Composition Closure of ε-free Linear Extended Top-down Tree Transducers

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Contents

Motivation

Tree transducer

- used in statistical machine translation [Knight, Graehl 2005]
- used in XML query processing [Benedikt et al. 2013]

Motivation

Tree transducer

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Compositions

- $\tau_1 ; \tau_2 = \{(s, u) \mid (s, t) \in \tau_1, (t, u) \in \tau_2\}$
- support modular development
- allow integration of external knowledge sources
- occur naturally in query rewriting

Problem

Question:

Given a class $\mathcal{C}$ of transformations, is there $n \in \mathbb{N}$ such that

$$\mathcal{C}^n = \bigcup_{k \geq 1} \mathcal{C}^k \qquad \mathcal{C}^k = \underbrace{\mathcal{C} \, ; \cdots ; \mathcal{C}}_{k \text{ times}}$$

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Given a class $\mathcal{C}$ of transformations, is there $n \in \mathbb{N}$ such that

$$\mathcal{C}^n = \bigcup_{k \geq 1} \mathcal{C}^k$$

$$\mathcal{C}^k = \underbrace{\mathcal{C} \,;\, \cdots \,;\, \mathcal{C}}_{k \text{ times}}$$

Note

- $\mathcal{C}^k \subseteq \mathcal{C}^{k+1}$ for our classes $\mathcal{C}$

→ we search least n such that $\mathcal{C}^n = \mathcal{C}^{n+1}$ (if it exists)

Extended Top-down Tree Transducer

Definition (XTOP)

Linear extended top-down tree transducer $(Q, \Sigma, \Delta, I, R)$

- finite set Q *states*
- ranked alphabets Σ and Δ *input and output symbols*
- $I \subseteq Q$ *initial states*
- finite set $R \subseteq T_\Sigma(Q) \times Q \times T_\Delta(Q)$ *rules*
  - each $q \in Q$ occurs at most once in ℓ and r $(\ell, q, r) \in R$
  - each $q \in Q$ that occurs in r also occurs in ℓ $(\ell, q, r) \in R$

Extended Top-down Tree Transducer

Example

XTOP $M_1 = (Q, \Sigma, \Sigma, \{\star\}, R)$

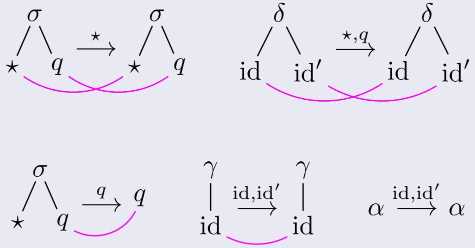
- $Q = \{\star, q, \mathrm{id}, \mathrm{id}'\}$
- $\Sigma = \{\sigma^{(2)}, \delta^{(2)}, \gamma^{(1)}, \alpha^{(0)}\}$
- the following rules in R:

$$\sigma(\star, q) \xrightarrow{\star} \sigma(\star, q) \qquad \sigma(\star, q) \xrightarrow{q} q$$

$$\delta(\mathrm{id}, \mathrm{id}') \xrightarrow{\star, q} \delta(\mathrm{id}, \mathrm{id}') \qquad \gamma(\mathrm{id}) \xrightarrow{\mathrm{id}, \mathrm{id}'} \gamma(\mathrm{id}) \qquad \alpha \xrightarrow{\mathrm{id}, \mathrm{id}'} \alpha$$

Extended Top-down Tree Transducer

Graphical representation



Extended Top-down Tree Transducer

Definition (Syntactic properties)

XTOP $(Q, \Sigma, \Delta, I, R)$ is

- linear top-down tree transducer (TOP)
  if ℓ contains exactly one element of Σ $\quad (\ell, q, r) \in R$

Extended Top-down Tree Transducer

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XTOP $(Q, \Sigma, \Delta, I, R)$ is

- linear top-down tree transducer (TOP)
  if ℓ contains exactly one element of Σ $\qquad (\ell, q, r) \in R$
- ε-free (resp. strict)
  if $\ell \notin Q$ (resp. $r \notin Q$) $\qquad (\ell, q, r) \in R$

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Extended Top-down Tree Transducer

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XTOP $(Q, \Sigma, \Delta, I, R)$ is

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- ε-free (resp. strict)
  if $\ell \notin Q$ (resp. $r \notin Q$) — $(\ell, q, r) \in R$
- delabeling if it is a TOP and
  r contains at most one element of Δ — $(\ell, q, r) \in R$
- nondeleting
  if the same elements of Q occur in ℓ and r — $(\ell, q, r) \in R$

Extended Top-down Tree Transducer

- linear top-down tree transducer (TOP) if ℓ contains exactly one element of Σ $\qquad (\ell, q, r) \in R$

$$\sigma(\star, q) \xrightarrow{\star} \sigma(\star, q) \qquad \sigma(\star, q) \xrightarrow{q} q$$

$$\delta(\mathrm{id}, \mathrm{id}') \xrightarrow{\star, q} \delta(\mathrm{id}, \mathrm{id}') \qquad \gamma(\mathrm{id}) \xrightarrow{\mathrm{id}, \mathrm{id}'} \gamma(\mathrm{id}) \qquad \alpha \xrightarrow{\mathrm{id}, \mathrm{id}'} \alpha$$

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Extended Top-down Tree Transducer

- linear top-down tree transducer (TOP)
  if ℓ contains exactly one element of Σ $\qquad M_1$: ✓
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Extended Top-down Tree Transducer

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  if ℓ contains exactly one element of Σ — M_1: ✓
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Extended Top-down Tree Transducer

- linear top-down tree transducer (TOP)
  if ℓ contains exactly one element of Σ — M_1: ✓
- ε-free (resp. strict)
  if $\ell \notin Q$ (resp. $r \notin Q$) — M_1: ✓(✗)
- delabeling if it is a TOP and
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- nondeleting
  if the same elements of Q occur in ℓ and r — $(\ell, q, r) \in R$

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Extended Top-down Tree Transducer

- linear top-down tree transducer (TOP)
  if ℓ contains exactly one element of Σ — M_1: ✓
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  if $\ell \notin Q$ (resp. $r \notin Q$) — M_1: ✓(✗)
- delabeling if it is a TOP and
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$$\sigma(\star, q) \xrightarrow{\star} \sigma(\star, q) \qquad \sigma(\star, q) \xrightarrow{q} q$$

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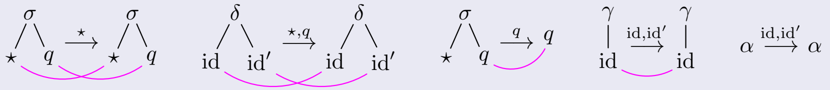
Extended Top-down Tree Transducer

Rules



Extended Top-down Tree Transducer

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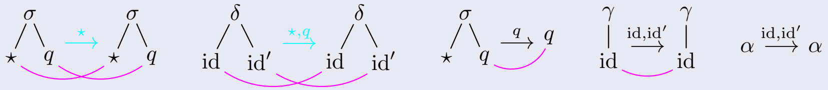

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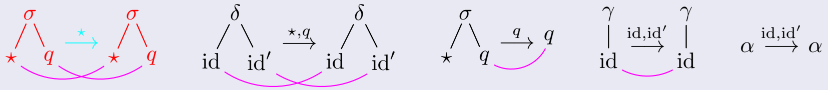

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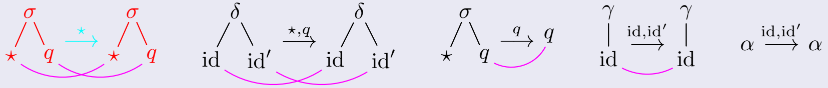

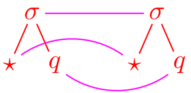

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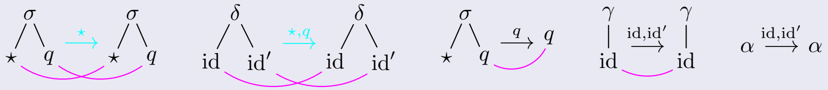

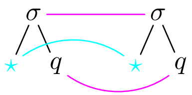

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Extended Top-down Tree Transducer

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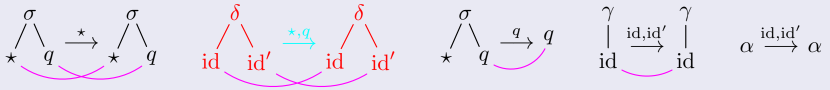

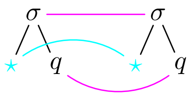

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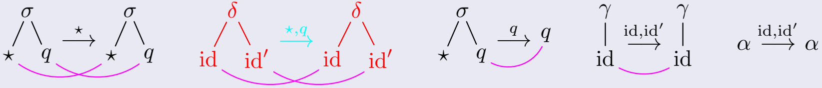

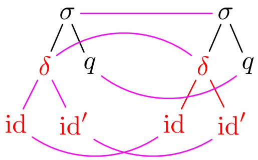

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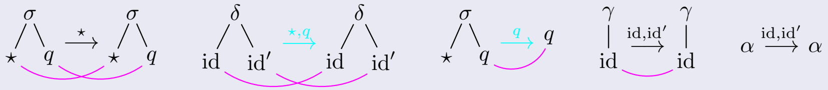

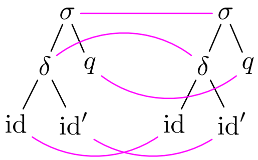

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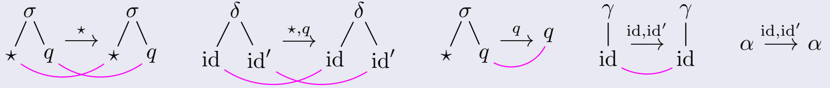

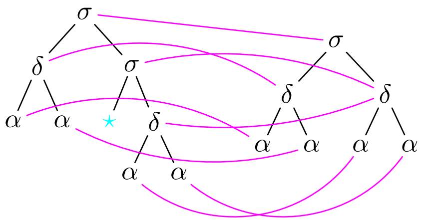

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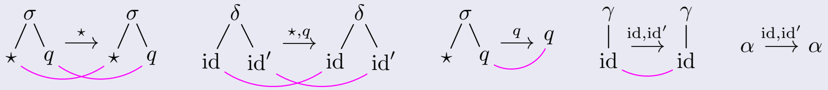
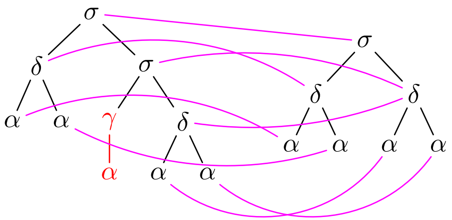
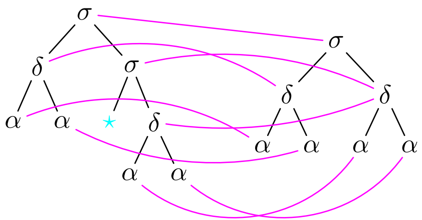




Extended Top-down Tree Transducer

Look-ahead

XTOP with regular look-ahead add map $c\colon Q \to \mathrm{Reg}(\Sigma)$ (regular tree language)



Only insertion of $t \in c(\star)$ possible

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Extended Top-down Tree Transducer

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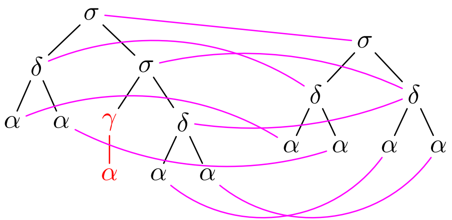

Only insertion of $t \in c(\star)$ possible

Extended Top-down Tree Transducer

Semantics

- Computed dependencies:

$$M_q = \{(t, D, u) \mid t \in T_\Sigma, u \in T_\Delta, (q, D_0, q) \Rightarrow_M^* (t, D, u)\}$$

- Computed transformation:

$$\tau_M = \{(t, u) \mid (t, D, u) \in \bigcup_{q \in I} M_q\}$$

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Contents

Overview

	TOP	XTOP
ε-free, strict, nondeleting	1	
ε-free, strict	2	
ε-free	2	
otherwise (without delabeling)	2	

Overview

	TOP	XTOP
ε-free, strict, nondeleting	1	2
ε-free, strict	2	??? (2)
ε-free	2	??? (4)
otherwise (without delabeling)	2	??? (∞)

Delabelings move around

$\not\varepsilon$ = ε-free; d = delabeling

s = strict; n = nondeleting

Theorem

Switch delabeling from back to front:

$$\not\varepsilon[\mathsf{s}]\text{-XTOP}^R \,;\, [\mathsf{s}]\mathsf{d}\text{-TOP}^R \subseteq \not\varepsilon[\mathsf{s}]\text{-XTOP}^R \subseteq [\mathsf{s}]\mathsf{d}\text{-TOP}^R \,;\, \not\varepsilon\mathsf{sn}\text{-XTOP}$$

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Notes

Delabelings move around

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s = strict; n = nondeleting

Theorem

Switch delabeling from back to front:

$$\not{e}[s]\text{-XTOP}^R ; [s]d\text{-TOP}^R \subseteq \not{e}[s]\text{-XTOP}^R \subseteq [s]d\text{-TOP}^R ; \not{e}sn\text{-XTOP}$$

Notes

- other transducer becomes strict and nondeleting

Delabelings move around

$\not\varepsilon$ = ε-free; d = delabeling
s = strict; n = nondeleting

Theorem

Switch delabeling from back to front:

$$\not\varepsilon\text{[s]-XTOP}^{\text{R}} \,;\, \text{[s]d-TOP}^{\text{R}} \subseteq \not\varepsilon\text{[s]-XTOP}^{\text{R}} \subseteq \text{[s]d-TOP}^{\text{R}} \,;\, \not\varepsilon\text{sn-XTOP}$$

Notes

- other transducer becomes strict and nondeleting
- other transducer looses look-ahead

ε-free and look-ahead

$\not\varepsilon$ = ε-free; d = delabeling

s = strict; n = nondeleting

Theorem

$(\not\varepsilon[\mathrm{s}]\text{-XTOP}^{\mathrm{R}})^n \subseteq [\mathrm{s}]\mathrm{d}\text{-TOP}^{\mathrm{R}}$; $\not\varepsilon\mathrm{sn}\text{-XTOP}^2 \subseteq (\not\varepsilon[\mathrm{s}]\text{-XTOP}^{\mathrm{R}})^3$

ε-free and look-ahead

$\not\varepsilon$ = ε-free; d = delabeling

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Theorem

$(\not\varepsilon[\mathsf{s}]\text{-XTOP}^{\mathsf{R}})^n \subseteq [\mathsf{s}]\mathsf{d}\text{-TOP}^{\mathsf{R}}$; $\not\varepsilon\mathsf{sn}\text{-XTOP}^2 \subseteq (\not\varepsilon[\mathsf{s}]\text{-XTOP}^{\mathsf{R}})^3$

Proof.

$$
\begin{aligned}
& (\not\varepsilon[\mathsf{s}]\text{-XTOP}^{\mathsf{R}})^{n+1} \\
\subseteq & \\
\subseteq & \\
\subseteq & \quad \square
\end{aligned}
$$

ε-free and look-ahead

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\subseteq\ & \not\varepsilon[\text{s}]\text{-XTOP}^{\text{R}} ; [\text{s}]\text{d-TOP}^{\text{R}} ; \not\varepsilon\text{sn-XTOP}^2 \\
\subseteq\ & \\
\subseteq\ &
\end{aligned}
$$

□

ε-free and look-ahead

$\not\varepsilon$ = ε-free; d = delabeling

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Proof.

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□

ε-free and look-ahead

$\not\varepsilon$ = ε-free; d = delabeling

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Theorem

$(\not\varepsilon[\mathrm{s}]\text{-XTOP}^{\mathrm{R}})^n \subseteq [\mathrm{s}]\mathrm{d}\text{-TOP}^{\mathrm{R}} ; \not\varepsilon\mathrm{sn}\text{-XTOP}^2 \subseteq (\not\varepsilon[\mathrm{s}]\text{-XTOP}^{\mathrm{R}})^3$

Proof.

$$\begin{aligned}
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\subseteq\ & [\mathrm{s}]\mathrm{d}\text{-TOP}^{\mathrm{R}} ; \not\varepsilon\mathrm{sn}\text{-XTOP}^3 \\
\subseteq\ & [\mathrm{s}]\mathrm{d}\text{-TOP}^{\mathrm{R}} ; \not\varepsilon\mathrm{sn}\text{-XTOP}^2
\end{aligned}$$

□

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ε-free, but no look-ahead

Corollary

$\not\varepsilon$[s]-XTOP$^n \subseteq$ QR ; [s]d-TOP ; $\not\varepsilon$sn-XTOP$^2 \subseteq \not\varepsilon$[s]-XTOP^4

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ε-free, but no look-ahead

Corollary

$\not\varepsilon$[s]-XTOP$^n \subseteq$ QR ; [s]d-TOP ; $\not\varepsilon$sn-XTOP$^2 \subseteq \not\varepsilon$[s]-XTOP^4

Proof.

Uses only standard encoding of look-ahead □

Partial results

	TOP	$\not\varepsilon$-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	
strict	2	
look-ahead	1	
—	2	

Partial results

	TOP	$\not\varepsilon$-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	
strict	2	
look-ahead	1	≤ 3
—	2	≤ 4

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Delabelings move around even more

Theorem

Delabeling homomorphism moving from front to back:

$$\text{sd-HOM} \mathbin{;} \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{sn-XTOP} \mathbin{;} \text{sd-HOM}$$

Delabelings move around even more

Theorem

Delabeling homomorphism moving from front to back:

$$\text{sd-HOM} ; \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{sn-XTOP} ; \text{sd-HOM}$$

Notes

Delabelings move around even more

Theorem

Delabeling homomorphism moving from front to back:

$$\text{sd-HOM} \,;\, \not{\epsilon}\text{s-XTOP} \subseteq \not{\epsilon}\text{s-XTOP} \subseteq \not{\epsilon}\text{sn-XTOP} \,;\, \text{sd-HOM}$$

Notes

- other transducer becomes nondeleting

Delabelings move around even more

Theorem

Delabeling homomorphism moving from front to back:

$$\text{sd-HOM} ; \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{sn-XTOP} ; \text{sd-HOM}$$

Notes

- other transducer becomes nondeleting
- other transducer needs to be strict and have no look-ahead

ε-free and strict

Theorem

$(\not\varepsilon\text{s-XTOP}^{\text{R}})^n \subseteq \not\varepsilon\text{sn-XTOP}$; $\not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP}^2$

ε-free and strict

Theorem

$(\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^n \subseteq \not\varepsilon\text{sn-XTOP} \; ; \; \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP}^2$

Proof.

$$(\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^{n+1} \subseteq (\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^n \; ; \; \not\varepsilon\text{s-XTOP}$$

$$\subseteq$$

$$\subseteq$$

$$\subseteq$$

$$\subseteq$$

$$\subseteq$$

□

ε-free and strict

Theorem

$(\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^n \subseteq \not\varepsilon\text{sn-XTOP} \,;\, \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP}^2$

Proof.

$$
\begin{aligned}
(\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^{n+1} &\subseteq (\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^n \,;\, \not\varepsilon\text{s-XTOP} \\
&\subseteq \not\varepsilon\text{sn-XTOP} \,;\, \text{sd-HOM} \,;\, \not\varepsilon\text{s-XTOP}^2 \\
&\subseteq \\
&\subseteq \\
&\subseteq \\
&\subseteq
\end{aligned}
$$

□

ε-free and strict

Theorem

$(\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^n \subseteq \not\varepsilon\text{sn-XTOP} \;;\; \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP}^2$

Proof.

$$\begin{aligned}
(\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^{n+1} &\subseteq (\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^n \;;\; \not\varepsilon\text{s-XTOP} \\
&\subseteq \not\varepsilon\text{sn-XTOP} \;;\; \text{sd-HOM} \;;\; \not\varepsilon\text{s-XTOP}^2 \\
&\subseteq \not\varepsilon\text{sn-XTOP}^3 \;;\; \text{sd-HOM} \\
&\subseteq \\
&\subseteq \\
&\subseteq
\end{aligned}$$

□

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ε-free and strict

Theorem

$(\not\varepsilon\text{s-XTOP}^{\text{R}})^n \subseteq \not\varepsilon\text{sn-XTOP} \,;\, \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP}^2$

Proof.

$$
\begin{aligned}
(\not\varepsilon\text{s-XTOP}^{\text{R}})^{n+1} &\subseteq (\not\varepsilon\text{s-XTOP}^{\text{R}})^n \,;\, \not\varepsilon\text{s-XTOP} \\
&\subseteq \not\varepsilon\text{sn-XTOP} \,;\, \text{sd-HOM} \,;\, \not\varepsilon\text{s-XTOP}^2 \\
&\subseteq \not\varepsilon\text{sn-XTOP}^3 \,;\, \text{sd-HOM} \\
&\subseteq \not\varepsilon\text{sn-XTOP}^2 \,;\, \text{sd-HOM} \\
&\subseteq \\
&\subseteq
\end{aligned}
$$

□

ε-free and strict

Theorem

$(\not\varepsilon\text{s-XTOP}^{\text{R}})^n \subseteq \not\varepsilon\text{sn-XTOP}$; $\not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP}^2$

Proof.

$$\begin{aligned}
(\not\varepsilon\text{s-XTOP}^{\text{R}})^{n+1} &\subseteq (\not\varepsilon\text{s-XTOP}^{\text{R}})^n \,;\, \not\varepsilon\text{s-XTOP} \\
&\subseteq \not\varepsilon\text{sn-XTOP} \,;\, \text{sd-HOM} \,;\, \not\varepsilon\text{s-XTOP}^2 \\
&\subseteq \not\varepsilon\text{sn-XTOP}^3 \,;\, \text{sd-HOM} \\
&\subseteq \not\varepsilon\text{sn-XTOP}^2 \,;\, \text{sd-HOM} \\
&\subseteq \not\varepsilon\text{sn-XTOP} \,;\, \not\varepsilon\text{s-XTOP}^{\text{R}} \\
&\subseteq
\end{aligned}$$

□

ε-free and strict

Theorem

$(\not\varepsilon\text{s-XTOP}^{\text{R}})^n \subseteq \not\varepsilon\text{sn-XTOP} \mathbin{;} \not\varepsilon\text{s-XTOP} \subseteq \not\varepsilon\text{s-XTOP}^2$

Proof.

$$
\begin{aligned}
(\not\varepsilon\text{s-XTOP}^{\text{R}})^{n+1} &\subseteq (\not\varepsilon\text{s-XTOP}^{\text{R}})^n \mathbin{;} \not\varepsilon\text{s-XTOP} \\
&\subseteq \not\varepsilon\text{sn-XTOP} \mathbin{;} \text{sd-HOM} \mathbin{;} \not\varepsilon\text{s-XTOP}^2 \\
&\subseteq \not\varepsilon\text{sn-XTOP}^3 \mathbin{;} \text{sd-HOM} \\
&\subseteq \not\varepsilon\text{sn-XTOP}^2 \mathbin{;} \text{sd-HOM} \\
&\subseteq \not\varepsilon\text{sn-XTOP} \mathbin{;} \not\varepsilon\text{s-XTOP}^{\text{R}} \\
&\subseteq \not\varepsilon\text{sn-XTOP} \mathbin{;} \not\varepsilon\text{s-XTOP}
\end{aligned}
$$

□

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Upper bounds

	TOP	$\not\varepsilon$-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	
strict	2	
look-ahead	1	≤ 3
—	2	≤ 4

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Upper bounds

	TOP	$\not\varepsilon$-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	≤ 2
strict	2	≤ 2
look-ahead	1	≤ 3
—	2	≤ 4

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Contents

Known result

Theorem

$\not\varepsilon\text{s-XTOP} \subsetneq \not\varepsilon\text{s-XTOP}^{\mathrm{R}} \subsetneq \not\varepsilon\text{s-XTOP}^2 = (\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^2$

Known result

Theorem

$\not\varepsilon\text{s-XTOP} \subsetneq \not\varepsilon\text{s-XTOP}^{\mathrm{R}} \subsetneq \not\varepsilon\text{s-XTOP}^2 = (\not\varepsilon\text{s-XTOP}^{\mathrm{R}})^2$

Proof.

- look-ahead adds power at first level
- none of the basic classes is closed under composition □

Upper bounds

	TOP	$\not\varepsilon$-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	≤ 2
strict	2	≤ 2
look-ahead	1	≤ 3
—	2	≤ 4

Upper bounds

	TOP	$\not\varepsilon$-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	2
strict	2	2
look-ahead	1	≤ 3
—	2	≤ 4

Properties of dependencies

Definition

A set $\mathcal{D} \subseteq \mathcal{L}$ of link structures

- is input hierarchical if for all $D \in \mathcal{D}$, $(v_1, w_1), (v_2, w_2) \in D$
  - $w_1 \preceq w_2$ if $v_1 \prec v_2$
  - $w_1 \preceq w_2$ or $w_2 \preceq w_1$ if $v_1 = v_2$

Properties of dependencies

Definition

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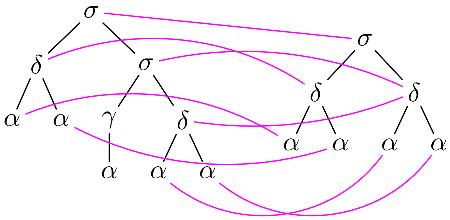
  - $w_1 \preceq w_2$ if $v_1 \prec v_2$
  - $w_1 \preceq w_2$ or $w_2 \preceq w_1$ if $v_1 = v_2$
- has bounded distance in the input
  if $\exists k \in \mathbb{N}$ s.t. for all $D \in \mathcal{D}$, $(v, w), (vv'', w'') \in D$
  there exists $(vv', w') \in D$ with $v' \prec v''$ and $|v'| \leq k$

Dependencies



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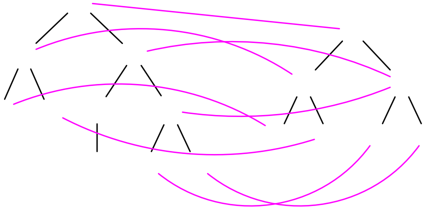
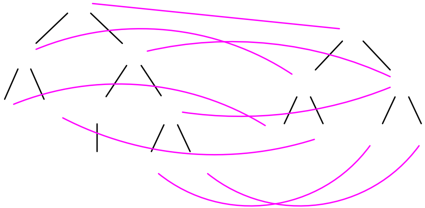
Dependencies

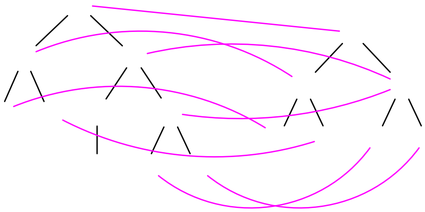


Dependencies



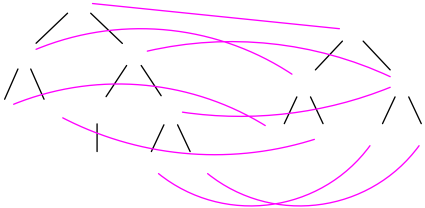
input hierarchical

Dependencies



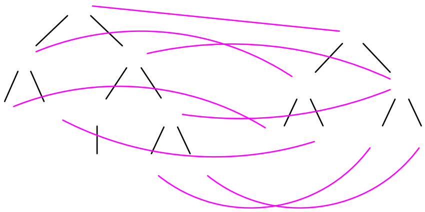
input hierarchical and output hierarchical

Dependencies



input hierarchical and output hierarchical
with bounded distance in the input

Dependencies



input hierarchical and output hierarchical
with bounded distance in the input and the output

Dependencies

Theorem

Any $XTOP^R$ computes

- *input and output hierarchical dependencies*
- *with bounded distance in the input and the output*

Main theorem

Theorem

$$\not\varepsilon\text{-XTOP}^2 \subseteq (\not\varepsilon\text{-XTOP}^R)^2 \subsetneq \not\varepsilon\text{-XTOP}^3 \subseteq (\not\varepsilon\text{-XTOP}^R)^3$$

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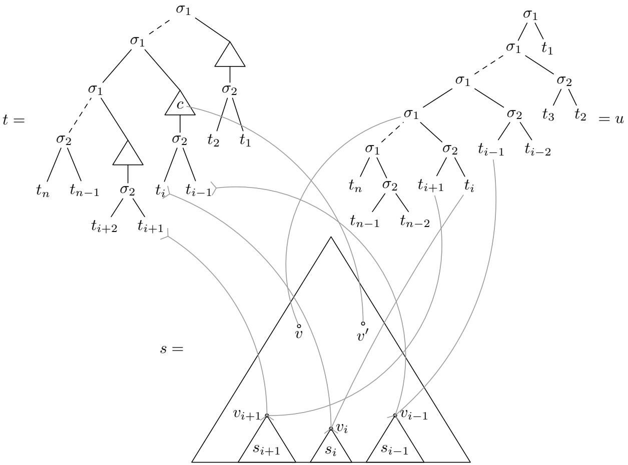
Sketch of proof



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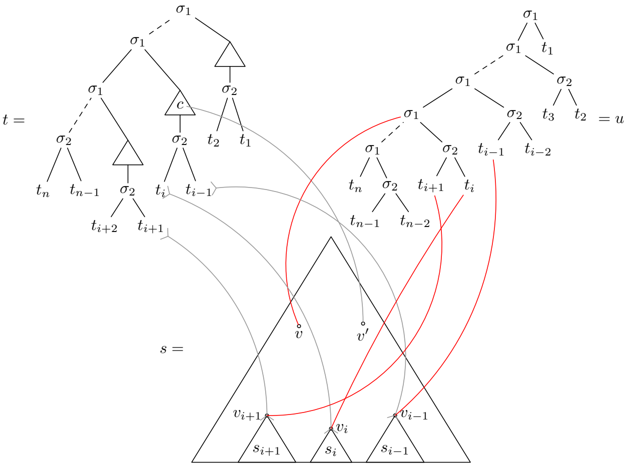
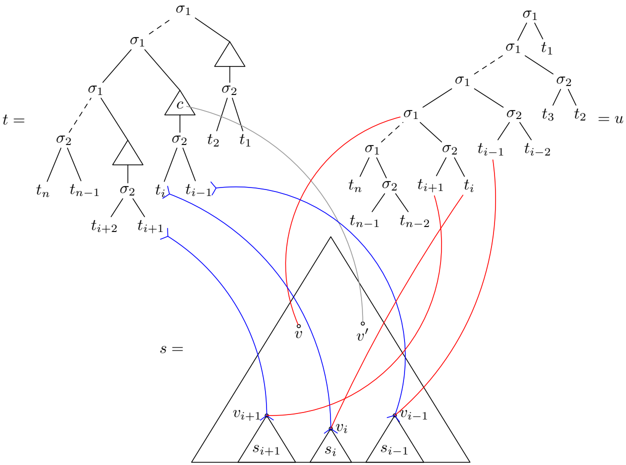
Sketch of proof

$v \not\preceq v_{i-1}$ and $v \preceq v_i$ and $v \preceq v_{i+1}$



Sketch of proof

$v \not\preceq v_{i-1}$ and $v \preceq v_i$ and $v \preceq v_{i+1}$



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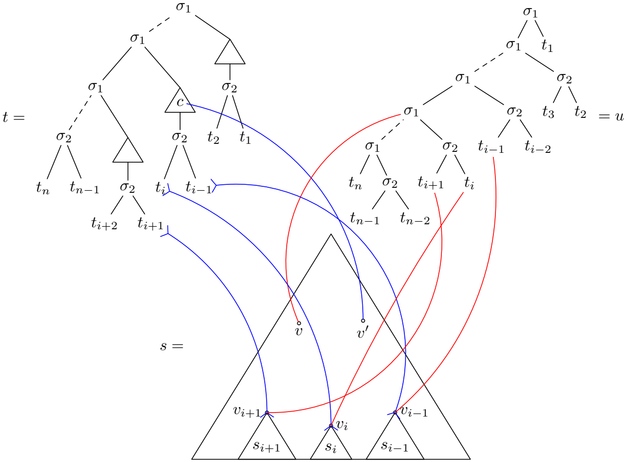
Sketch of proof

$v \not\preceq v_{i-1}$ and $v \preceq v_i$ and $v \preceq v_{i+1}$

$v' \preceq v_{i-1}$ and $v' \preceq v_i$ and $v' \not\preceq v_{i+1}$



Summary

	TOP	$\not\varepsilon$-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	2
strict	2	2
look-ahead	1	3
—	2	3–4 (4)