Linking Theorems for Tree Transducers

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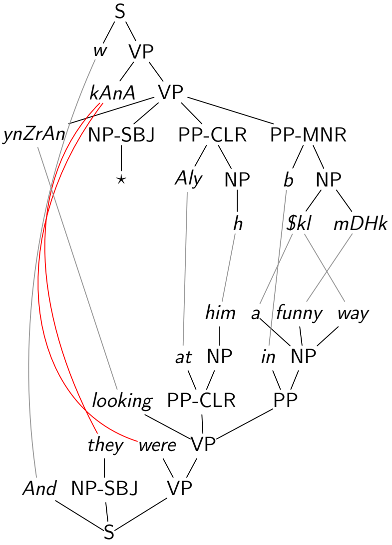
Statistical Machine Translation



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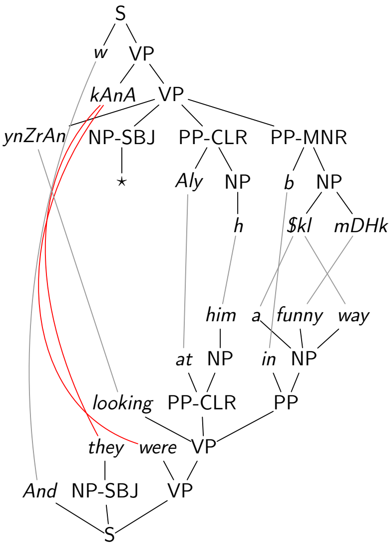

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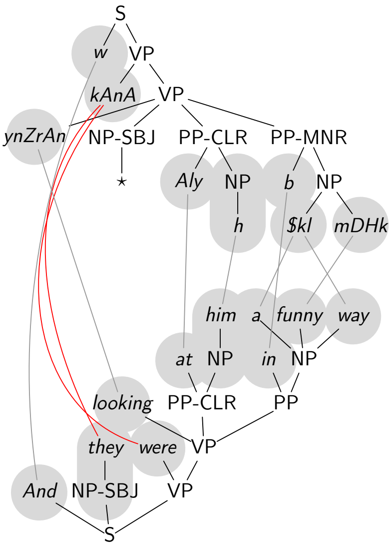

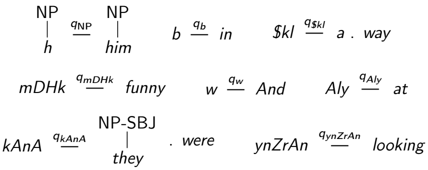

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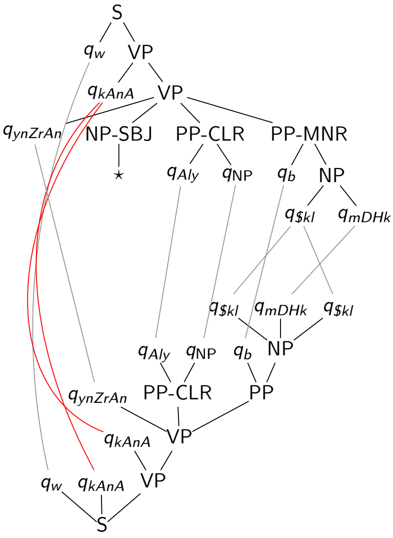

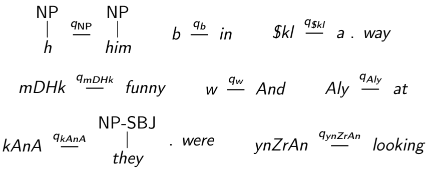

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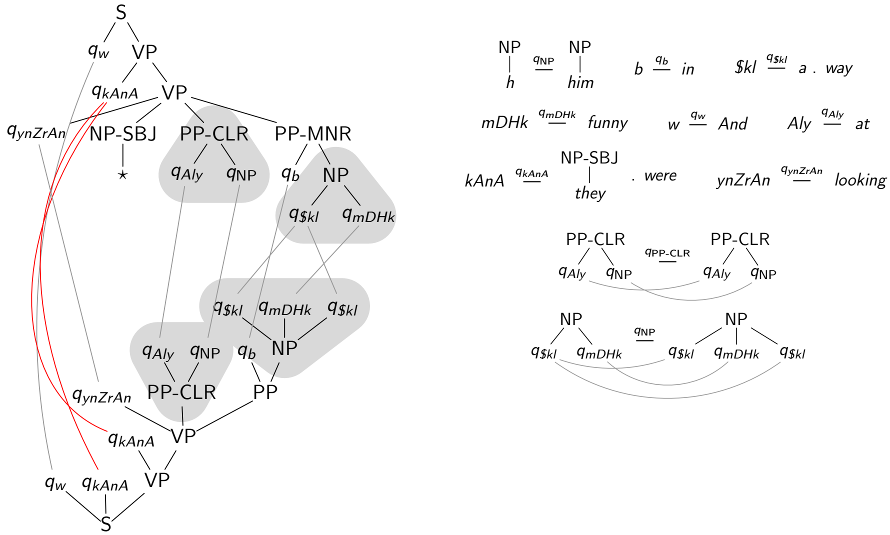

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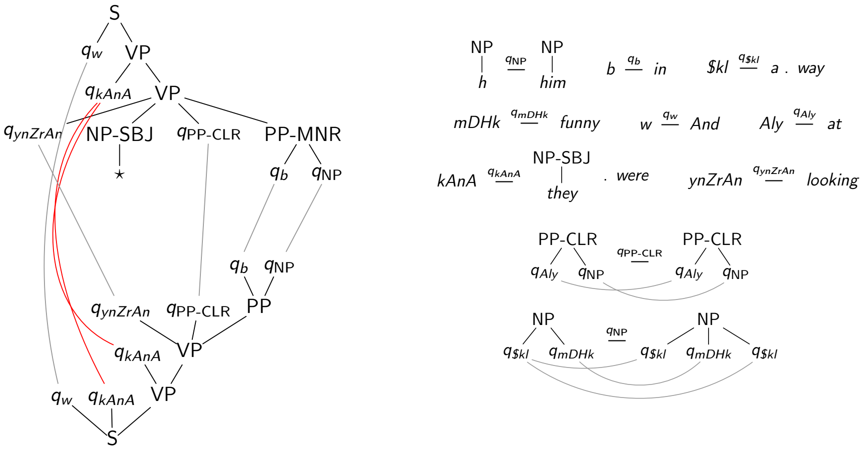

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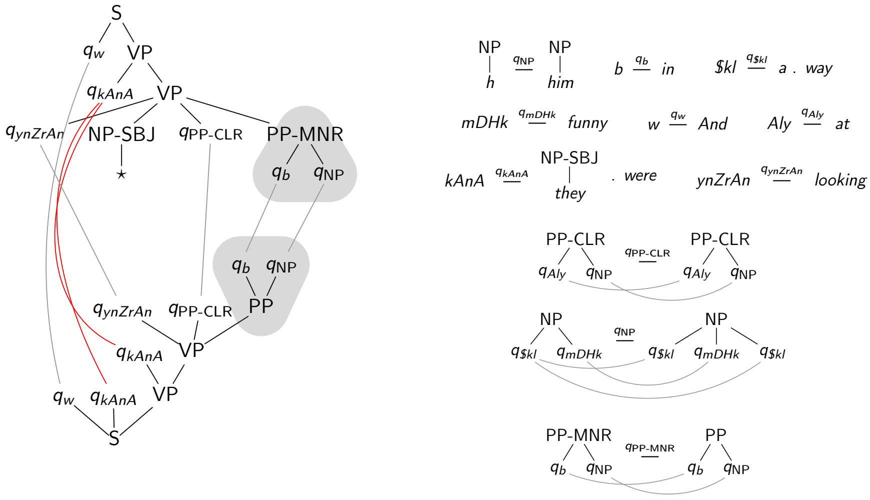
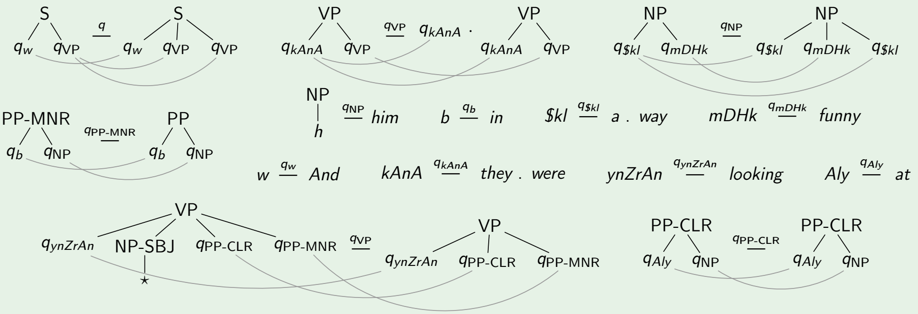


Statistical Machine Translation

Extracted rules



Linear Multi Tree Transducer

MBOT

linear multi tree transducer (Q, Σ, I, R)

- finite set Q — states
- alphabet Σ — input and output symbols
- $I \subseteq Q$ — initial states
- finite set $R \subseteq T_\Sigma(Q) \times Q \times T_\Sigma(Q)^*$ — rules
  - each $q \in Q$ occurs at most once in ℓ — $(\ell, q, \vec{r}) \in R$
  - each $q \in Q$ that occurs in $\vec{r}$ also occurs in ℓ — $(\ell, q, \vec{r}) \in R$

Linear Multi Tree Transducer

Syntactic properties

MBOT (Q, Σ, I, R) is

- linear tree transducer with regular look-ahead (XTOP$^{\text{R}}$)
  if $|\vec{r}| \leq 1$ $\qquad \forall(\ell, q, \vec{r}) \in R$
- linear tree transducer (XTOP)
  if $|\vec{r}| = 1$ $\qquad \forall(\ell, q, \vec{r}) \in R$

Linear Multi Tree Transducer

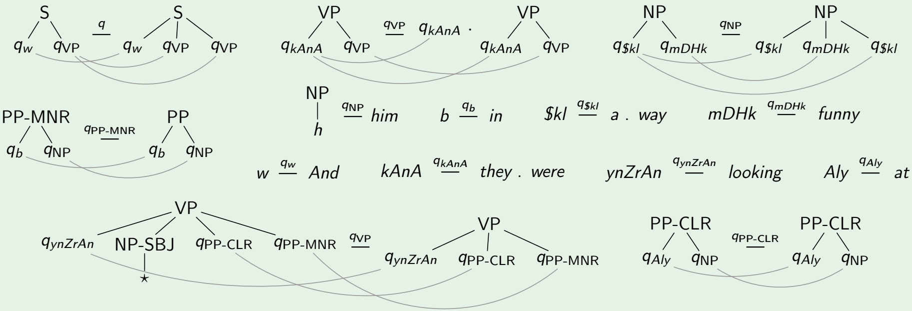
Syntactic properties

MBOT (Q, Σ, I, R) is

- linear tree transducer with regular look-ahead ($\mathrm{XTOP^R}$)
  if $|\vec{r}| \leq 1$ $\quad \forall(\ell, q, \vec{r}) \in R$
- linear tree transducer (XTOP)
  if $|\vec{r}| = 1$ $\quad \forall(\ell, q, \vec{r}) \in R$
- ε-free if $\ell \notin Q$ $\quad \forall(\ell, q, \vec{r}) \in R$

Linear Multi Tree Transducer

Extracted rules



Properties

XTOP^R: ✗ XTOP: ✗ ε-free: ✓

Another Example

Textual example

MBOT $M = (Q, \Sigma, \{\star\}, R)$

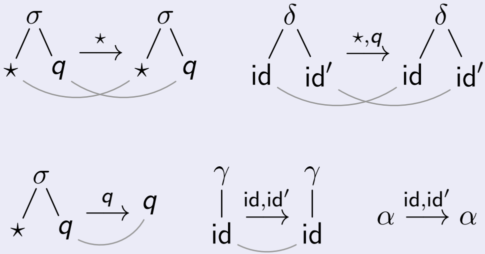
- $Q = \{\star, q, \mathrm{id}, \mathrm{id}'\}$
- $\Sigma = \{\sigma, \delta, \gamma, \alpha\}$
- the following rules in R:

$$\sigma(\star, q) \xrightarrow{\star} \sigma(\star, q) \qquad \sigma(\star, q) \xrightarrow{q} q$$

$$\delta(\mathrm{id}, \mathrm{id}') \xrightarrow{\star, q} \delta(\mathrm{id}, \mathrm{id}') \qquad \gamma(\mathrm{id}) \xrightarrow{\mathrm{id}, \mathrm{id}'} \gamma(\mathrm{id}) \qquad \alpha \xrightarrow{\mathrm{id}, \mathrm{id}'} \alpha$$

Another Example

Graphical representation

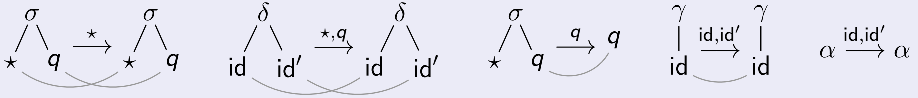


Properties

XTOP^{R}: ✓ XTOP: ✓ ε-free: ✓

Semantics

Rules



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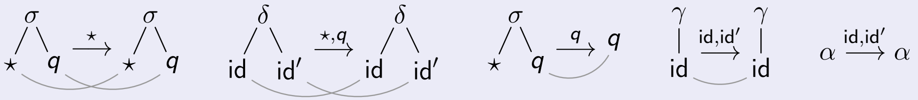

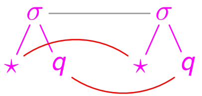

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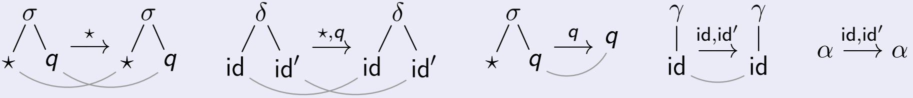

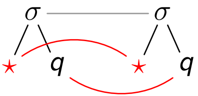

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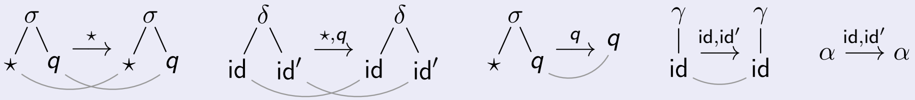

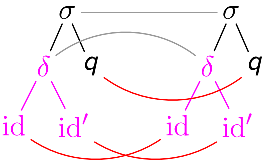

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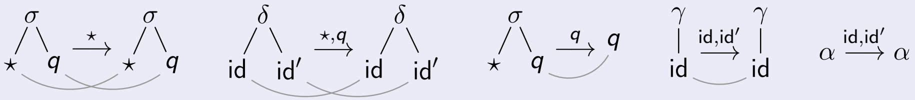

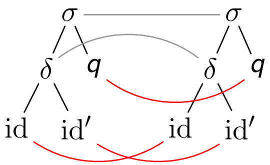

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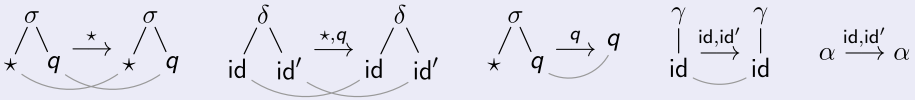

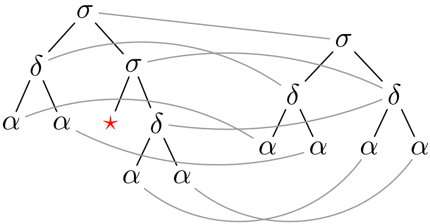

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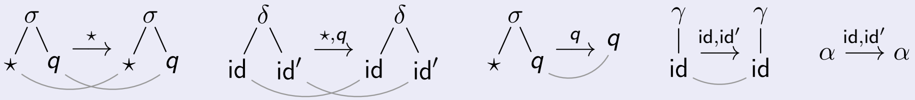

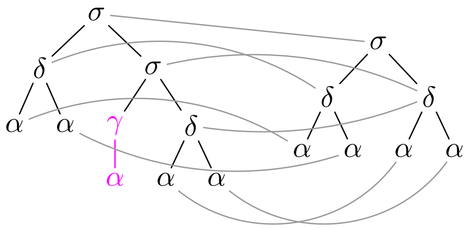

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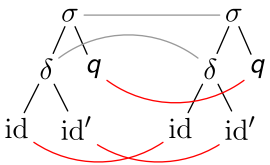
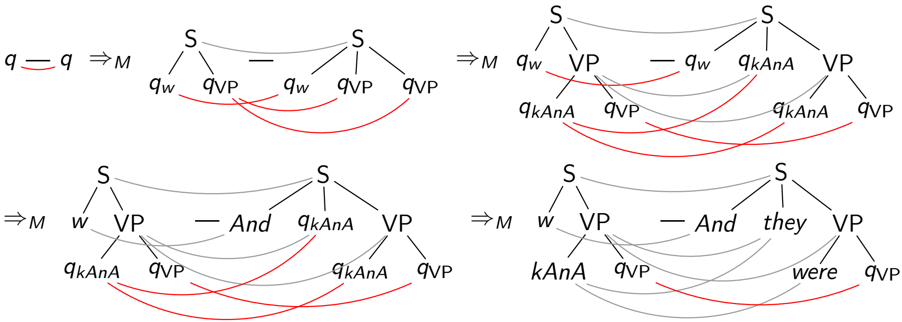


Sentential forms

$$\langle t, A, D, u \rangle$$

- $t \in T_\Sigma(Q)$ — input tree
- $A \subseteq \mathbb{N}^* \times \mathbb{N}^*$ — active links (red)
- $D \subseteq \mathbb{N}^* \times \mathbb{N}^*$ — disabled links (gray)
- $u \in T_\Sigma(Q)$ — output tree

Semantics


q — q ⇒M
S qw qVP — S qw qVP qVP
⇒M S qw VP qkAnA qVP — S qw qkAnA VP qkAnA qVP
⇒M S w VP qkAnA qVP — S And qkAnA VP qkAnA qVP
⇒M S w VP kAnA qVP — S And they VP were qVP

Semantics

Dependencies and relation

- state-computed dependencies:

$$M_q = \{\langle t, D, u\rangle \mid t, u \in T_\Sigma,\ \langle q, \{(\varepsilon, \varepsilon)\}, \emptyset, q\rangle \Rightarrow_M^* \langle t, \emptyset, D, u\rangle\}$$

- computed dependencies:

$$\mathrm{dep}(M) = \bigcup_{q \in I} M_q$$

Semantics

Dependencies and relation

- state-computed dependencies:

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- computed dependencies:

$$\mathrm{dep}(M) = \bigcup_{q \in I} M_q$$

- computed transformation:

$$\tau_M = \{(t, u) \mid \langle t, D, u\rangle \in \mathrm{dep}(M)\}$$

Further Properties

Regularity-preserving

transformation $\tau \subseteq T_\Sigma \times T_\Sigma$ preserves regularity
if $\tau(L) = \{u \mid (t, u) \in \tau,\ t \in L\}$ is regular for all regular $L \subseteq T_\Sigma$

rp-MBOT = regularity preserving transformations computable by MBOT

Compositions

- $\tau_1 \,;\, \tau_2 = \{(s, u) \mid \exists t\colon (s, t) \in \tau_1, (t, u) \in \tau_2\}$
- support modular development
- allow integration of external knowledge sources
- occur naturally in query rewriting

Contents

Dependencies

Recent research

- Bojańczyk, ICALP 2014
- Maneth et al., ICALP 2015

on models with dependencies

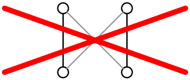
Dependencies

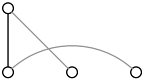
Hierarchical properties

A dependency $\langle t, D, u \rangle$ is

- input hierarchical if
  1. $w_2 \not< w_1$
  2. $\exists (v_1, w_1') \in D$ with $w_1' \leq w_2$

  for all $(v_1, w_1), (v_2, w_2) \in D$ with $v_1 < v_2$





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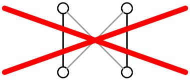

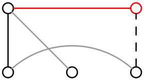

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  for all $(v_1, w_1), (v_2, w_2) \in D$ with $v_1 < v_2$
- strictly input hierarchical if
  1. $v_1 < v_2$ implies $w_1 \leq w_2$
  2. $v_1 = v_2$ implies $w_1 \leq w_2$ or $w_2 \leq w_1$

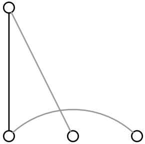
  for all $(v_1, w_1), (v_2, w_2) \in D$

Dependencies

Distance properties

A dependency $\langle t, D, u \rangle$ is

- input link-distance bounded by $b \in \mathbb{N}$
  if for all $(v_1, w_1), (v_1 v', w_2) \in D$ with $|v'| > b$
  $\exists (v_1 v, w_3) \in D$ such that $v < v'$ and $1 \leq |v| \leq b$



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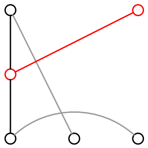

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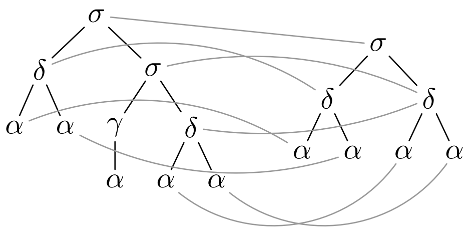
  $\exists (v_1 v, w_3) \in D$ such that $v < v'$ and $1 \leq |v| \leq b$
- strict input link-distance bounded by b
  if for all $v_1, v_1 v' \in \text{pos}(t)$ with $|v'| > b$
  $\exists (v_1 v, w_3) \in D$ such that $v < v'$ and $1 \leq |v| \leq b$

Dependencies



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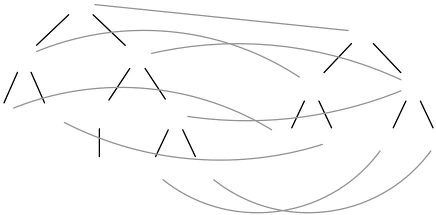

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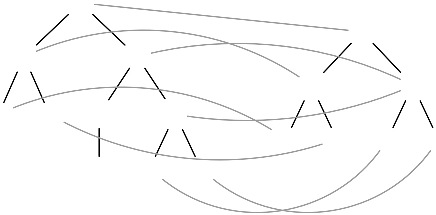
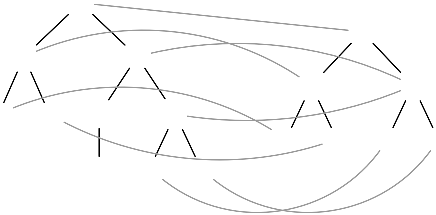


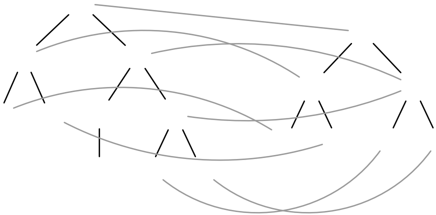
strictly input hierarchical

Dependencies



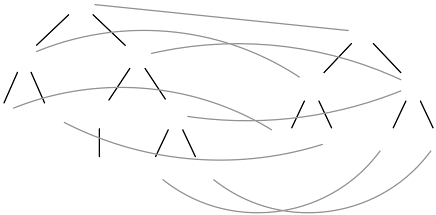
strictly input hierarchical and strictly output hierarchical

Dependencies



strictly input hierarchical and strictly output hierarchical
with strict input link-distance 2

Dependencies



strictly input hierarchical and strictly output hierarchical
with strict input link-distance 2 and strict output link-distance 1

Dependencies

	hierarchical		link-distance bounded	
Model \ Property	input	output	input	output
XTOP^{R}	strictly	strictly	✓	strictly
MBOT	✓	strictly	✓	strictly

Linking Theorem

Theorem

Let $M_1, \ldots, M_k$ *be* ε*-free* $XTOP^R$ *over* Σ *such that*

$$\{(c[t_1, \ldots, t_n]\,,\, c'[t_1, \ldots, t_n]) \mid t_1, \ldots, t_n \in T\} \subseteq \tau_{M_1} \,;\, \cdots \,;\, \tau_{M_k}$$

for some contexts $c, c' \in C_\Sigma(X_n)$ *and special* $T \subseteq T_\Sigma$*.*

$\forall 1 \leq i \leq k$, $\forall 1 \leq j \leq n$

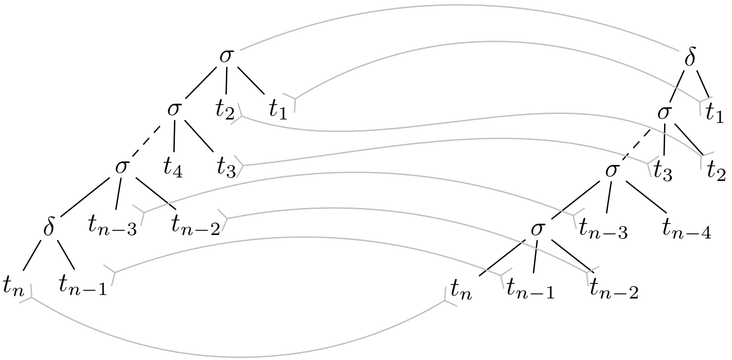
$\exists t_j \in T$, $\exists \langle u_{i-1}, D_i, u_i \rangle \in \mathrm{dep}(M_i)$, $\exists (v_{ji}, w_{ji}) \in D_i$ *such that*

- $u_0 = c[t_1, \ldots, t_n]$ *and* $u_k = c'[t_1, \ldots, t_n]$
- $\mathrm{pos}_{x_j}(c') \leq w_{jk}$
- $v_{ji} \leq w_{j(i-1)}$ *if* $i \geq 2$
- $\mathrm{pos}_{x_j}(c) \leq v_{j1}$

Linking Theorem

Corollary [Arnold, Dauchet, TCS 1982]

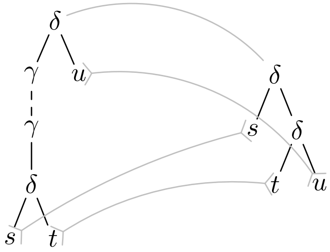
Illustrated relation τ cannot be computed by any ε-free XTOP^R



Linking Theorem

Corollary [M. et al., SICOMP 2009]

Illustrated relation τ cannot be computed by any ε-free $\mathsf{XTOP}^{\mathsf{R}}$



Topicalization

Example

- *It rained yesterday night.*

  Topicalized: *Yesterday night, it rained.*

Topicalization

Example

- *It rained yesterday night.*

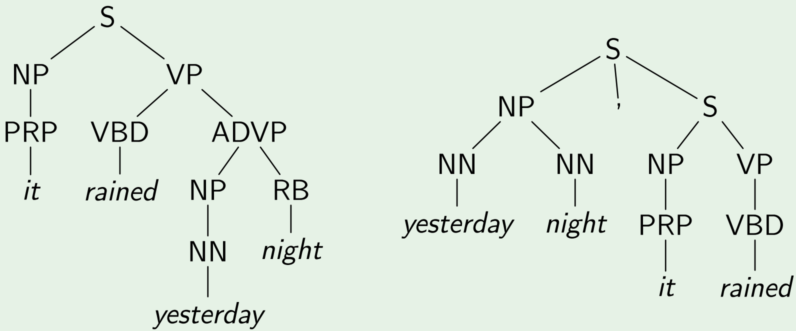
  Topicalized: *Yesterday night, it rained.*

- *We toiled all day yesterday at the restaurant that charges extra for clean plates.*

  Topicalized: *At the restaurant that charges extra for clean plates, we toiled all day yesterday.*

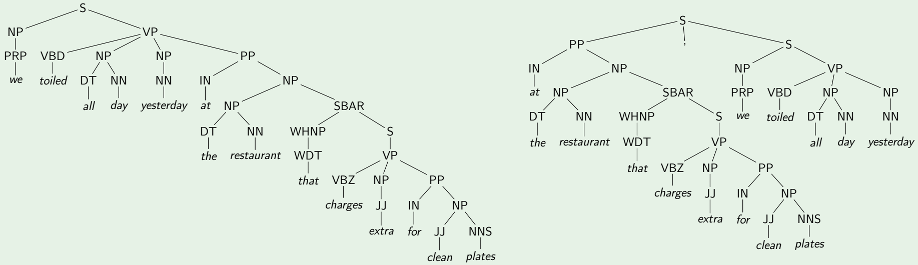
Topicalization

On the tree level

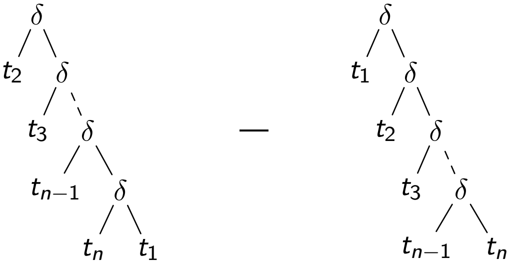


Topicalization

On the tree level



Topicalization



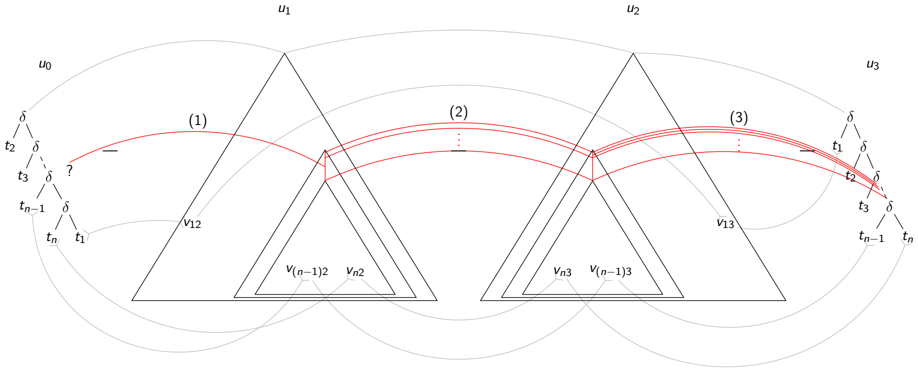
Theorem

Topicalization is in rp-MBOT

Topicalization

Theorem

Topicalization cannot be computed by any composition of ε-free XTOP^R



3 ε-free XTOP^R sufficient to simulate any composition of ε-free XTOP^R

Topicalization

Corollary

$$(\mathsf{XTOP}^{\mathsf{R}})^{*} \subsetneq \text{rp-MBOT}$$

Linking Theorem

Theorem

Let $M = (Q, \Sigma, I, R)$ *be an* ε*-free MBOT such that*

$$\{(c[t_1, \ldots, t_n]\,,\, c'[t_1, \ldots, t_n]) \mid t_1, \ldots, t_n \in T\} \subseteq \tau_M$$

for some contexts $c, c' \in C_\Sigma(X_n)$ *and special* $T \subseteq T_\Sigma$*.*

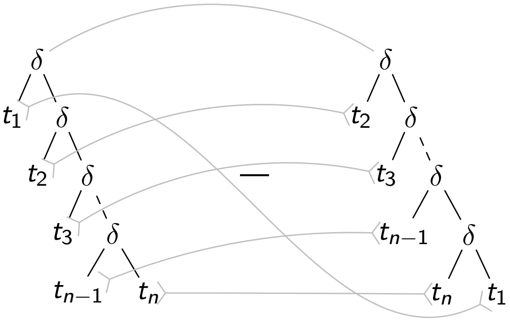
$\forall 1 \leq j \leq n$, $\exists t_j \in T$, $\exists \langle u, D, u' \rangle \in \mathrm{dep}(M)$, $\exists (v_j, w_j) \in D$ *with*

- $u = c[t_1, \ldots, t_n]$ *and* $u' = c'[t_1, \ldots, t_n]$
- $\mathrm{pos}_{x_j}(c) \leq v_j$
- $\mathrm{pos}_{x_j}(c') \leq w_j$

Linking Theorem

Corollary

Inverse of topicalization cannot be computed by any ε-free MBOT



Summary & References

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1. $(\mathrm{XTOP}^{\mathrm{R}})^* \subsetneq$ rp-MBOT
2. rp-MBOT not closed under inverses
3. What happens to invertable MBOT?

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