

Compositions of Weighted Extended Top-down Tree Transducers

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Contents

- 1 Weighted Extended Top-down Tree Transducer
- 2 General Composition
- 3 Composition with a WTT
- 4 Allowing ε -rules

Weight structure

Definition

Commutative semiring $(C, +, \cdot, 0, 1)$ if

- $(C, +, 0)$ and $(C, \cdot, 1)$ commutative monoids
- \cdot distributes over finite (incl. empty) sums

Idempotent if $c + c = c$

Example

- BOOLEAN semiring $(\{0, 1\}, \max, \min, 0, 1)$ (idempotent)
- semiring $(\mathbb{N}, +, \cdot, 0, 1)$ of non-negative integers
- tropical semiring $(\mathbb{N} \cup \{\infty\}, \min, +, \infty, 0)$ (idempotent)
- any field, ring, etc.

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Syntax

Definition (ARNOLD, DAUCHET 1976, GRAEHL, KNIGHT 2004)

Weighted extended top-down tree transducer (WXTT) is a system

$(Q, \Sigma, \Delta, I, R, \chi)$

- Q : finite set of *states*
- Σ and Δ : ranked alphabets of *input* and *output symbols*
- $I \subseteq Q$: *initial states*
- R : finite set of *rule identifiers*

- $\chi: R \rightarrow Q(T_\Sigma(X)) \times C \times T_\Delta(Q(X))$ such that
 - ▶ $\{\ell, r\} \not\subseteq Q(X)$
 - ▶ ℓ is linear (in X)
 - ▶ $\text{var}(r) \subseteq \text{var}(\ell)$

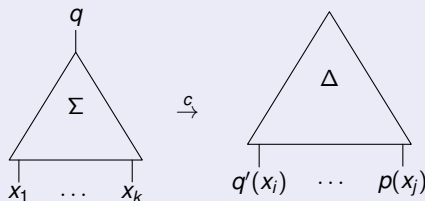
for every $(\ell, c, r) = \chi(\rho)$ with $\rho \in R$

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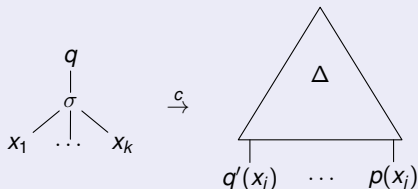


Syntactic Restrictions

Definition (ROUNDS 1970, THATCHER 1970)

- **Weighted top-down tree transducer (WTT)** if

$$\ell = q(\sigma(x_1, \dots, x_k))$$



- **linear** if r is linear
- **nondeleting** if $\text{var}(r) = \text{var}(\ell)$
- **ε -free** if $\ell \in Q(X)$
- **producing** if $r \notin Q(X)$

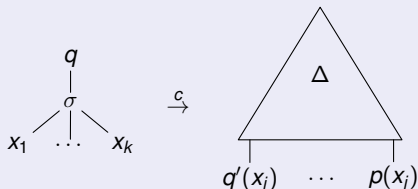
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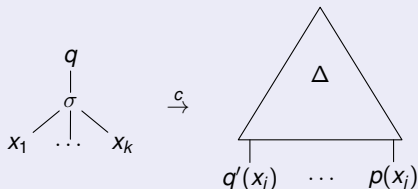
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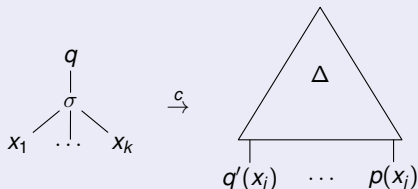
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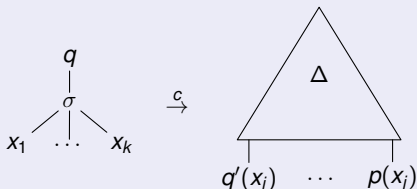
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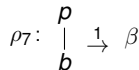
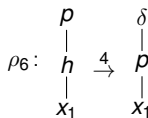
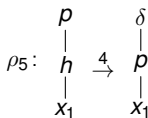
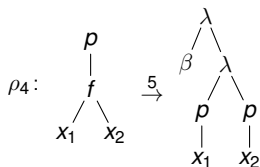
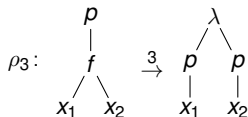
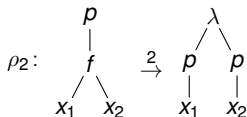
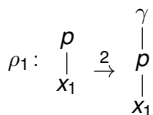
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Example



ϵ -rule	producing	linear	nondeleting
ρ_1	$\rho_1 - \rho_7$	$\rho_1 - \rho_7$	$\rho_1 - \rho_7$

Semantics

WXTT $M = (Q, \Sigma, \Delta, I, R, \chi)$, sentential forms $\xi, \zeta \in T_{\Delta'}(Q(T_{\Sigma'}(X)))$
with $\Sigma \subseteq \Sigma'$ and $\Delta \subseteq \Delta'$

Definition

- position $w \in \text{pos}_Q(\xi)$ in ξ **reducible** if $\xi|_w = \ell\theta$ for some rule $\ell \xrightarrow{c} r \in \chi(R)$ and substitution $\theta: X \rightarrow T_{\Sigma'}(X)$

- Let $\rho \in R$ with $\chi(\rho) = \ell \xrightarrow{c} r$.

ξ **rewrites to ζ using ρ** ($\xi \Rightarrow_M^\rho \zeta$) if

- $\xi|_w = \ell\theta$
- $\zeta = \xi[r\theta]_w$

for some substitution $\theta: X \rightarrow T_{\Sigma'}(X)$ and the least reducible position $w \in \text{pos}_Q(\xi)$

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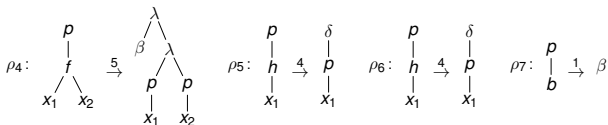
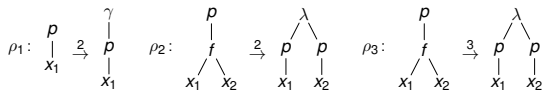
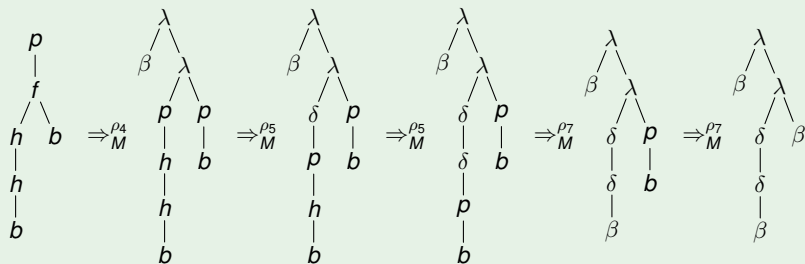
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Semantics

Example



Semantics

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Definition

- **Rule weight:** $\text{wt}_M(\rho) = c$ iff $\chi(\rho) = \ell \xrightarrow{c} r$
- $\text{wt}_M(\rho_1 \cdots \rho_k) = \prod_{i=1}^k \text{wt}_M(\rho_i)$
- **Derivation weight:**

$$\text{wt}_M(\xi, \zeta) = \sum_{\substack{\rho_1, \dots, \rho_k \in R \\ \xi \xRightarrow{p_1}_M \dots \xRightarrow{p_k}_M \zeta}} \text{wt}_M(\rho_1 \cdots \rho_k)$$

- **Semantics:** $M: T_{\Sigma} \times T_{\Delta} \rightarrow \mathcal{C}$ with $M(t, u) = \sum_{q \in I} \text{wt}_M(q(t), u)$

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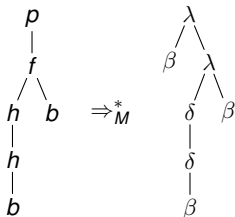
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Example



realized by the rule sequences:

$\rho_4 \rho_5 \rho_5 \rho_7 \rho_7$	$\text{wt}_M(\rho_4) \cdot \text{wt}_M(\rho_5) \cdot \text{wt}_M(\rho_5) \cdot \text{wt}_M(\rho_7)^2$
$\rho_4 \rho_5 \rho_6 \rho_7 \rho_7$	$\text{wt}_M(\rho_4) \cdot \text{wt}_M(\rho_5) \cdot \text{wt}_M(\rho_6) \cdot \text{wt}_M(\rho_7)^2$
$\rho_4 \rho_6 \rho_5 \rho_7 \rho_7$	$\text{wt}_M(\rho_4) \cdot \text{wt}_M(\rho_6) \cdot \text{wt}_M(\rho_5) \cdot \text{wt}_M(\rho_7)^2$
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$$M(f(h(h(b)), b), \lambda(\beta, \lambda(\delta(\delta(\beta)), \beta))) = 80 + 80 + 80 + 80 = 360$$

Semantic Properties

Definition

WXTT $M = (Q, \Sigma, \Gamma, I, R, \chi)$

- **functional (total)** if for every $q \in Q$ and $t \in T_\Sigma$ there exists at most (at least) one $u \in T_\Gamma$ such that

$$\text{wt}_M(q(t), u) \neq 0$$

Remark

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Example Composition

Example

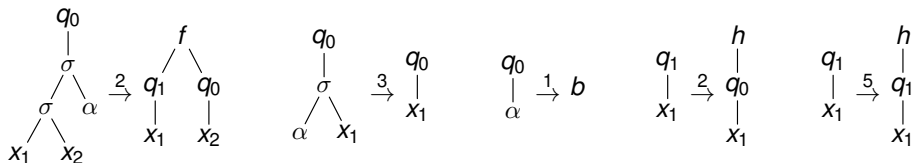
semiring $(\mathbb{R}, +, \cdot, 0, 1)$ and WXTT $M = (Q, \Sigma, \Gamma, l, R, \chi)$

- $Q = \{q_0, q_1\}$ and $l = \{q_0\}$
- $\Sigma = \{\sigma, \alpha\}$ and $\Gamma = \{f, h, b\}$
- the following rules:

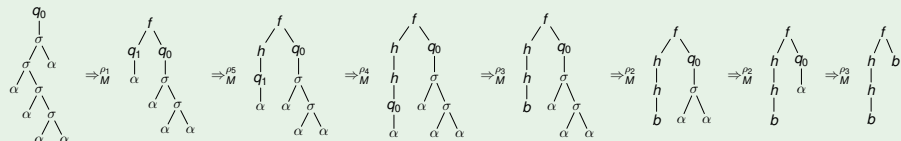
$$\begin{array}{ll} \rho_1 : & q_0(\sigma(\sigma(x_1, x_2), \alpha)) \xrightarrow{2} f(q_1(x_1), q_0(x_2)) \\ \rho_2 : & q_0(\sigma(\alpha, x_1)) \xrightarrow{3} q_0(x_1) \\ \rho_3 : & q_0(\alpha) \xrightarrow{1} b \\ \rho_4 : & q_1(x_1) \xrightarrow{2} h(q_0(x_1)) \\ \rho_5 : & q_1(x_1) \xrightarrow{5} h(q_1(x_1)) \end{array}$$

ε -rule	producing	linear	nondeleting
ρ_4, ρ_5	$\rho_1, \rho_3 - \rho_5$	$\rho_1 - \rho_5$	$\rho_1 - \rho_5$

Example Composition



Example (Derivation)



$$\text{wt}_M(\rho_1 \rho_5 \rho_4 \rho_3 \rho_2 \rho_2 \rho_3) = 2 \cdot 5 \cdot 2 \cdot 1 \cdot 3 \cdot 3 \cdot 1 = 180$$

A Second WXTT

Example

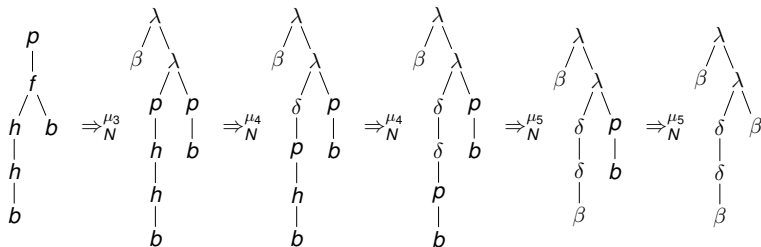
WXTT $N = (\{p\}, \Gamma, \Delta, \{p\}, R', \chi')$

- $\Gamma = \{f, h, b\}$ and $\Delta = \{\lambda, \gamma, \delta, \beta\}$
- the following rules:

$$\begin{array}{l} \mu_1: \quad p(x_1) \xrightarrow{2} \gamma(p(x_1)) \\ \mu_2: \quad p(f(x_1, x_2)) \xrightarrow{5} \lambda(p(x_1), p(x_2)) \\ \mu_3: \quad p(f(x_1, x_2)) \xrightarrow{3} \lambda(\beta, \lambda(p(x_1), p(x_2))) \\ \mu_4: \quad p(h(x_1)) \xrightarrow{8} \delta(p(x_1)) \\ \mu_5: \quad p(b) \xrightarrow{1} \beta \end{array}$$

ε -rule	producing	linear	nondeleting
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A Second WXTT



$$\text{wt}_N(\mu_3\mu_4\mu_4\mu_5\mu_5) = 5 \cdot 8 \cdot 8 \cdot 1 \cdot 1 = 320$$

Formal Definition

$\tau_1: T_\Sigma \times T_\Gamma \rightarrow A$ and $\tau_2: T_\Gamma \times T_\Delta \rightarrow A$

$$(\tau_1 ; \tau_2)(s, u) = \sum_{t \in T_\Gamma} \tau_1(s, t) \cdot \tau_2(t, u)$$

Restriction

- (i) to permit infinite sums
- (ii) to restrict the weighted relations

Here:

For all $s \in T_\Sigma$ and $u \in T_\Delta$

$$\{t \mid (s, t) \in \text{supp}(\tau_1)\} \quad \text{or} \quad \{t \mid (t, u) \in \text{supp}(\tau_2)\}$$

is finite

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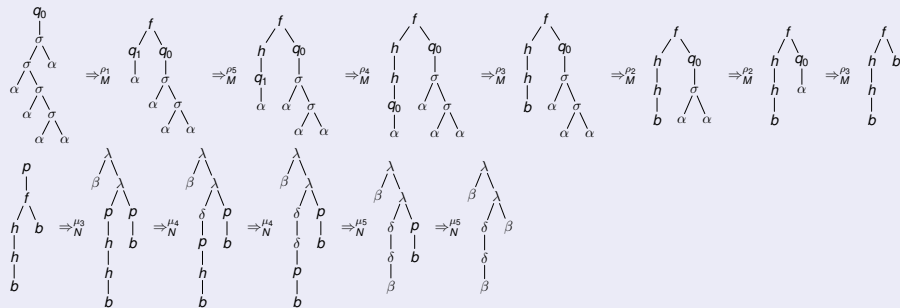
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Composition Strategy

$\rho_1 \rho_5 \rho_4 \rho_3 \rho_2 \rho_2 \rho_3 \mu_3 \mu_4 \mu_4 \mu_5 \mu_5$

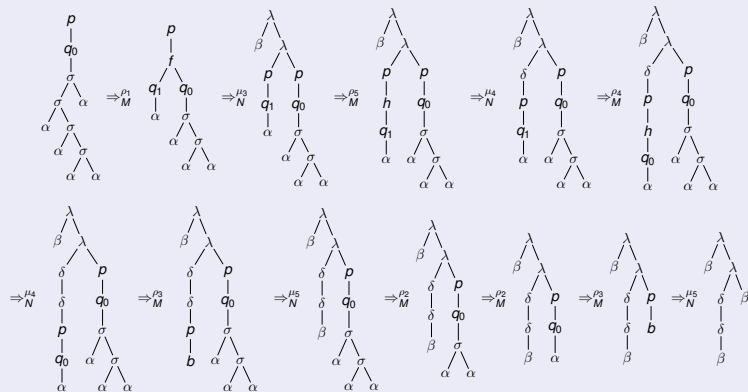
Original order



Composition Strategy

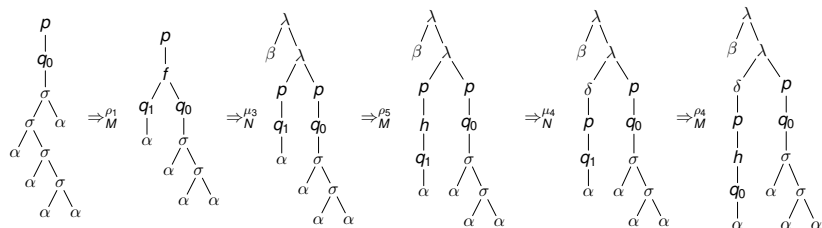
$\rho_1 \rho_5 \rho_4 \rho_3 \rho_2 \rho_2 \rho_3 \mu_3 \mu_4 \mu_4 \mu_5 \mu_5$

Reordered

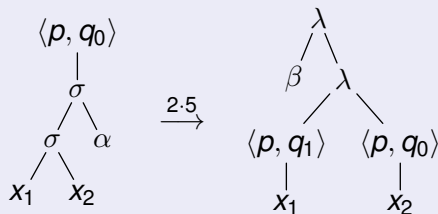


$\rho_1 \mu_3 \rho_5 \mu_4 \rho_4 \mu_4 \rho_3 \mu_5 \rho_2 \rho_2 \rho_3 \mu_5$

Composition Strategy



Composed rule



The Unweighted Case

Composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
(c)	functional	nondeleting
(d)	functional and total	

References:

[[ENGELFRIET](#): *Bottom-up and top-down tree transformations — A comparison.* Math. Syst. Theory 9(3): 198-231 (1975)]

[[BAKER](#): *Composition of top-down and bottom-up tree transductions.* Inf. Control 41(2): 186-213 (1979)]

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Overview

Requirements

- WXTT $M = (Q, \Sigma, \Gamma, l_1, R_1, \chi_1)$ without ε -rules
- WTT $N = (P, \Gamma, \Delta, l_2, R_2, \chi_2)$

Constants

- Let $c \geq |\text{pos}_x(r)|$ for all rules $l \rightarrow r$ of N and $x \in X$
- Let $s \geq |\text{pos}_\Gamma(r)|$ for all rules $l \rightarrow r$ of M
- Let $m \geq c^s$

Construction

Definition

Composed WXTT $M ; N = (P \times Q, \Sigma, \Delta, l_2 \times l_1, R, \chi)$

- $R = \{ \langle \rho, p, w \rangle \mid \rho \in R_1, p \in P, w \in R_2^*, |w| \leq m \}$
- $\chi(\rho, p, \mu_1 \cdots \mu_k) = (p(\ell), a, r')$
 - ▶ for every $\rho \in R_1$ with $\chi_1(\rho) = \ell \rightarrow r$
 - ▶ $p \in P$
 - ▶ $\mu_1, \dots, \mu_k \in R_2$ with $k \leq m$
 - ▶ $r' \in T_\Delta(P(Q(X)))$ and

$$a = \begin{cases} \text{wt}_M(\rho) \cdot \prod_{i=1}^k \text{wt}_N(\mu_i) & \text{if } p(\ell) \Rightarrow_M^\rho ; \Rightarrow_N^{\mu_1 \cdots \mu_k} r' \\ 0 & \text{otherwise.} \end{cases}$$

Construction — Example

Example

- M has the rules

$$\rho_1: q(\gamma(x_1)) \xrightarrow{2} \gamma(\gamma(q(x_1)))$$

$$\rho_2: q(\alpha) \xrightarrow{2} \alpha$$

- N has the rules

$$\mu_1: p_0(\gamma(x_1)) \xrightarrow{4} \sigma(p_0(x_1), p_0(x_1))$$

$$\mu_6: p(\gamma(x_1)) \xrightarrow{1} \gamma(p(x_1))$$

$$\mu_2: p_0(\gamma(x_1)) \xrightarrow{2} \sigma(p_0(x_1), p(x_1))$$

$$\mu_7: p(\gamma(x_1)) \xrightarrow{3} \alpha$$

$$\mu_3: p_0(\gamma(x_1)) \xrightarrow{2} \sigma(p(x_1), p_0(x_1))$$

$$\mu_8: p(\alpha) \xrightarrow{1} \alpha$$

$$\mu_4: p_0(\gamma(x_1)) \xrightarrow{1} \sigma(p(x_1), p(x_1))$$

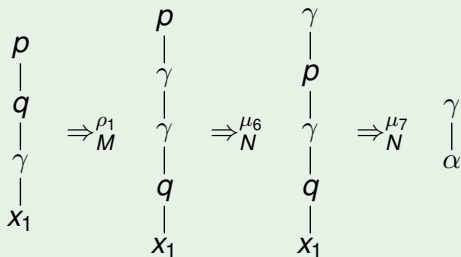
$$\mu_5: p_0(\alpha) \xrightarrow{1} \alpha$$

ε -rule	producing	linear	nondeleting
	ρ_1, ρ_2	ρ_1, ρ_2	ρ_1, ρ_2
	$\mu_1 - \mu_8$	$\mu_5 - \mu_8$	$\mu_1 - \mu_6, \mu_8$

Construction — Example

Example

rule for identifier $\iota = \langle \rho_1, \rho, \mu_6 \mu_7 \rangle$

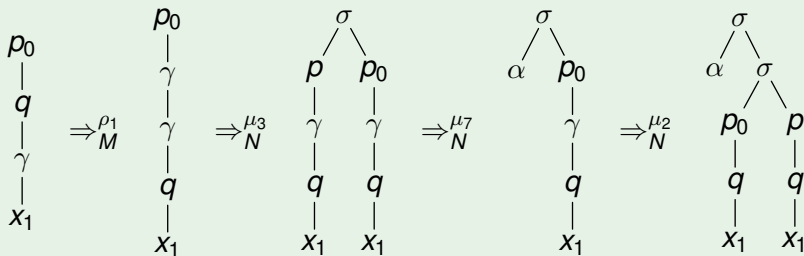


$$\chi(\iota) = \left(\langle p, q \rangle(\gamma(x_1)), 2 \cdot 1 \cdot 3, \gamma(\alpha) \right)$$

Construction — Example

Example

rule for identifier $\iota = \langle \rho_1, \rho_0, \mu_3 \mu_7 \mu_2 \rangle$



$$\chi(\iota) = \left(\langle \rho_0, q \rangle(\gamma(x_1)), 2 \cdot 2 \cdot 3 \cdot 2, \sigma(\alpha, \sigma(\langle \rho_0, q \rangle(x_1), \langle p, q \rangle(x_1))) \right)$$

First Result

Unweighted composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
(c)	functional	nondeleting
(d)	functional and total	

Theorem

If the WTT N is linear and nondeleting, then $\tau_{M;N} = \tau_M ; \tau_N$.

[\sim : *Compositions of tree series transformations.*

Theor. Comput. Sci. 366(3): 248–271 (2006)]

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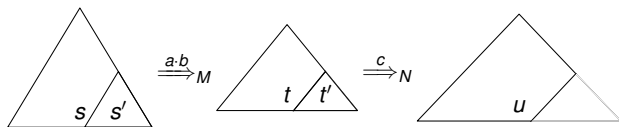
Theor. Comput. Sci. 366(3): 248–271 (2006)]

Towards a Second Result

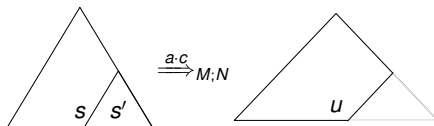
Unweighted composition results

Case	M	N
(a)		linear and nondeleting
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Composition:



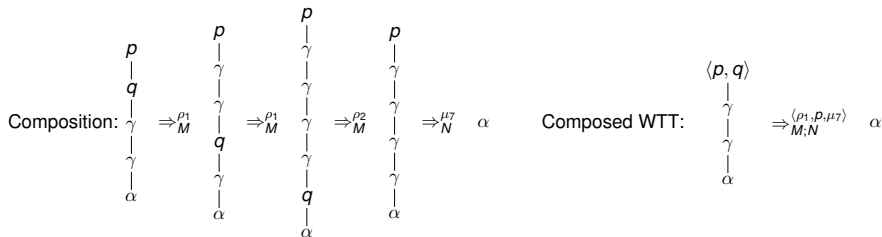
Composed WTT:



Towards a Second Result

Unweighted composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
(c)	functional	nondeleting
(d)	functional and total	



Towards a Second Result

Definition

$q \in Q$ **constant** if there is $c \in C$

$$\sum_{t \in T_{\Gamma}} \sum_{\substack{\rho_1, \dots, \rho_k \in R_1 \\ q(s) \Rightarrow_M^{\rho_1}; \dots; \Rightarrow_M^{\rho_k} t}} \text{wt}_M(\rho_1 \cdots \rho_k) = c$$

for every $s \in T_{\Sigma}$

Example

1-constant states:

- in every total WTT over the BOOLEAN semiring
- in every BOOLEAN and total WTT over an idempotent semiring
- in every functional, total, and BOOLEAN WTT

Towards a Second Result

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1-constant states:

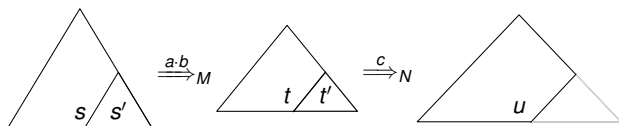
- in every total WTT over the BOOLEAN semiring
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A Second Result

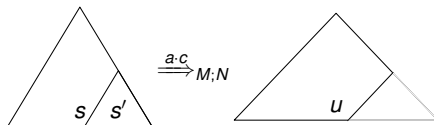
Conjecture

If the WXTT M is constant and the WTT N is linear, then $\tau_M; \tau_N$ can be computed by a WXTT.

Composition:



Composed WTT:

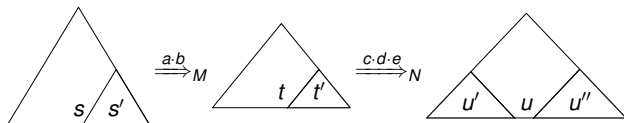


The Third Case

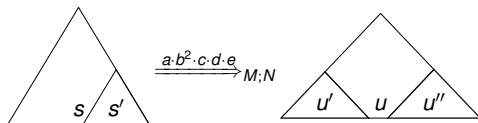
Unweighted composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
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(d)	functional and total	

Composition:

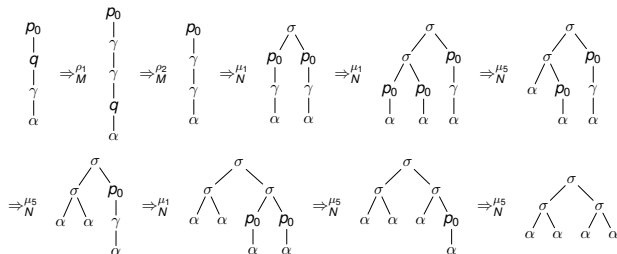


Composed WTT:

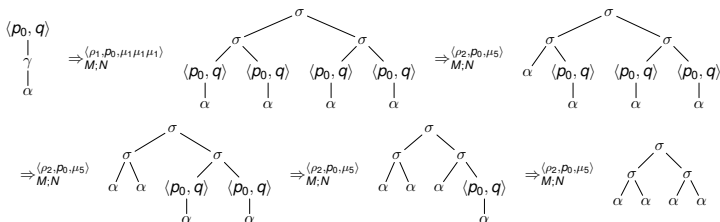


The Third Case

Composition:

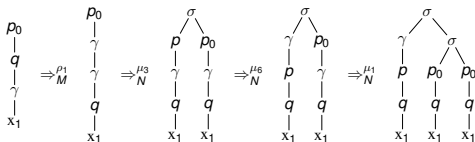


Composed WTT:

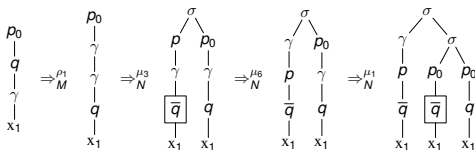


The Third Case

Original derivation:



Modified derivation:



Conjecture

If the WXTT M is functional and the WTT N is nondeleting, then $\tau_M ; \tau_N$ can be computed by a WXTT.

The Last Case

Unweighted composition results

Case	M	N
(a)		linear and nondeleting
(b)	total	linear
(c)	functional	nondeleting
(d)	functional and total	

Conjecture

If the WXTT M is functional and constant and N is a WTT, then $\tau_M ; \tau_N$ can be computed by an WXTT.

The Last Case

Unweighted composition results

Case	M	N
(a)		linear and nondeleting
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Conjecture

If the WXTT M is functional and constant and N is a WTT, then $\tau_M ; \tau_N$ can be computed by an WXTT.

essentially a combination of Case (b) and (c)

Summary

Case	M	N	Reference
(a)	linear, nondel. functional	linear, nondel. func., lin., nondel. linear, nondel.	[1] Thm. 2.4 [2] Thm. 5.18 [3] Thm. 26
(b)	constant	linear	Conj.
(c)	functional	nondeleting	Conj.
(d)	BOOL., func., total BOOL., func., total constant, func.	functional linear	[2] Thm. 5.18 [3] Thm. 30 Conj.

- [1] **KUICH**: *Full Abstract Families of Tree Series I*.
In “Jewels are Forever”, pp. 145–156 (1999)
- [2] **ENGELFRIET, FÜLÖP, VOGLER**: *Bottom-up and top-down tree series transformations*. *J. Autom. Lang. Combin.* 7(1): 11–70 (2002)
- [3] \sim : *Compositions of tree series transformations*.
Theor. Comput. Sci. 366(3): 248–271 (2006)

Contents

- 1 Weighted Extended Top-down Tree Transducer
- 2 General Composition
- 3 Composition with a WTT
- 4 Allowing ε -rules

Requirements

Definition

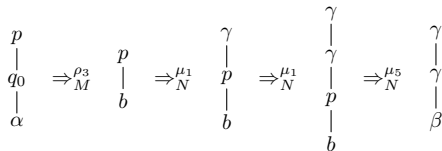
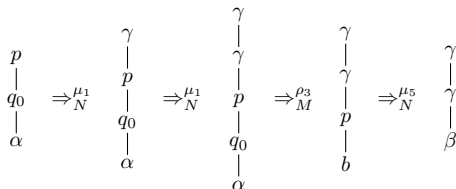
- WXTT M **shallow** if $|\text{pos}_\Gamma(r)| \leq 1$ for every rule $\ell \rightarrow r$ of M
- WXTT N **WTT with ε -rules** if $|\text{pos}_\Gamma(\ell)| \leq 1$ for every rule $\ell \rightarrow r$ of N

[\sim , **VOGLER**: *Compositions of top-down tree transducers with ε -rules*.
In Proc. FSMNLP 2009, pp. 69–80 (2010)]

Why Another Construction

Example

This derivation cannot be simulated by $M ; N$ because the rules constructed for $M ; N$ always trigger a rule of M first



New Construction

Definition

$$M ;_{\varepsilon} N = (P \times Q, \Sigma, \Delta, l_2 \times l_1, R, \chi)$$

- rule identifiers:

$$R = \{\langle \rho, p, \varepsilon \rangle \mid \text{erasing } \rho \in R_1, p \in P\} \cup \\ \cup \{\langle \rho, p, \mu \rangle \mid \text{producing } \rho \in R_1, p \in P, \text{ consuming } \mu \in R_2\} \cup \\ \cup \{\langle \varepsilon, q, \mu \rangle \mid q \in Q, \varepsilon\text{-rule } \mu \in R_2\}$$

- $\chi(\langle \rho, p, \varepsilon \rangle) = (p(\ell), \text{wt}_M(\rho), p(r))$ for all erasing $\rho = \ell \rightarrow r, p \in P$
- $\chi(\langle \rho, p, \mu \rangle) = (p(\ell), a, r')$

$$a = \begin{cases} \text{wt}_M(\rho) \cdot \text{wt}_N(\mu) & \text{if } p(\ell) \Rightarrow_M^{\rho} ; \Rightarrow_N^{\mu} r' \\ 0 & \text{otherwise} \end{cases}$$

for producing $\rho = \ell \rightarrow r$ of $M, p \in P$, and consuming $\mu \in R_2$

- $\chi(\langle \varepsilon, q, \mu \rangle) = \dots$

New Construction

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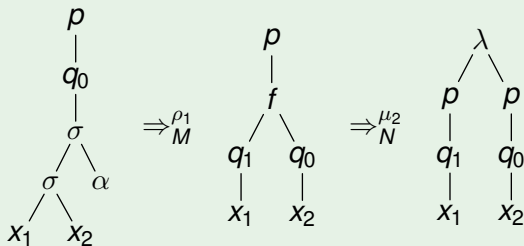
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for producing $\rho = \ell \rightarrow r$ of $M, p \in P$, and consuming $\mu \in R_2$

- $\chi(\langle \varepsilon, q, \mu \rangle) = \dots$

Construction — Example

Example



Unweighted Setting

Theorem

M shallow, *N* is a WTT with ε -rules. If

- *N* is linear, and
- *M* is total or *N* is nondeleting,

then $\tau_M ; \tau_N$ can be computed by a WXTT.

Case	<i>M</i>	<i>N</i>	Reference
(a)		linear, nondeleting	[1] Thm. 17
(b)	total	linear	[1] Thm. 17

[1] \sim , [VOGLER](#): *Compositions of top-down tree transducers with ε -rules.*

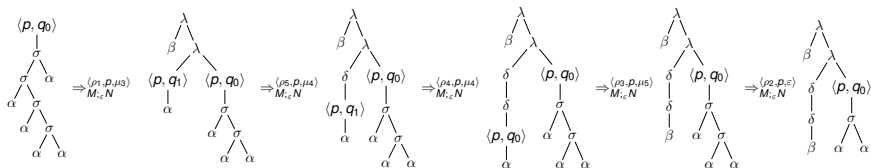
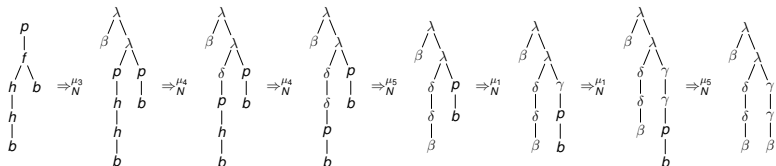
In Proc. FSMNLP 2009, pp. 69–80 (2010)

The First Case

Theorem

If M is shallow and N is a linear and nondeleting WTT with ε -rules, then $\tau_{M;\varepsilon}N = \tau_M ; \tau_N$.

The First Case



Step	1	2	3	4	5	6	7
d_M :	ρ_1	ρ_5	ρ_4	ρ_3	ρ_2	ρ_2	ρ_3
d_N :	μ_3	μ_4	μ_4	μ_5			$\mu_1 \mu_1 \mu_5$
d :	$\langle \rho_1, p, \mu_3 \rangle$	$\langle \rho_5, p, \mu_4 \rangle$	$\langle \rho_4, p, \mu_4 \rangle$	$\langle \rho_3, p, \mu_5 \rangle$	$\langle \rho_2, p, \varepsilon \rangle$	$\langle \rho_2, p, \varepsilon \rangle$	$\langle \varepsilon, q_0, \mu_1 \rangle \langle \varepsilon, q_0, \mu_1 \rangle \langle \rho_3, p, \mu_5 \rangle$

The Second Case

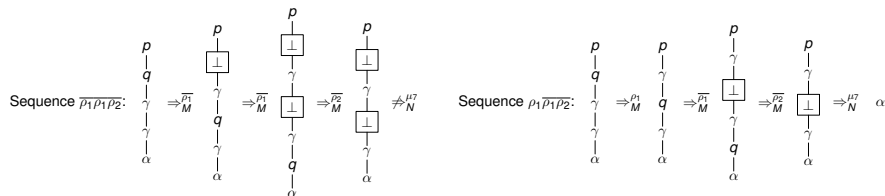
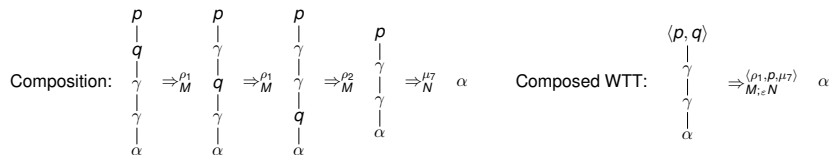
Theorem

*If the shallow WXTT M is total and **BOOLEAN**, the WTT N with ε -rules is linear, and the semiring C is idempotent, then $\tau_{M;\varepsilon N} = \tau_M ; \tau_N$.*

The Second Case

Theorem

If the shallow WXTT M is total and **BOOLEAN**, the WTT N with ε -rules is linear, and the semiring C is idempotent, then $\tau_{M;\varepsilon N} = \tau_M ; \tau_N$.



Conjecture

If the shallow WXTT M is constant and the WTT N with ε -rules is linear, then $\tau_M ; \tau_N$ can be computed by a WXTT.

Case	M	N
(a)		linear, nondeleting
(b)	total, BOOLEAN constant	linear linear

It's over!

Thank you for your attention!