Efficiency Analysis for the Elimination of Intermediate Results in Functional Programs by Compositions of Attributed Tree Transducers

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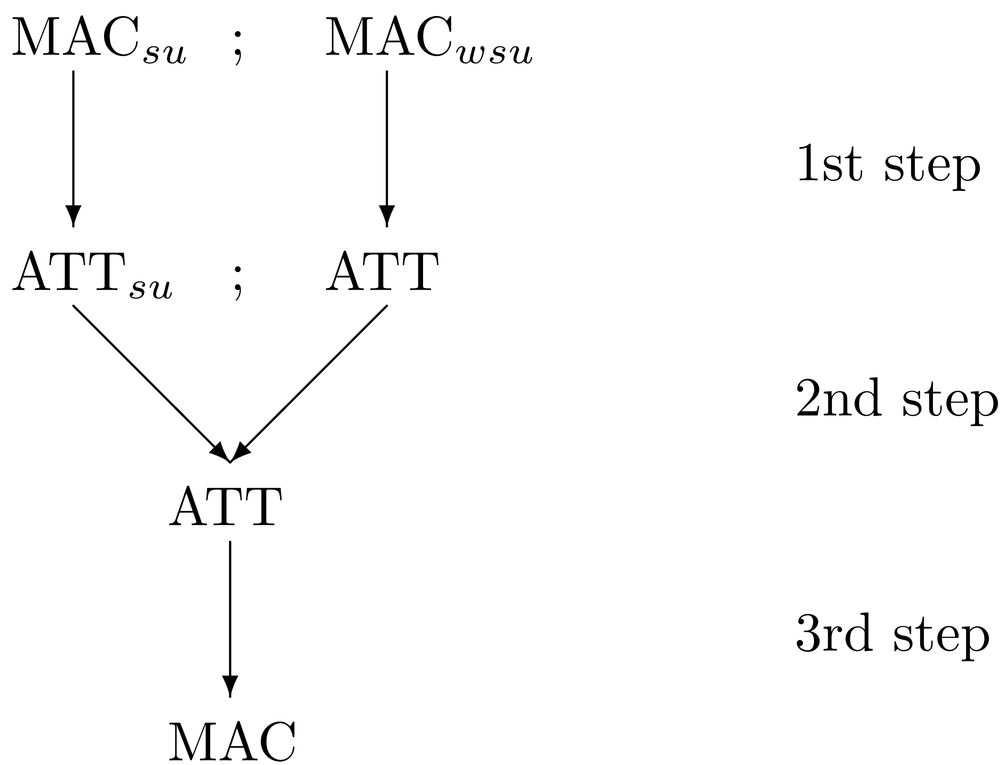
Itinerary

- Motivation and introduction
- Step 1: To Attributed Tree Transducers
- Step 2: Composing Attributed Tree Transducers
- Step 3: Back to Macro Tree Transducers
- Conclusions

Motivation

- Intermediate results are ubiquitous in functional programs.
- Elimination of such results **might** therefore
  - save memory and
  - speed up computation of the final result.
- *Major question:* How can we compose functions symbolically?
- *Minor question:* Can we guarantee a speed-up?

Introduction



Macro Tree Transducers

- Special (restricted) functional programs

Example:

$$M_{\text{pal}} = \left(\{A^{(1)}, B^{(1)}, N^{(0)}\}, \{A^{(1)}, B^{(1)}, N^{(0)}\}, \{s^{(1)}\}, (s\,x_1\,N), R\right)$$

$$\begin{array}{lllll} R = \{ & s\,(A\,x_1)\,y_1 & = & A\left(s\,x_1\,(A\,y_1)\right) & , \\ & s\,(B\,x_1)\,y_1 & = & B\left(s\,x_1\,(B\,y_1)\right) & , \\ & s\,N\,y_1 & = & y_1 & \} \end{array}$$

- Appends the reversed input list to the input list; constructs a palindrome

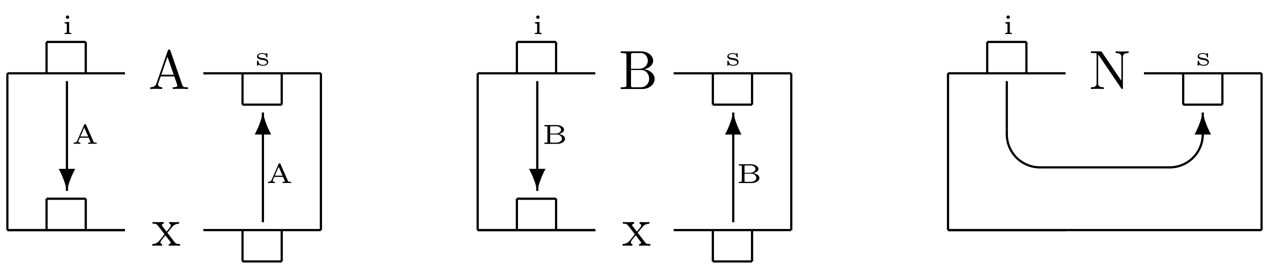
Attributed Tree Transducers

- Special (restricted) attribute grammars

Example:

$$M'_{\mathrm{pal}} = \big(\{A^{(1)}, B^{(1)}, N^{(0)}\}, \{A^{(1)}, B^{(1)}, N^{(0)}\}, \{s\}, \{i\}, \widehat{s}, \widehat{\sigma}, R\big)$$

Depiction of some rules in the rule-set R:



$\mathbf{MAC}_{wsu} \subseteq \mathbf{ATT}$

- Synthesized attributes instead of function symbols
- Simulate context parameters by inherited attributes
- Associate a set of inherited attributes to every synthesized attribute

$\mathbf{MAC}_{wsu} \subseteq$ ATT (cont'd)

- Macro Tree Transducer M,
  Attributed Tree Transducer $M' = C[M]$ ($\tau_{M'} = \tau_M$)
- Established efficiency relation:

$$\text{count}(M) = \text{count}(M') - i - 1$$

- i : number of reduction steps invested to reduce inherited attribute instances

$\Rightarrow$ only count the (non-root) synthesized attribute instances

$\mathbf{MAC}_{wsu} \subseteq$ ATT (Example)

Macro Tree Transducer:

$\underline{s\,(ABN)\,N}$

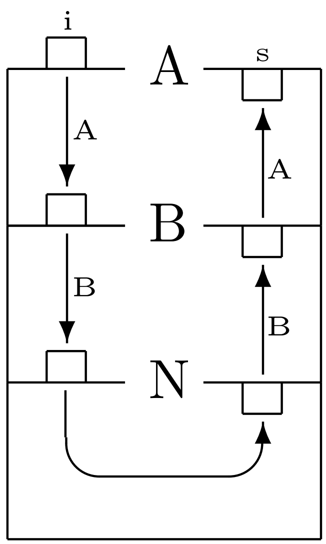
$\Rightarrow A\,(\underline{s\,BN(AN)})$

$\Rightarrow AB\,(\underline{s\,N(BAN)})$

$\Rightarrow ABBAN$

3 reduction steps

Attributed Tree Transducer:



3 synthesized attribute instances

$\mathbf{ATT}_{su}$; ATT $\subseteq$ ATT

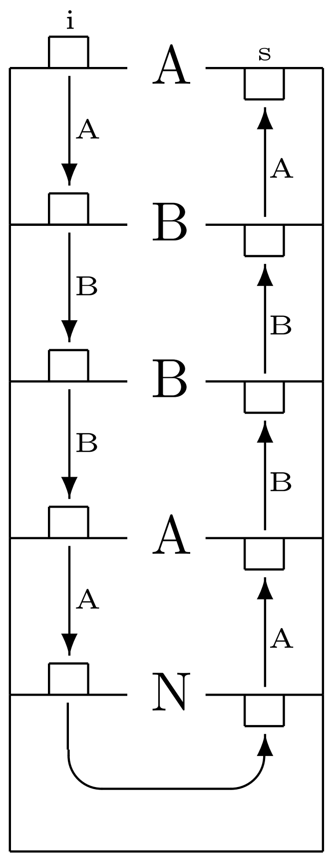
- Core idea of construction: *pairing of attributes*
- Efficiency considerations

M_1, M_2	syntactic single-use atts
F_1, F_2	attribute sets of M_1, M_2
$M = C[M_1, M_2]$	att with $\tau_M = \tau_{M_1} ; \tau_{M_2}$
t_1, t_2	input tree for M_1 and $t_2 = \tau_{M_1}(t_1)$

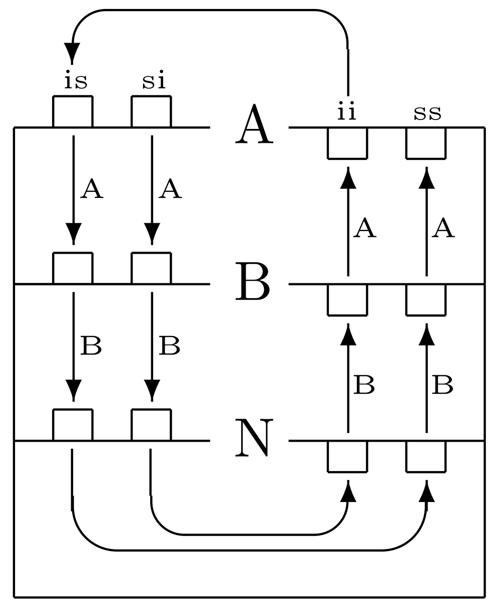
M is more efficient than $(M_1 ; M_2)$, iff

$$\text{size}(t_1)|F_1|\,|F_2| + 1 \;<\; \text{size}(t_1)|F_1| + \text{size}(t_2)|F_2| + 2$$

$\mathbf{ATT}_{su}$; $\mathbf{ATT} \subseteq \mathbf{ATT}$ (Example)



Composition result $(M'_{\mathrm{pal}}; M'_{\mathrm{pal}})$:



$\mathbf{ATT}_{su}$; ATT ⊆ ATT (Example cont'd)

$$\text{size}(t_1)|F_1|\,|F_2| + 1 < \text{size}(t_1)|F_1| + \text{size}(t_2)|F_2| + 2$$

- in our example:

$$3 \cdot 2 \cdot 2 + 1 = 13 \;<\; 18 = 3 \cdot 2 + 5 \cdot 2 + 2$$

- in general for $(M'_{\text{pal}}; M'_{\text{pal}})$:

$$n \cdot 2 \cdot 2 + 1 = 4n + 1 \;<\; 6n = n \cdot 2 + (2n - 1) \cdot 2 + 2$$

$n = \text{size}(t_1) > 0$

ATT $\subset$ MAC

- Function symbols replace synthesized attributes
- Operations on inherited attributes simulated in context parameters
- Every function symbol has as many context parameters as there are inherited attributes

ATT $\subset$ MAC (cont'd)

- single-use Attributed Tree Transducer M',
  Macro Tree Transducer $M = C[M']$ $(\tau_M = \tau_{M'})$
- Established efficiency relation:

$$\text{count}(M) = \text{count}(M') - i - 1$$

- i : number of reduction steps invested to reduce inherited attribute instances

$\Rightarrow$ only count the (non-root) synthesized attribute instances

ATT $\subset$ MAC (Example)

- Running example (let $\Sigma = \{A^{(1)}, B^{(1)}, N^{(0)}\}$):

$$M_{\text{pal;pal}} = \big(\Sigma, \Sigma, \{ss^{(2)}, ii^{(2)}\}, (ss\,x_1\,(ii\,x_1\,N\,N)\,N), R\big)$$

$$
\begin{array}{llcl}
R = \{ & ss\,(A\,x_1)\,y_1\,y_2 & = & A\,\big(ss\,x_1\,(A\,y_1)\,(A\,y_2)\big)\;, \\
& ss\,(B\,x_1)\,y_1\,y_2 & = & B\,\big(ss\,x_1\,(B\,y_1)\,(B\,y_2)\big)\;, \\
& ss\,N\,y_1\,y_2 & = & y_1 \\
& ii\,(A\,x_1)\,y_1\,y_2 & = & A\,\big(ii\,x_1\,(A\,y_1)\,(A\,y_2)\big)\;, \\
& ii\,(B\,x_1)\,y_1\,y_2 & = & B\,\big(ii\,x_1\,(B\,y_1)\,(B\,y_2)\big)\;, \\
& ii\,N\,y_1\,y_2 & = & y_2 \qquad \}
\end{array}
$$

ATT $\subset$ MAC (Example cont'd)

- Reduction:

$$\underline{ss\,(ABN)\,(ii\,(ABN)\,N\,N)\,N}$$
$$\Rightarrow A\,(\underline{ss\,(BN)\,(A\,[ii\,(ABN)\,N\,N])\,(AN)})$$
$$\Rightarrow AB\,(\underline{ss\,N\,(BA\,[ii\,(ABN)\,N\,N])\,(BAN)})$$
$$\Rightarrow ABBA\,[\underline{ii\,(ABN)\,N\,N}]$$
$$\Rightarrow ABBAA\,[\underline{ii\,(BN)\,(AN)\,(AN)}]$$
$$\Rightarrow ABBAAB\,[\underline{ii\,N\,(BAN)\,(BAN)}]$$
$$\Rightarrow ABBAABBAN$$

6 reduction steps

Main theorem

M_1, M_2	syntactic single-use and preserving macs
F_1, F_2	set containing the function symbols of M_1, M_2
$M = C[M_1, M_2]$	mac with $\tau_M = \tau_{M_1}; \tau_{M_2}$
t_1, t_2	input tree for M_1 and $t_2 = \tau_{M_1}(t_1)$

M is more efficient than $(M_1; M_2)$, iff

$$\mathrm{size}(t_1)\Big(|F_1|\,|F_2| + \sum_{f \in F_1} \mathrm{rank}_{F_1}(f) \sum_{f \in F_2} \mathrm{rank}_{F_2}(f)\Big) < \mathrm{size}(t_1)|F_1| + \mathrm{size}(t_2)|F_2|$$

Main theorem (cont'd)

- Running example with $n = \text{size}(t_1)$:

$$n \cdot (1 \cdot 1 + 1 \cdot 1) = 2n \; < \; 3n - 1 = n \cdot 1 + (2n-1) \cdot 1$$

- Some derived results (only additional properties listed):

M_1	M_2	$M = C[M_1, M_2]$ more efficient, if
	tdtt, $\lvert F_2 \rvert = 1$	always
producing		$\lvert F_1 \rvert \; > \; \text{rsum}(F_1) \cdot \text{rsum}(F_2)$

Conclusions

- Composition result seems to suffer heavily from the explosion in the number of attributes.
- From the theorem several small classes can be derived.
- Further studies (especially in connection with further optimization techniques like copy rules elimination) for other composition techniques are necessary.
- Implementations!!