the non-logical constants (in our case these would be just the sentence letters $q_1, q_2, \ldots$);
ii) the definition specifies that $w \approx_t w'$ provided each logical constant gets the same value in w and w' at all times t' less than t (and not: less than or equal to t).
The first one to have worked with models of this kind appears to have been Dana Scott, who launched the idea as far back as 1965 (see e.g. [12]). Scott too defined $\approx$ in terms of the truth values of atomic sentences, and also requires for w w' only that these values coincide at times preceding t. He moreover considers only integer-like time (in what is to follow here it will be dense time that plays a dominant role; this, as will emerge, in part for opportunistic reasons); and he seems to have avoided, at least at first, the introduction of tense operators into the object language. In stead he combines D with a deontic operator, a combination also to be found in Chellas [5], where tense operators are added in addition. (Chellas has in particular an operator 'it was the case the last time before now', which of course makes sense only within the context of discrete time). The same ideas are also to be found in the work of Aquist (see [1], [2]). Wiggins, in [14], proposes a characterization of 'it is inevitable that q' which is in the spirit of Montague, Scott and Chellas, rather than of the present paper, in as much as the quoted expression ~~it is~~ to be taken as true as long as q holds in all worlds which coincide with the world of evaluation up to, but not necessarily including, the time of evaluation. (Wiggins' notation, it should be added here, is in the spirit of the language $L(D_t)$, discussed in section VI below, rather than of the language $L(P,F,D)$ under discussion now. Nevertheless it seems preferable to already refer to his work at this point.)

If we forbear defining the relation $\approx$ in the way indicated above the difference between ~~all~~ these proposals and the present one disappears. But it reëmerges when one tries to give an intuitively correct definition of well-foundedness. The view represented by the mentioned authors suggests not the definition given in the main text but rather:

M is <u>well-founded</u> iff for all w, w', t, q_1 if $w \approx_t w'$ then for all $t' < t$ $A(q_1, w, t') = A(q_1, w', t')$.

Scott established completeness for his system; but as he leaves the tense-operators out of it his result is incommensurable with the problem that will occupy us through much of this paper. Chellas suggests axioms for his system, but neither he nor, for that matter, Montague or Wiggins seem to have broached the problem of completeness.

The ideas of Scott are manifest not only in the present paper, but also in my 'Semantics vs. Pragmatics' (in: Guenthner & Schmidt (eds.) <u>Formal Semantics and Pragmatics of Natural Languages</u>. Dordrecht '78), where I failed to include an appropriate acknowledgement. I hereby wish to apologize for that omission.

Wiggins' essay, as well as all I have learned from him over the years concerning the issues with which his essay deals or this one tries to deal, have helped me very much in my efforts to think about the questions to some of which I have endeavored here to find ~~more~~ formal answers.

My indebtedness to Montague is so pervasive that it would be almost misleading to reaffirm it here in the specific context of the reference to his work that appears in this footnote.

M is called <u>perfect</u> iff for all $w, w' \in W$ $\mathcal{T}_w = \mathcal{T}_{w'}$. Structures $\langle W, \mathcal{T}, \approx \rangle$ satisfying the conditions i) - iii) above will be called L(P,F,D)-<u>frames.</u>

¶ The <u>truth value of</u> a sentence φ of L(P,F,D) <u>in</u> a model $\mathcal{M}$ <u>in</u> w <u>at</u> t , $[\varphi]_{\mathcal{M},w,t}$, is defined much like we defined truth for L(P,F). The important clauses of the definition are:

$$[P\varphi]_{\mathcal{M},w,t} = 1 \text{ iff } (\exists t' \in T_w)(t' <_w t \wedge [\varphi]_{\mathcal{M},w,t'} = 1)$$

$$[F\varphi]_{\mathcal{M},w,t} = 1 \text{ iff } (\exists t' \ T_w)(t <_w t' \wedge [\varphi]_{\mathcal{M},w,t'} = 1)$$

$$[D\varphi]_{\mathcal{M},w,t} = 1 \text{ iff } (\forall w' \in W)(w' \approx_t w \rightarrow [\varphi]_{\mathcal{M},w',t} = 1)$$

~~I will also refer to triples ⟨W, , ⟩ satisfying the conditions i) - iii) of the Definition 2 as frames.~~ As before we define: φ is <u>valid with respect to</u> the time structure $\mathcal{T}$ iff for all models $\mathcal{M} = \langle W, \mathcal{T}', \approx, A \rangle$ such that for all $w \in W$ $\mathcal{T}'_w = \mathcal{T}$: for all $w \in W$ and for all $t \in T$, $[\varphi]_{\mathcal{M},w,t} = 1$; and φ <u>is valid with respect to</u> the class $\mathcal{K}$ of time structures iff for every model $\mathcal{M} = \langle W, \mathcal{T}, \approx, A \rangle$ such that for all $w \in W$ $\mathcal{T}_w \in \mathcal{K}$: for all $w \in W$ and for all $t \in T_w$ $[\varphi]_{\mathcal{M},w,t} = 1$. Both these notions are special cases of the mere general concept - is <u>valid with respect to</u> the class of models $\mathfrak{M}$ if for each $\mathcal{M} = \langle W, \mathcal{T}, \approx, A \rangle \in \mathfrak{M}$, each $w \in W$ and each $t \in T_w$ $[\varphi]_{\mathcal{M},w,t} = 1$.

¶ We are now in a position to elucidate an earlier remark, to the effect that not every L(P,F,D)-frame is obtainable from a partially ordered structure of world-states. To facilitate the discussion let us agree to call such world-state structures T-structures *) **). To be precise a <u>T-structure</u> is a structure $\langle S, \propto \rangle$ where i) S is a non-empty set of instantaneous world-states).

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*) By a fortunate coincidence 'T' is the first letter both of the word 'tree' and the word 'Thomason'. See [13]

**) Remarks similar to those found on the next to pages have also been made by Nishimura. See [10].