Composition Closure of Linear Extended Top-down Tree Transducers

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Leipzig — April 8, 2014

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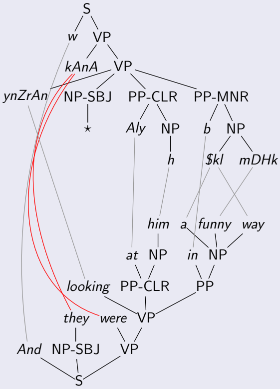
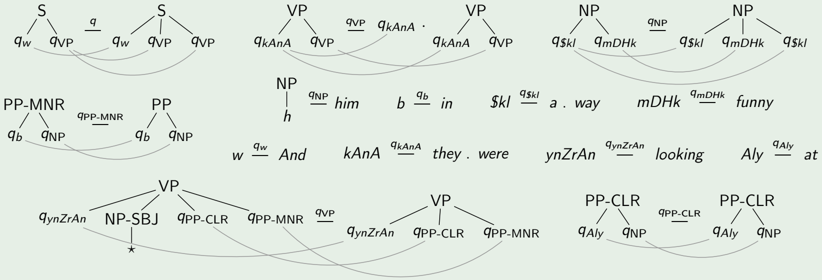
Syntax-based Statistical Machine Translation

Input data



Syntax-based Statistical Machine Translation

Extracted rules



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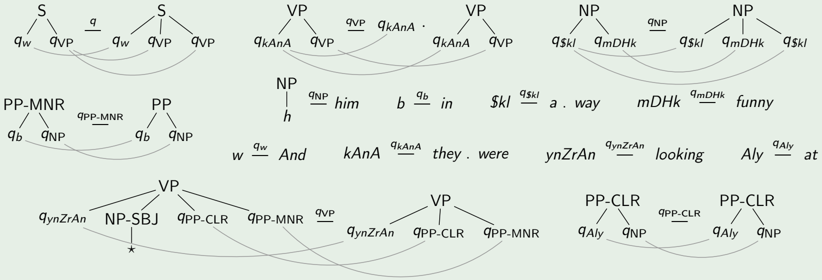

- for a tree-to-tree transformation device = tree transducer
- here: for a linear extended multi bottom-up tree transducer

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Motivation

Tree transducer

- used in statistical machine translation [Knight, Graehl 2005]
- used in XML query processing [Benedikt et al. 2013]

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Compositions

- $\tau_1 \,;\, \tau_2 = \{(s, u) \mid \exists t\colon (s, t) \in \tau_1, (t, u) \in \tau_2\}$
- support modular development
- allow integration of external knowledge sources
- occur naturally in query rewriting

Problem

Question:

given a class $\mathcal{C}$ of transformations, is there $n \in \mathbb{N}$ such that

$$\mathcal{C}^n = \bigcup_{k \geq 1} \mathcal{C}^k$$

$$\mathcal{C}^k = \underbrace{\mathcal{C} ; \cdots ; \mathcal{C}}_{k \text{ times}}$$

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given a class $\mathcal{C}$ of transformations, is there $n \in \mathbb{N}$ such that

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Note

- $\mathcal{C}^k \subseteq \mathcal{C}^{k+1}$ for our classes $\mathcal{C}$

$\rightarrow$ we search least n such that $\mathcal{C}^n = \mathcal{C}^{n+1}$ (if it exists)

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Linear Extended Multi Bottom-up Tree Transducer

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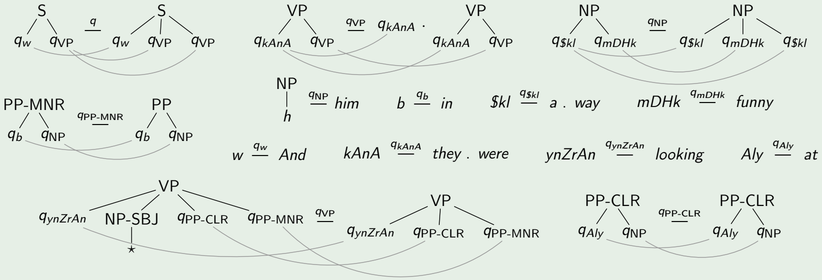

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Linear Extended Multi Bottom-up Tree Transducer

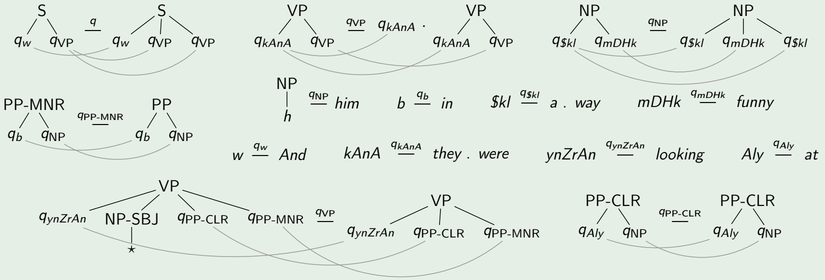
Definition (MBOT)

linear extended multi bottom-up tree transducer (Q, Σ, I, R)

- finite set Q — *states*
- alphabet Σ — *input and output symbols*
- $I \subseteq Q$ — *initial states*
- finite set $R \subseteq T_\Sigma(Q) \times Q \times T_\Sigma(Q)^*$ — *rules*
  - each $q \in Q$ occurs at most once in ℓ — $(\ell, q, \vec{r}) \in R$
  - each $q \in Q$ that occurs in $\vec{r}$ also occurs in ℓ — $(\ell, q, \vec{r}) \in R$

Linear Extended Multi Bottom-up Tree Transducer

Extracted rules



Linear Extended Multi Bottom-up Tree Transducer

Definition (Syntactic properties)

MBOT (Q, Σ, I, R) is

- linear extended top-down tree transducer with regular look-ahead (XTOP^R) if $|\vec{r}| \leq 1$ $\quad \forall(\ell, q, \vec{r}) \in R$
- linear extended top-down tree transducer (XTOP) if $|\vec{r}| = 1$

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- linear extended top-down tree transducer (XTOP) if $|\vec{r}| = 1$
- linear top-down tree transducer ($\text{TOP}/\text{TOP}^\text{R}$) if $\text{XTOP}/\text{XTOP}^\text{R}$ and ℓ contains exactly one element of Σ
- ε-free (resp. strict) if $\ell \notin Q$ (resp. $\vec{r} \notin Q^+$)

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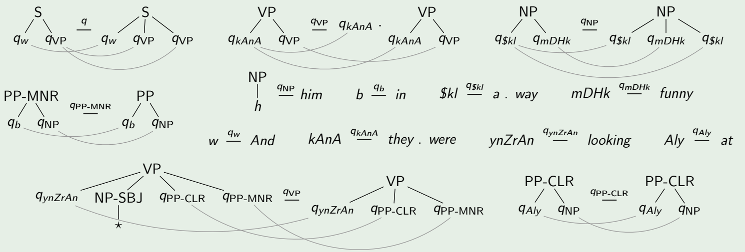
- ε-free (resp. strict) if $\ell \notin Q$ (resp. $\vec{r} \notin Q^+$)
- delabeling if it is a TOP and $\vec{r}$ contains at most one element of Σ
- nondeleting if the same elements of Q occur in ℓ and $\vec{r}$

Linear Extended Multi Bottom-up Tree Transducer

Extracted rules



Properties

XTOP^{R}: ✗ XTOP: ✗ TOP^{R}: ✗ TOP: ✗

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Linear Extended Multi Bottom-up Tree Transducer

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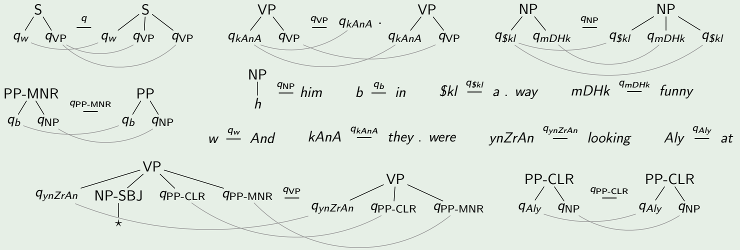

Properties

XTOP^R:	✗	XTOP:	✗	TOP^R:	✗	TOP:	✗
ε-free:	✓	strict:	✓	delabeling:	✗	nondeleting:	✓

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Another Example

Example (textual)

MBOT $M = (Q, \Sigma, \{\star\}, R)$

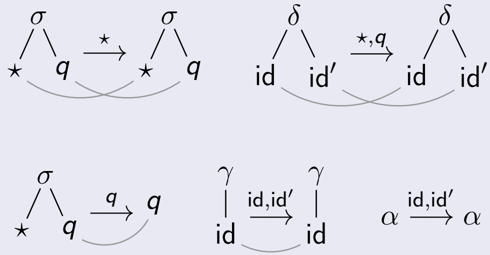
- $Q = \{\star, q, \mathrm{id}, \mathrm{id}'\}$
- $\Sigma = \{\sigma, \delta, \gamma, \alpha\}$
- the following rules in R:

$$\sigma(\star, q) \xrightarrow{\star} \sigma(\star, q) \qquad \sigma(\star, q) \xrightarrow{q} q$$

$$\delta(\mathrm{id}, \mathrm{id}') \xrightarrow{\star, q} \delta(\mathrm{id}, \mathrm{id}') \qquad \gamma(\mathrm{id}) \xrightarrow{\mathrm{id}, \mathrm{id}'} \gamma(\mathrm{id}) \qquad \alpha \xrightarrow{\mathrm{id}, \mathrm{id}'} \alpha$$

Another Example

Graphical representation



Properties

XTOP^R: ✓ XTOP: ✓ TOP^R: ✓ TOP: ✓

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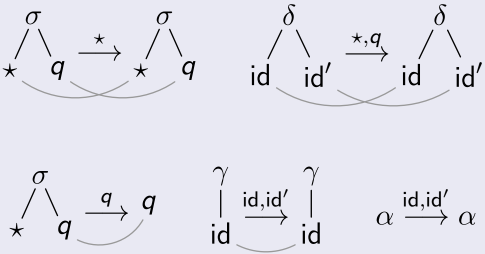
Another Example

Graphical representation



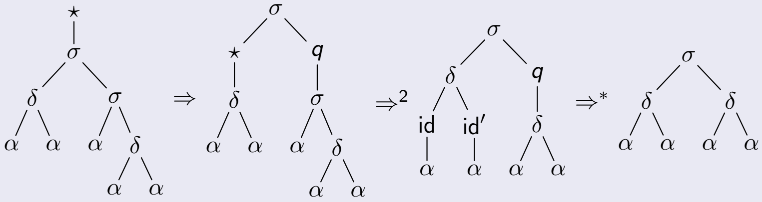
Properties

$\mathrm{XTOP}^{\mathrm{R}}$:	✓	XTOP:	✓	$\mathrm{TOP}^{\mathrm{R}}$:	✓	TOP:	✓
ε-free:	✓	strict:	✗	delabeling:	✓	nondeleting:	✗

Semantics — Synchronous Generation

Discussion

- typical semantics: derivation semantics; input-driven



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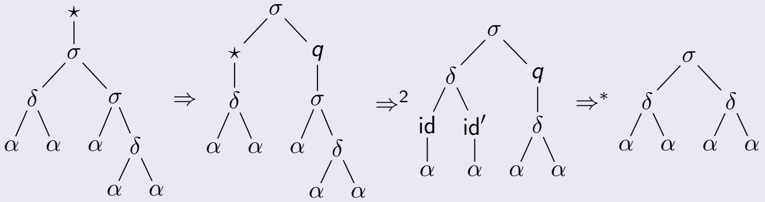
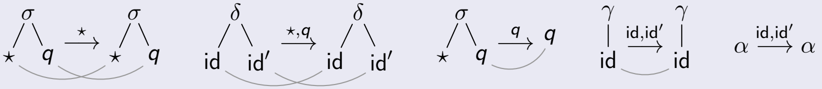


- unsuitable for our purposes
- input and output fragments should always be visible

Semantics — Synchronous Generation

Rules



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Semantics — Synchronous Generation

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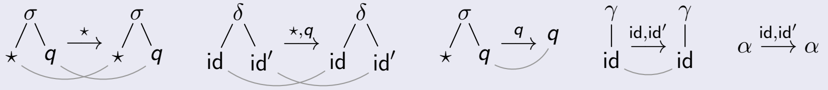

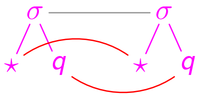

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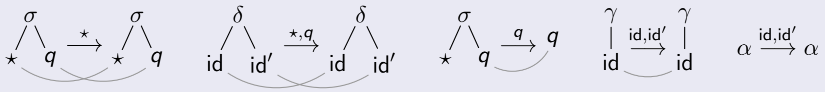

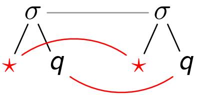

Semantics — Synchronous Generation

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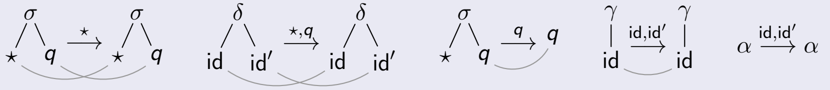

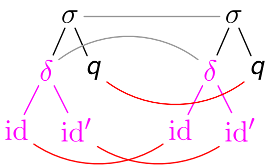

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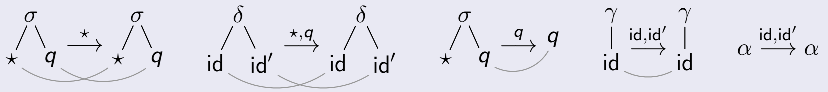

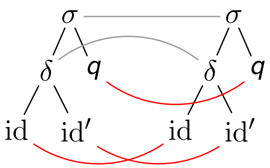

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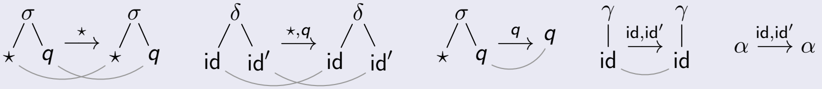

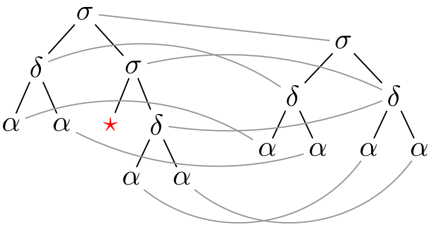

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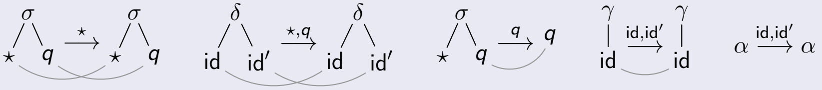
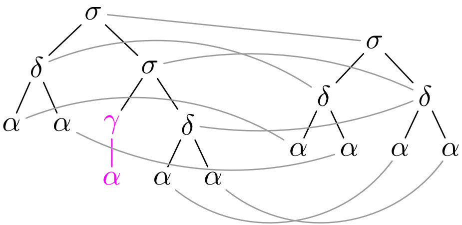




Semantics — Synchronous Generation

Definition (Generation step)

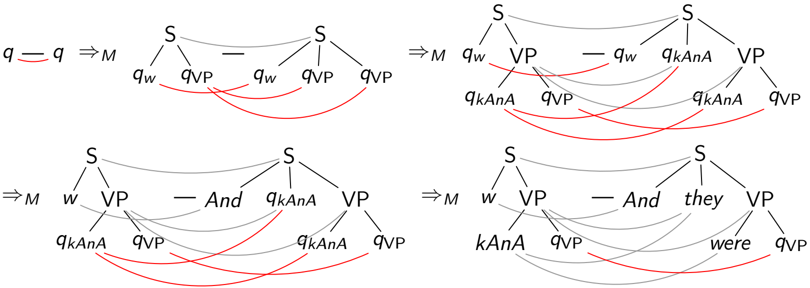
$$\langle t, A, D, u \rangle \Rightarrow_M \langle t', A', D', u' \rangle$$

if and only if $\exists q \in Q$, $\exists v \in \text{pos}(t)$ labeled by q, and $\exists \ell \stackrel{q}{\to} \vec{r} \in P$

- $|\vec{r}| = |A(v)|$ and $\vec{w} = \overrightarrow{A(v)}$
- $t' = t[\ell]_v$ and $u' = u[\vec{r}]_{\vec{w}}$
- $A' = (A \setminus L) \cup \text{links}_{v,\vec{w}}(\ell \stackrel{q}{\to} \vec{r})$ and $D' = D \cup L$ with

$$L = \{(v, w) \mid w \in A(v)\}$$

Semantics — Synchronous Generation



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Semantics — Synchronous Generation

Definition

- state-computed dependencies:

  $$M_q = \{\langle t, D, u\rangle \mid t, u \in T_\Sigma,\ \langle q, \{(\varepsilon, \varepsilon)\}, \emptyset, q\rangle \Rightarrow^*_M \langle t, \emptyset, D, u\rangle\}$$

- computed dependencies: $\text{dep}(M) = \bigcup_{q \in I} M_q$

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$$\tau_M = \{(t, u) \mid \langle t, D, u\rangle \in \mathrm{dep}(M)\}$$

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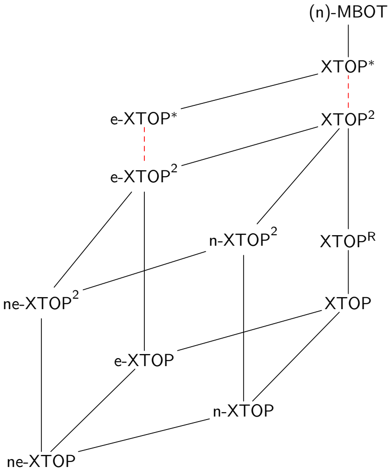
- (can be made to) coincide with traditional semantics

Known Relations



Contents

Known Results on Composition Closure

	TOP	XTOP	MBOT
ε-free, strict, nondeleting	1		1
ε-free, strict	2		1
ε-free	2		1
otherwise (without delabeling)	2		1

Known Results on Composition Closure

	TOP	XTOP	MBOT
ε-free, strict, nondeleting	1	2	1
ε-free, strict	2	?	1
ε-free	2	?	1
otherwise (without delabeling)	2	?	1

Delabelings Move Around

e = ε-free; d = delabeling

s = strict; n = nondeleting

Theorem

switch delabeling from back to front:

$$\text{e[s]-XTOP}^R \,;\, \text{[s]d-TOP}^R \subseteq \text{e[s]-XTOP}^R \subseteq \text{[s]d-TOP}^R \,;\, \text{esn-XTOP}$$

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Notes

- other transducer becomes strict and nondeleting

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Notes

- other transducer becomes strict and nondeleting
- other transducer looses look-ahead

ε-free and Look-ahead

e = ε-free; d = delabeling

s = strict; n = nondeleting

Theorem

$(\text{e[s]-XTOP}^{\text{R}})^n \subseteq \text{[s]d-TOP}^{\text{R}}$; $\text{esn-XTOP}^2 \subseteq (\text{e[s]-XTOP}^{\text{R}})^3$

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Proof.

$$(\text{e[s]-XTOP}^{\text{R}})^{n+1}$$

$$\subseteq$$

$$\subseteq$$

$$\subseteq$$

□

ε-free and Look-ahead

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\subseteq\ & \\
\subseteq\ &
\end{aligned}
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□

ε-free and Look-ahead

e = ε-free; d = delabeling

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□

ε-free, but no Look-ahead

Corollary

$\text{e[s]-XTOP}^n \subseteq \text{QR} \mathbin{;} \text{[s]d-TOP} \mathbin{;} \text{esn-XTOP}^2 \subseteq \text{e[s]-XTOP}^4$

ε-free, but no Look-ahead

Corollary

$\text{e[s]-XTOP}^n \subseteq \text{QR} ; \text{[s]d-TOP} ; \text{esn-XTOP}^2 \subseteq \text{e[s]-XTOP}^4$

Proof.

uses only standard encoding of look-ahead □

Results so far

	TOP	e-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	
strict	2	
look-ahead	1	
—	2	

Results so far

	TOP	e-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	
strict	2	
look-ahead	1	≤ 3
—	2	≤ 4

Delabelings Move Around Even More

Theorem

delabeling homomorphism moving from front to back:

$$\text{sd-HOM} \,;\, \text{es-XTOP} \subseteq \text{es-XTOP} \subseteq \text{esn-XTOP} \,;\, \text{sd-HOM}$$

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Delabelings Move Around Even More

Theorem

delabeling homomorphism moving from front to back:

sd-HOM ; es-XTOP ⊆ es-XTOP ⊆ esn-XTOP ; sd-HOM

Notes

Delabelings Move Around Even More

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Notes

- other transducer becomes nondeleting

Delabelings Move Around Even More

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Notes

- other transducer becomes nondeleting
- other transducer needs to be strict and have no look-ahead

ε-free and Strict

Theorem

$(\text{es-XTOP}^{\text{R}})^n \subseteq \text{esn-XTOP} \mathbin{;} \text{es-XTOP} \subseteq \text{es-XTOP}^2$

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ε-free and Strict

Theorem

$(\text{es-XTOP}^{\text{R}})^n \subseteq \text{esn-XTOP} \;;\; \text{es-XTOP} \subseteq \text{es-XTOP}^2$

Proof.

$$
\begin{aligned}
(\text{es-XTOP}^{\text{R}})^{n+1} &\subseteq (\text{es-XTOP}^{\text{R}})^n \;;\; \text{es-XTOP} \\
&\subseteq \\
&\subseteq \\
&\subseteq \\
&\subseteq \\
&\subseteq
\end{aligned}
$$

□

ε-free and Strict

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□

ε-free and Strict

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&\subseteq \text{esn-XTOP}^3 \,;\, \text{sd-HOM} \\
&\subseteq \\
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\end{aligned}$$

□

ε-free and Strict

Theorem

$(\text{es-XTOP}^{\text{R}})^n \subseteq \text{esn-XTOP} \mathbin{;} \text{es-XTOP} \subseteq \text{es-XTOP}^2$

Proof.

$$\begin{aligned}
(\text{es-XTOP}^{\text{R}})^{n+1} &\subseteq (\text{es-XTOP}^{\text{R}})^n \mathbin{;} \text{es-XTOP} \\
&\subseteq \text{esn-XTOP} \mathbin{;} \text{sd-HOM} \mathbin{;} \text{es-XTOP}^2 \\
&\subseteq \text{esn-XTOP}^3 \mathbin{;} \text{sd-HOM} \\
&\subseteq \text{esn-XTOP}^2 \mathbin{;} \text{sd-HOM} \\
&\subseteq \\
&\subseteq
\end{aligned}$$

□

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ε-free and Strict

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&\subseteq \text{esn-XTOP} ; \text{es-XTOP}^{\text{R}} \\
&\subseteq
\end{aligned}
$$

□

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&\subseteq \text{esn-XTOP} \mathbin{;} \text{es-XTOP}
\end{aligned}
$$

□

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Upper Bounds

	TOP	e-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	
strict	2	
look-ahead	1	≤ 3
—	2	≤ 4

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Upper Bounds

	TOP	e-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	≤ 2
strict	2	≤ 2
look-ahead	1	≤ 3
—	2	≤ 4

Contents

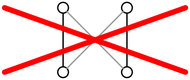
Properties of Dependencies

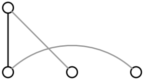
Definition (Hierarchy properties)

A dependency $\langle t, D, u \rangle$ is

- input hierarchical if
  1. $w_2 \not< w_1$
  2. $\exists(v_1, w_1') \in D$ with $w_1' \leq w_2$

  for all $(v_1, w_1), (v_2, w_2) \in D$ with $v_1 < v_2$





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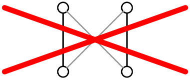

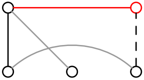

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- strictly input hierarchical if
  1. $v_1 < v_2$ implies $w_1 \leq w_2$
  2. $v_1 = v_2$ implies $w_1 \leq w_2$ or $w_2 \leq w_1$

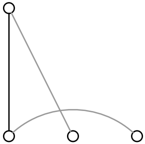
  for all $(v_1, w_1), (v_2, w_2) \in D$

Properties of Dependencies

Definition (Distance properties)

A dependency $\langle t, D, u \rangle$ is

- input link-distance bounded by $b \in \mathbb{N}$ if
  for all $(v_1, w_1), (v_1 v', w_2) \in D$ with $|v'| > b$
  $\exists (v_1 v, w_3) \in D$ such that $v < v'$ and $1 \leq |v| \leq b$



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Definition (Distance properties)

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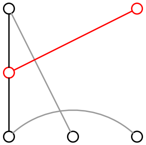


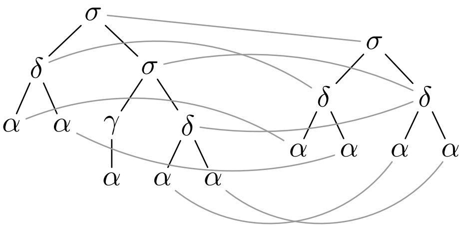
Properties of Dependencies

Definition (Distance properties)

A dependency $\langle t, D, u \rangle$ is

- input link-distance bounded by $b \in \mathbb{N}$ if
  for all $(v_1, w_1), (v_1 v', w_2) \in D$ with $|v'| > b$
  $\exists (v_1 v, w_3) \in D$ such that $v < v'$ and $1 \leq |v| \leq b$
- strict input link-distance bounded by b if for all
  $v_1, v_1 v' \in \text{pos}(t)$ with $|v'| > b$
  $\exists (v_1 v, w_3) \in D$ such that $v < v'$ and $1 \leq |v| \leq b$

Dependencies



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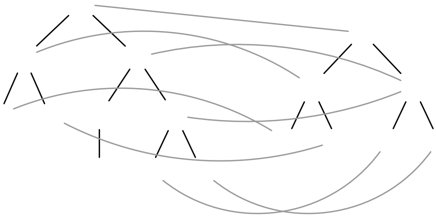
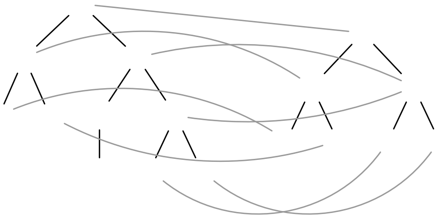
Dependencies



Dependencies



strictly input hierarchical

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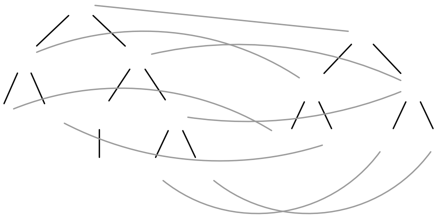
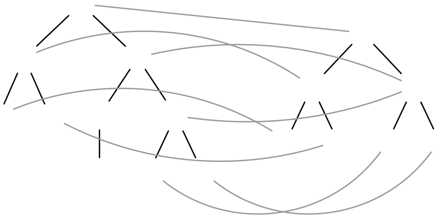
Dependencies



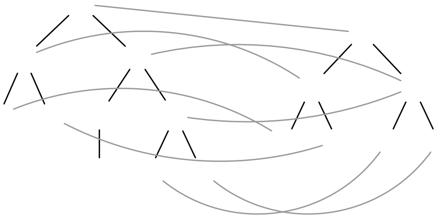
strictly input hierarchical and strictly output hierarchical

Dependencies



strictly input hierarchical and strictly output hierarchical
with strict input link-distance 2

Dependencies



strictly input hierarchical and strictly output hierarchical
with strict input link-distance 2 and strict output link-distance 1

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Theorem on Dependency Properties

Model \ Property	hierarchical input	hierarchical output	link-distance bounded input	link-distance bounded output
n-XTOP	strictly	strictly	strictly	strictly
$XTOP^R$	strictly	strictly	✓	strictly
MBOT	✓	strictly	✓	strictly

Linking Theorem for ε-free XTOP$^{\mathrm{R}}$

Theorem

Let $M_1, \dots, M_k$ be ε-free XTOP^R over Σ such that

$$\{(c[t_1, \dots, t_n]\,,\, c'[t_1, \dots, t_n]) \mid t_1, \dots, t_n \in T\} \subseteq \tau_{M_1} ; \cdots ; \tau_{M_k}$$

for some contexts $c, c' \in C_\Sigma(X_n)$ and special $T \subseteq T_\Sigma$.

$\forall 1 \leq i \leq k, \forall 1 \leq j \leq n$

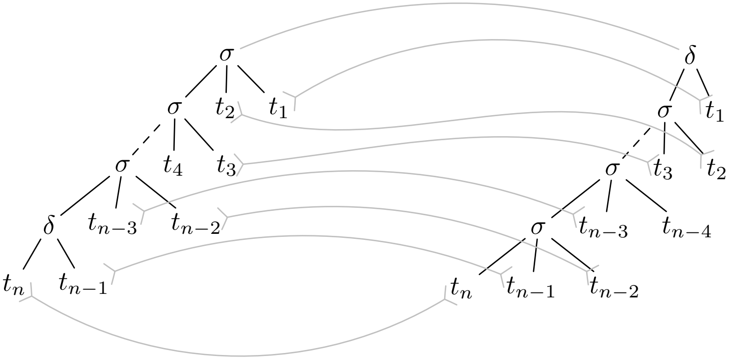
$\exists t_j \in T, \exists \langle u_{i-1}, D_i, u_i \rangle \in \mathrm{dep}(M_i), \exists (v_{ji}, w_{ji}) \in D_i$ *such that*

- $u_0 = c[t_1, \dots, t_n]$ *and* $u_k = c'[t_1, \dots, t_n]$
- $\mathrm{pos}_{x_j}(c') \leq w_{jk}$
- $v_{ji} \leq w_{j(i-1)}$ *if* $i \geq 2$
- $\mathrm{pos}_{x_j}(c) \leq v_{j1}$

Linking Theorem for ε-free XTOP^{R}

Corollary [see Sect. 3.4 in [Arnold, Dauchet 1982]]

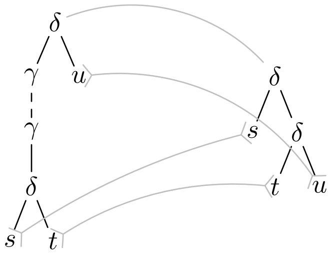
The illustrated tree transformation τ cannot be computed by any ε-free XTOP^{R}



Linking Theorem for ε-free XTOP^R

Corollary [see Thm. 5.2 in [Maletti et al. 2009]]

The illustrated tree transformation τ cannot be computed by any ε-free XTOP^R



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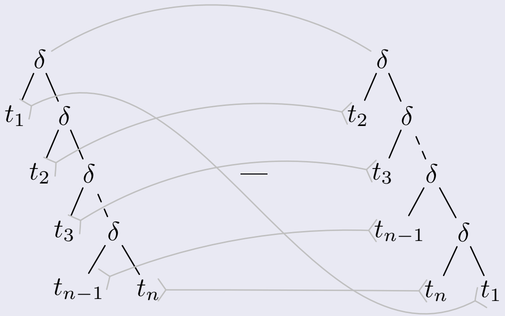
Linking Theorem for ε-free XTOP^R

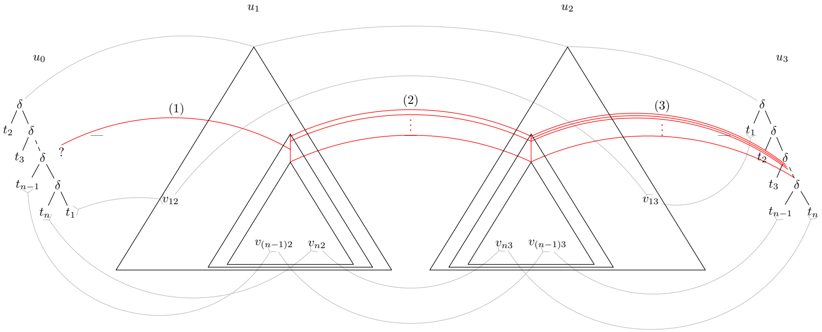
Inverse of topicalization



Linking Theorem for ε-free XTOP^R

Corollary

Topicalization cannot be computed by any composition chain of ε-free XTOP^R



Linking Theorem for ε-free MBOT

Theorem

Let $M = (Q, \Sigma, I, R)$ *be an* ε*-free MBOT such that*

$$\{(c[t_1, \ldots, t_n]\,,\, c'[t_1, \ldots, t_n]) \mid t_1, \ldots, t_n \in T\} \subseteq \tau_M$$

for some contexts $c, c' \in C_\Sigma(X_n)$ *and special* $T \subseteq T_\Sigma$*.*

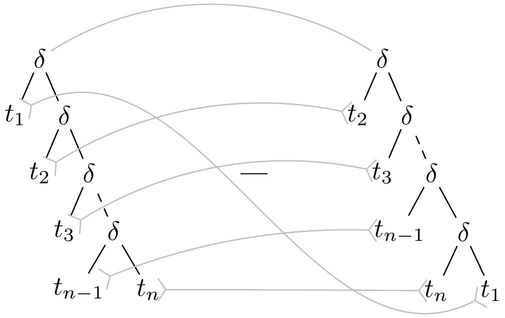
$\forall 1 \leq j \leq n$, $\exists t_j \in T$, $\exists \langle u, D, u' \rangle \in \mathrm{dep}(M)$, $\exists (v_j, w_j) \in D$ *with*

- $u = c[t_1, \ldots, t_n]$ *and* $u' = c'[t_1, \ldots, t_n]$
- $\mathrm{pos}_{x_j}(c) \leq v_j$
- $\mathrm{pos}_{x_j}(c') \leq w_j$

Linking Theorem for ε-free MBOT

Corollary

Inverse of topicalization cannot be computed by any ε-free MBOT



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Known Result

Theorem [see [Maletti et al. 2009]]

es-XTOP $\subsetneq$ es-XTOP^{R} $\subsetneq$ es-XTOP^{2} $=$ (es-XTOP^{R})^{2}

Known Result

Theorem [see [Maletti et al. 2009]]

$\text{es-XTOP} \subsetneq \text{es-XTOP}^{\mathrm{R}} \subsetneq \text{es-XTOP}^2 = (\text{es-XTOP}^{\mathrm{R}})^2$

Proof.

- look-ahead adds power at first level
- none of the basic classes is closed under composition □

Lower Bounds

	TOP	e-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	≤ 2
strict	2	≤ 2
look-ahead	1	≤ 3
—	2	≤ 4

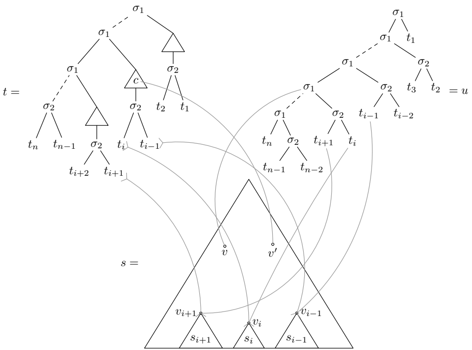
Lower Bounds

	TOP	e-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	2
strict	2	2
look-ahead	1	≤ 3
—	2	≤ 4

Main theorem

Theorem

$$\text{e-XTOP}^2 \subseteq (\text{e-XTOP}^{\mathsf{R}})^2 \subsetneq \text{e-XTOP}^3 \subseteq (\text{e-XTOP}^{\mathsf{R}})^3$$



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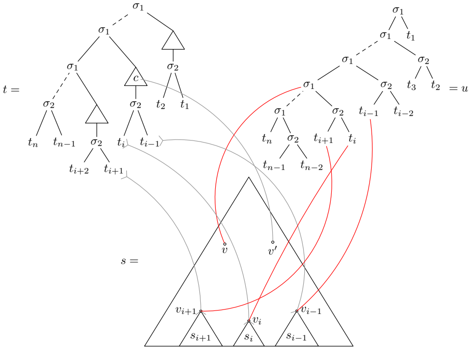
Main theorem

Theorem

$$\text{e-XTOP}^2 \subseteq (\text{e-XTOP}^R)^2 \subsetneq \text{e-XTOP}^3 \subseteq (\text{e-XTOP}^R)^3$$

$v \not\preceq v_{i-1}$ and $v \preceq v_i$ and $v \preceq v_{i+1}$



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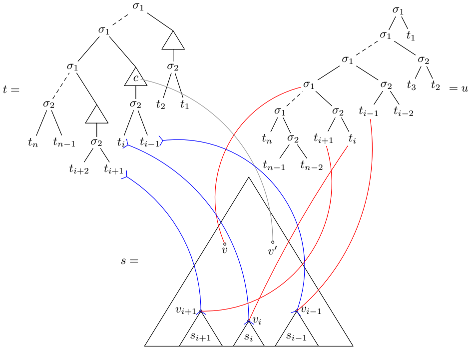

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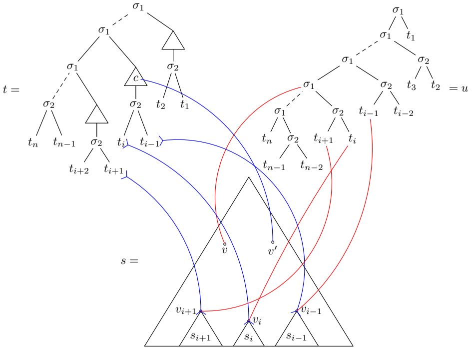
Main theorem

Theorem

e-XTOP^2 $\subseteq$ (e-XTOP^R)^2 $\subsetneq$ e-XTOP^3 $\subseteq$ (e-XTOP^R)^3

$v \not\preceq v_{i-1}$ and $v \preceq v_i$ and $v \preceq v_{i+1}$

$v' \preceq v_{i-1}$ and $v' \preceq v_i$ and $v' \not\preceq v_{i+1}$



Lower Bounds

	TOP	e-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	2
strict	2	2
look-ahead	1	≤ 3
—	2	≤ 4

Lower Bounds

	TOP	e-XTOP
strict, nondeleting	1	2
strict, look-ahead	1	2
strict	2	2
look-ahead	1	3
—	2	3–4 (4)

Missing Cases

	TOP	XTOP	XTOP^R
ε-free, nondeleting	1	∞	∞
strict	2	∞	∞
nondeleting	1	∞	∞
strict, nondeleting	1	∞	∞
—	2	∞	∞

Proof.

- completely different technique [Fülöp, Maletti, 2013] □

Summary

	TOP	XTOP	XTOP^R	MBOT
ε-free, strict, nondeleting	1	2	2	1
ε-free, strict	2	2	2	1
ε-free	2	4	3	1
otherwise (without delabeling)	2	∞	∞	1

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Summary