A Roadmap of my Thesis:

"The Power of Tree Series Transducers"

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April 12, 2006

Applications

. . . of (weighted/probabilistic) tree automata:

- Syntactic Pattern Matching (e.g. handwritten digit recognition)
  [López, Piñaga: Syntactic Pattern Recognition by Error Correcting Analysis on Tree Automata, 2000]
- Tree Banks
  [Liakata, Pulman: Learning Theories from Text, 2004]

. . . of tree series transducers:

- Code Selection
  [Borchardt: Code Selection by Tree Series Transducers, 2004]
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Motivation

Tree Series Substitution

Tree Series Transducers

Deterministic Tree Series Transducers

Nondeterministic Tree Series Transducers

Compositions of Transformations

Trees

Σ ranked alphabet, Z set

Definition:

Set $T_\Sigma(\mathrm{Z})$ of trees is smallest T such that

- $\mathrm{Z} \subseteq T$
- $\sigma(t_1, \ldots, t_k) \in T$ for all k, $\sigma \in \Sigma^{(k)}$, and $t_1, \ldots, t_k \in T$

Example:

ranked alphabet $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$ and $\mathrm{Z} = \{z_1, z_2, \ldots\}$

- z_1 z_2 $\cdots$ α $\sigma(z_1, z_1)$ $\cdots$ $\sigma(\alpha, \alpha)$ $\cdots$
- $T = \{z_1, z_2, \ldots\}$

Abbreviation:

$T_\Sigma := T_\Sigma(\emptyset)$

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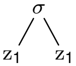
- z_1 z_2 $\cdots$ α 

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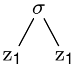
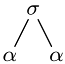
- z_1 z_2 $\cdots$ α  $\cdots$  $\cdots$
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Semirings

Definition:

algebraic structure $(A, +, \cdot, 0, 1)$ such that

- $(A, +, 0)$ commutative monoid
- $(A, \cdot, 1)$ monoid
- $\cdot$ distributes (both-sided) over $+$
- 0 is absorbing wrt. $\cdot$

Examples:

- natural numbers $(\mathbb{N}, +, \cdot, 0, 1)$
- reals $(\mathbb{R}, +, \cdot, 0, 1)$
- subsets $(\mathcal{P}(S), \cup, \cap, \emptyset, S)$
- any ring, field, distributive complete lattice

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Complete Semirings

Definition:

Semiring is complete, if there exists $\sum$ such that

- $\sum_{i \in \{j_1, j_2\}} a_{j_1} + a_{j_2}$ with $j_1 \neq j_2$
- $\sum_{i \in I} a_i = \sum_{j \in J} (\sum_{i \in I_j} a_i)$ with $I = \bigcup_{j \in J} I_j$ such that $I_{j_1} \cap I_{j_2} = \emptyset$ $(j_1 \neq j_2)$
- $(\sum_{i \in I} a_i) \cdot (\sum_{j \in J} a_j) = \sum_{(i,j) \in I \times J} (a_i \cdot a_j)$

Examples:

- natural numbers $(\mathbb{N} \cup \{\infty\}, +, \cdot, 0, 1)$
- tropical semiring $(\mathbb{N} \cup \{\infty\}, \min, +, \infty, 0)$
- subsets $(\mathcal{P}(S), \cup, \cap, \emptyset, S)$
- any distributive complete lattice (but no ring or field)

Note:

It follows that $\sum_{i \in \emptyset} a_i = 0$ and $\sum_{i \in \{j\}} a_i = a_j$.

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Tree Series

Σ ranked alphabet, $\mathcal{A} = (A, +, \cdot, 0, 1)$ semiring

Definition:

mapping $\psi \colon S \longrightarrow A$ with $S \subseteq T_\Sigma(\mathrm{Z})$

Notation:

- (ψ, s) denotes $\psi(s)$
- $\mathrm{supp}(\psi) = \{ s \in S \mid (\psi, s) \neq 0 \}$
- $\widetilde{0}$ such that $\mathrm{supp}(\widetilde{0}) = \emptyset$
- $(\psi + \varphi, s) = (\psi, s) + (\varphi, s)$
- $A\langle\langle S \rangle\rangle$ set of all tree series over $\mathcal{A}$ and S
- $A\langle S \rangle$ set of all polynomial (i.e., finite support) tree series

Definition:

- ψ linear, if t linear for all $t \in \mathrm{supp}(\psi)$
- ψ nondeleting, if t nondeleting for all $t \in \mathrm{supp}(\psi)$

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Let $\psi, \psi_1, \ldots, \psi_n \in A\langle\langle T_\Sigma(\mathrm{Z}_n)\rangle\rangle$.

$$\psi \leftarrow (\psi_1, \ldots, \psi_n) = \sum_{\substack{t \in \mathrm{supp}(\psi), \\ t_1 \in \mathrm{supp}(\psi_1), \\ \ldots, \\ t_n \in \mathrm{supp}(\psi_n)}} (\psi, t) \cdot (\psi_1, t_1) \cdot \ldots \cdot (\psi_n, t_n)\, t[t_1, \ldots, t_n]$$

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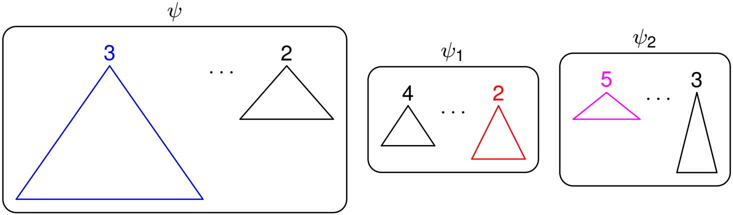
Illustration



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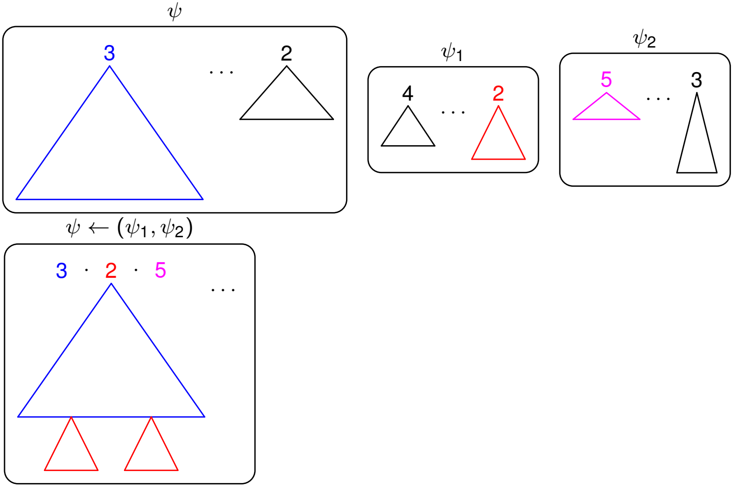

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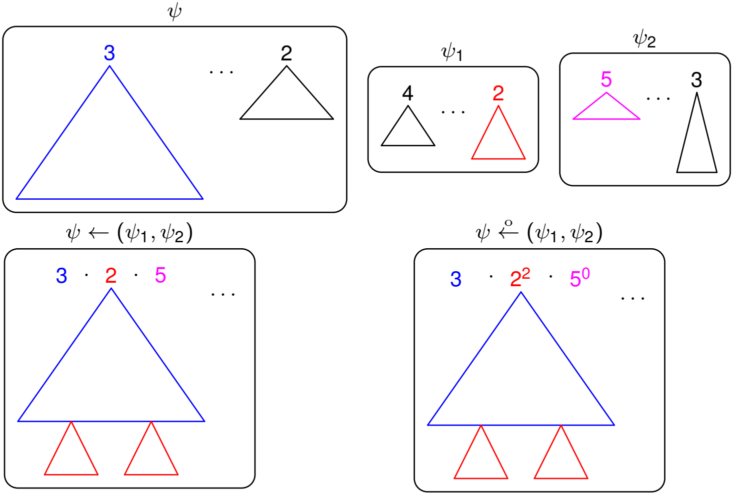

Notes on Substitution

- pure substitution introduced in
  [Bozapalidis: Context-free Series on Trees, 2001]
- o-substitution introduced in
  [Fülöp, Vogler: Tree Series Transformations that Respect Copying, 2003]
- potentially infinite sum
- usually only considered for polynomial tree series or in complete semirings

Properties:

- distributive:

$$\sum_{\substack{i \in I, \\ i_1 \in I_1, \dots, i_n \in I_n}} \psi_i \overset{?}{\longleftarrow} (\psi_{i_1}, \dots, \psi_{i_n}) = \Big(\sum_{i \in I} \psi_i\Big) \overset{?}{\longleftarrow} \Big(\sum_{i_1 \in I_1} \psi_{i_1}, \dots, \sum_{i_n \in I_n} \psi_{i_n}\Big)$$

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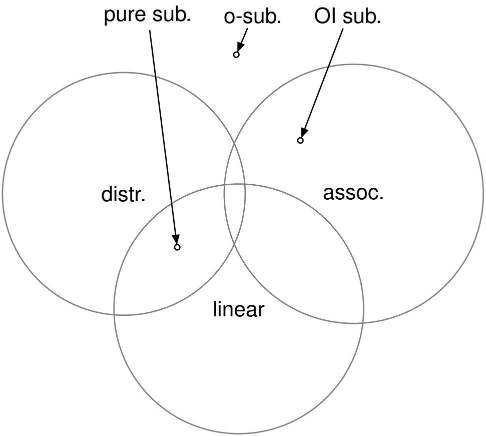
Overview



Some Results

Theorem:

	pure	
distributive	yes	
linear	commutative $\mathcal{A}$	

Theorem:

- $\mathcal{A}$ commutative, continuous, and idempotent; ψ linear

$$\psi \overset{\circ}{\leftarrow} (\psi_1, \ldots, \psi_n) \quad \text{recognizable}$$

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Syntax

Definition:

$(Q, \Sigma, \Delta, \mathcal{A}, F, \mu)$ tree series transducer (tst), if

- Q finite set (of *states*)
- Σ and Δ ranked alphabets
- $\mathcal{A}$ semiring
- $F \in A\langle\langle C_\Delta(\mathrm{Z}_1)\rangle\rangle^Q$
- $\mu = (\mu_k)_{k \in \mathbb{N}}$ with $\mu_k \colon \Sigma_k \longrightarrow A\langle\langle T_\Delta(\mathrm{Z})\rangle\rangle^{Q \times Q(\mathrm{Z}_k)^*}$ such that
  - $\mu_k(\sigma)_{q,w} \in A\langle\langle T_\Delta(\mathrm{Z}_{|w|})\rangle\rangle$
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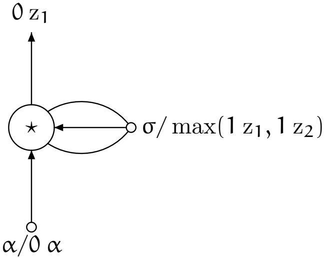

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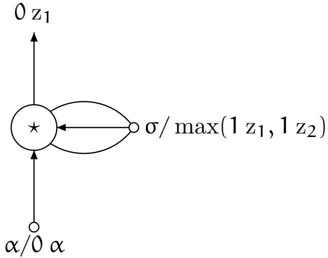

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Bottom-up and Top-down

Definition:

Tst $(Q, \Sigma, \Delta, \mathcal{A}, F, \mu)$ is

- bottom-up (bu), if $w = q_1(z_1) \cdots q_k(z_k)$ for every k, $\sigma \in \Sigma^{(k)}$, $q \in Q$, and $w \in Q(Z_k)^*$ such that $\mu_k(\sigma)_{q,w} \neq \widetilde{0}$
- top-down (td), if $\mu_k(\sigma)_{q,w}$ nondeleting and linear for every k, $\sigma \in \Sigma^{(k)}$, $q \in Q$, and $w \in Q(Z_k)^*$

Example:

is bottom-up, but not top-down!

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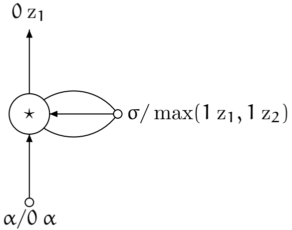

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Semantics

Tst $(Q, \Sigma, \Delta, \mathcal{A}, F, \mu)$

Definition:

$h_\mu^{?} \colon T_\Sigma \longrightarrow A\langle\!\langle T_\Delta \rangle\!\rangle^Q$

$$h_\mu^{?}(\sigma(t_1, \dots, t_k))_q = \sum_{\substack{w \in Q(\mathrm{Z}_k)^* \\ w = q_1(\mathrm{z}_{i_1}) \cdots q_n(\mathrm{z}_{i_n})}} \mu_k(\sigma)_{q,w} \overset{?}{\leftarrow} (h_\mu^{?}(t_{i_1})_{q_1}, \dots, h_\mu^{?}(t_{i_n})_{q_n})$$

Example:

Input tree: $\sigma(\sigma(\alpha, \alpha), \alpha)$

$$h_\mu^{o}(\alpha)_\star = 0\ \alpha$$

$$h_\mu^{o}(\sigma(\alpha, \alpha))_\star = \max(1\ \mathrm{z}_1, 1\ \mathrm{z}_2) \overset{o}{\leftarrow} (0\ \alpha, 0\ \alpha) = 1\ \alpha$$

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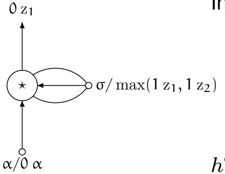

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$$h^{\text{o}}_\mu(\sigma(\alpha, \alpha))_\star = \max(1\ z_1, 1\ z_2) \overset{\text{o}}{\longleftarrow} (0\ \alpha, 0\ \alpha) = 1\ \alpha$$

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Semantics

Tst $M = (Q, \Sigma, \Delta, \mathcal{A}, F, \mu)$

Definition:

Transformation computed by M

Tree Level $\|M\|^?: T_\Sigma \longrightarrow A\langle\langle T_\Delta \rangle\rangle$:

$$\|M\|^?(t) = \sum_{q \in Q} F_q \xleftarrow{?} (h_\mu^?(t)_q)$$

Series Level $\|M\|^?: A\langle\langle T_\Sigma \rangle\rangle \longrightarrow A\langle\langle T_\Delta \rangle\rangle$:

$$\|M\|^?(\psi) = \sum_{t \in \mathrm{supp}(\psi)} (\psi, t) \cdot \|M\|^?(t)$$

Example:

Let $M =$

Then $\|M\|^{\text{o}}(t) = \mathrm{height}(t)\ \alpha$

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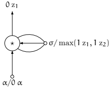
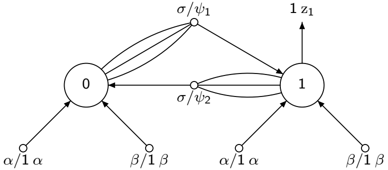
Let $M =$ 

Then $\|M\|^{\text{o}}(t) = \text{height}(t)\ \alpha$

A More Complex Example



Bottom-up vs. Top-down Determinism

Definition:

$(Q, \Sigma, \Delta, \mathcal{A}, F, \mu)$ bu-tst bu-deterministic, if

- for every k, $\sigma \in \Sigma^{(k)}$, and $q_1, \ldots, q_k \in Q$ there exists at most one $(q, u) \in Q \times T_\Delta(\mathrm{Z}_k)$ such that $(\mu_k(\sigma)_{q,w}, u) \neq 0$

I.e., deterministic state and output behavior

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- there exists at most one $(q, u) \in Q \times T_\Delta(\mathrm{Z}_1)$ such that $(F_q, u) \neq 0$

I.e., deterministic state and output behavior plus single initial state

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Summary of Properties

Definition:

$M = (Q, \Sigma, \Delta, \mathcal{A}, F, \mu)$ x-tst x-homomorphism, if

- $Q = \{\star\}$
- M is x-deterministic and x-total
- $F_\star \in \{\widetilde{0}, 1\ z_1\}$ (i.e., final state; no final weight)

Abbrev.	Property	Short description
d	determinism	unambiguous state and output behavior
t	totality	nonblocking state and output behavior
n	nondeletion	whole input tree is processed (td) and no processed part is deleted (bu)
l	linearity	each part of input tree processed only once (td) and no processed part is duplicated (bu)
h	homomorphism	single state, total and deterministic with final state

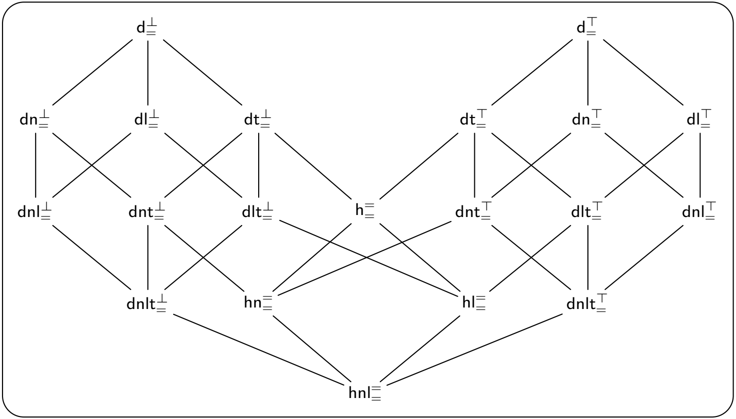
Semiring Properties

Definition:

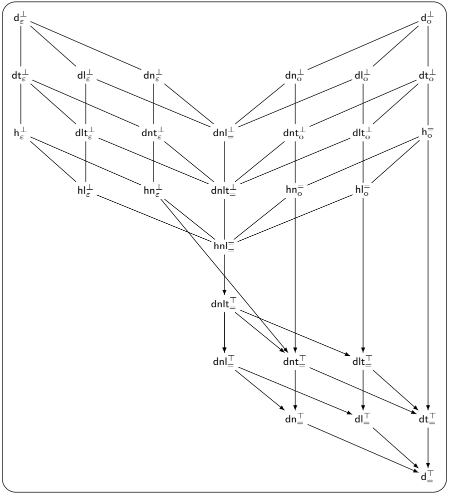
Semiring $\mathcal{A} = (A, +, \cdot, 0, 1)$

- commutative, if $a \cdot b = b \cdot a$ for every $a, b \in A$
- (mult.) periodic, if $\{ a^n \mid n \in \mathbb{N} \}$ is finite for every $a \in A$
- zero-divisor free, if $a \cdot b = 0$ implies that $0 \in \{a, b\}$
- (mult.) idempotent, if $a \cdot a = a$ for every $a \in A$

Boolean Semiring



Nonperiodic Semirings without Zero-Divisors



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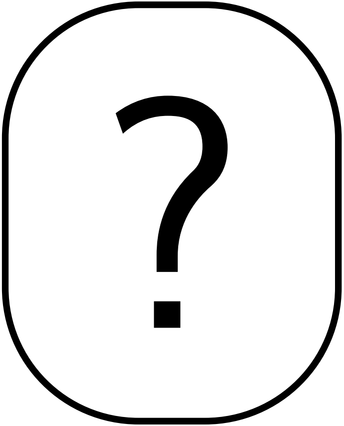
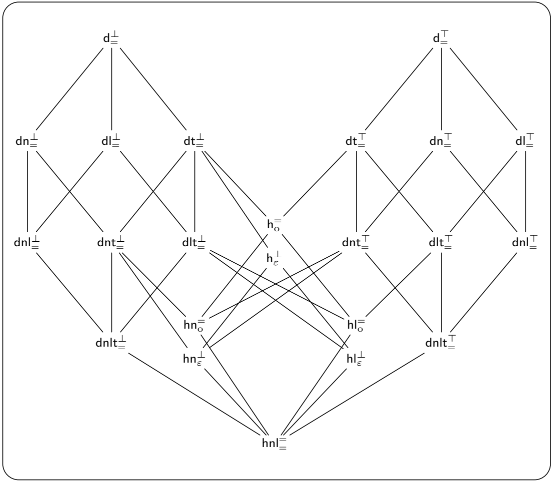


Periodic Semirings without Zero-Divisors



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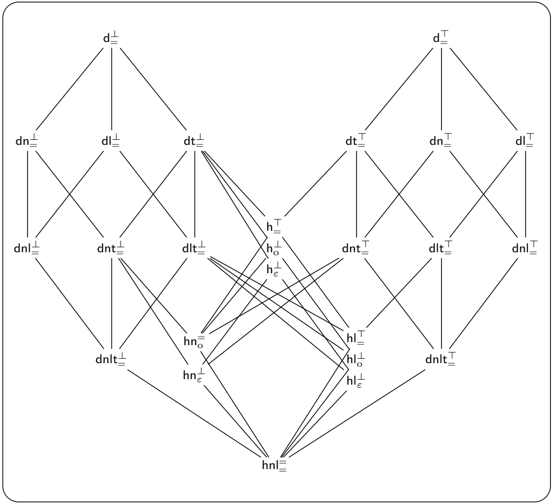
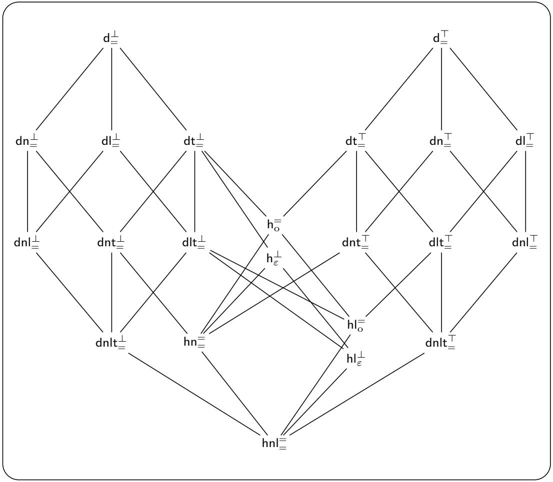


Idempotent Semirings without Zero-Divisors



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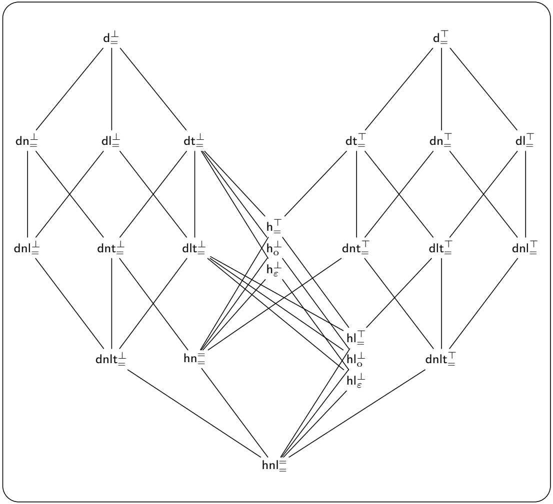

Main Result

$\mathbb{A} = (\mathbb{N} \cup \{-\infty\}, \max, +, -\infty, 0)$ and $\mathbb{L}_S = (\mathcal{P}(S^*), \cup, \circ, \emptyset, \{\varepsilon\})$

Theorem:

[Fülöp, Vogler: Tree Series Transformations that Respect Copying, 2003]

$$\text{p-BOT}(\mathbb{N}) \bowtie \text{p-BOT}^{\text{o}}(\mathbb{N})$$
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$$\text{p-BOT}(\mathbb{L}_S) \bowtie \text{p-BOT}^{\text{o}}(\mathbb{L}_S)$$

Definition:

semiring $(A, +, \cdot, 0, 1)$ ordered by $\leq \subseteq A^2$ if

- $a + a' \leq b + b'$ provided that $a \leq b$ and $a' \leq b'$
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General Result

Definition:

$(A, +, \cdot, 0, 1)$ weakly growing if there exists $a \in A$ such that

- $a^0 < a^1 < a^2 < a^3 < \cdots$
- if $a^n = c + (b_1 \cdot b \cdot b_2)$ then there exists m such that $b \leq a^m$

Theorem:

$\mathcal{A}$ (add.) idempotent and weakly growing wrt. $\leq$

$$\text{p-BOT}(\mathcal{A}) \bowtie \text{p-BOT}^{\text{o}}(\mathcal{A})$$

Open:

How to prove a general statement including $\mathbb{N}$?

Possible approach: Use (bounded) closure of sets instead of partial order!

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Composition

$\tau_1 : T_\Sigma \longrightarrow A\langle\langle T_\Delta \rangle\rangle$ and $\tau_2 : T_\Delta \longrightarrow A\langle\langle T_\Gamma \rangle\rangle$; complete semiring $\mathcal{A} = (A, +, \cdot, 0, 1)$

Definition:

composition of τ_1 and τ_2; denoted by $\tau_1 \,;\, \tau_2$

$$(\tau_1 \,;\, \tau_2, t) = \sum_{u \in T_\Delta} (\tau_1(t), u) \cdot \tau_2(u)$$

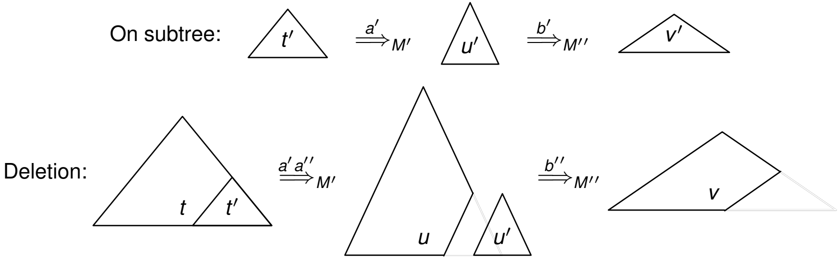
Theorem:

[Engelfriet, Fülöp, Vogler: Bottom-up and Top-down Tree Series Transformations, 2002]

$\mathcal{A}$ commutative

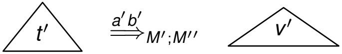
- nlp-BOT$(\mathcal{A})$; h-BOT$(\mathcal{A})$ = p-BOT$(\mathcal{A})$
- p-BOT$(\mathcal{A})$; bh-BOT$(\mathcal{A})$ = p-BOT$(\mathcal{A})$

The Problem

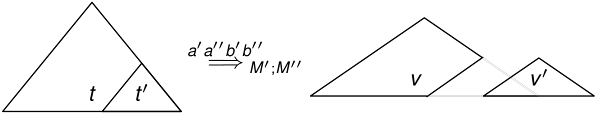


The Problem

On subtree:



Deletion:



Main Theorem

Theorem:

$\mathcal{A}$ commutative

$$\text{lp-BOT}(\mathcal{A}) \,;\, \text{p-BOT}(\mathcal{A}) = \text{p-BOT}(\mathcal{A})$$
$$\text{lp-BOT}(\mathcal{A}) \,;\, \text{lp-BOT}(\mathcal{A}) = \text{lp-BOT}(\mathcal{A})$$
$$\text{nlp-BOT}(\mathcal{A}) \,;\, \text{nlp-BOT}(\mathcal{A}) = \text{nlp-BOT}(\mathcal{A})$$

Theorem:

$\mathcal{A}$ commutative

$$\text{p-BOT}(\mathcal{A}) \,;\, \text{bd-BOT}(\mathcal{A}) = \text{p-BOT}(\mathcal{A})$$

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Thank you for your attention!