# Compositions of Bottom-Up Tree Series Transformations

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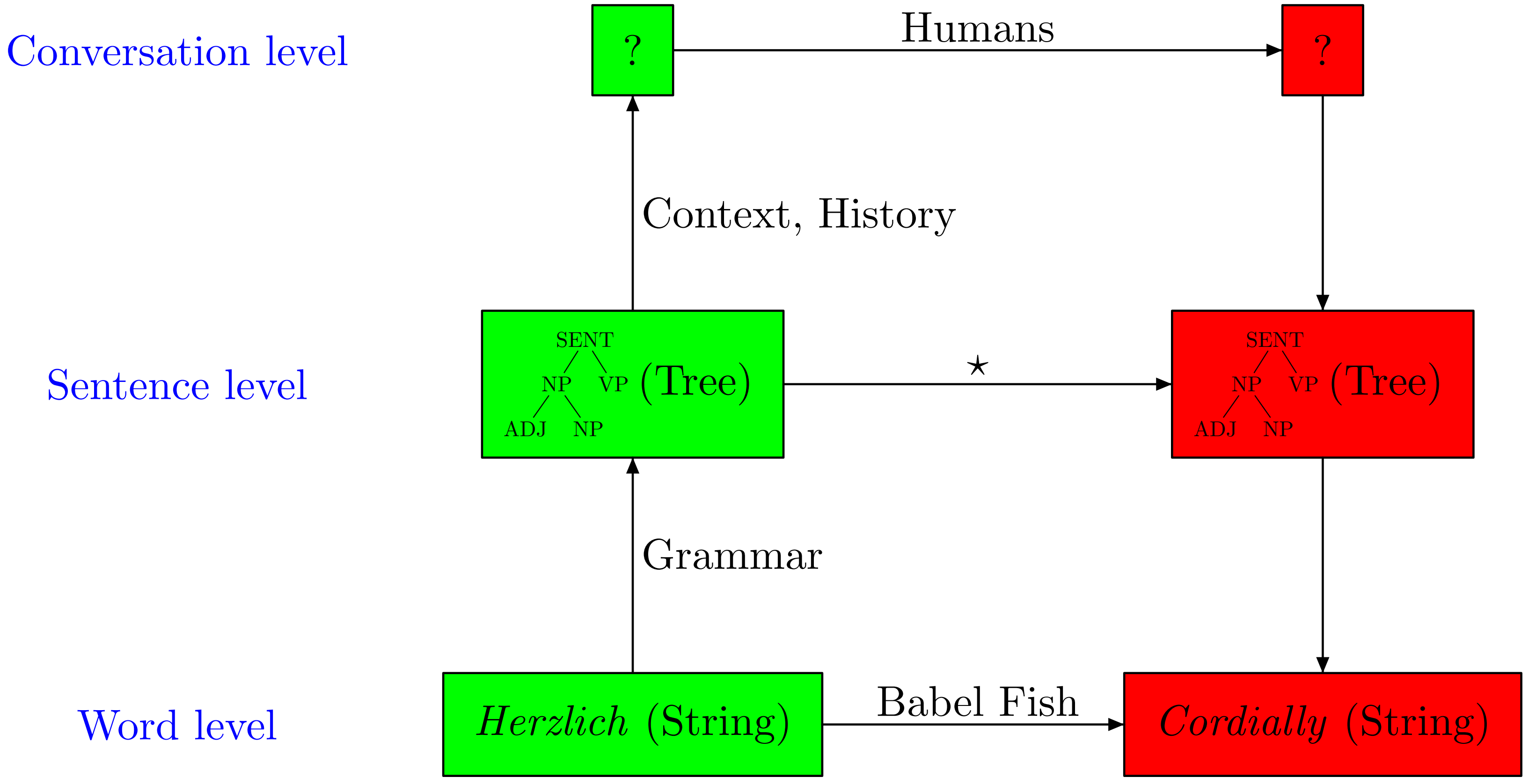
# Motivation

## Babel Fish Translation

| German | English |
|---|---|
| Herzlich willkommen meine sehr geehrten Damen und Herren. Ich möchte mich vorab bei den Organisatoren für die vortrefflich geleistete Arbeit bedanken. | Cordially welcomely my very much honoured ladies and gentlemen. I would like to thank you first the supervisors for the splendid carried out work. |
| Herzlich willkommen meine sehr geehrten Damen und Herren. Ich möchte mich vorab bei den Organisatoren, *die diese Veranstaltung erst ermöglicht haben*, für die vortrefflich geleistete Arbeit bedanken. | Cordially welcomely my very much honoured ladies and gentlemen. I would like *me* first the supervisors, *who made this meeting possible only*, for *whom* splendid carried out work *thank you*. |

# Motivation



Conversation level
?
Humans
?
Context, History
Sentence level
SENT
NP
VP
(Tree)
ADJ
NP
⋆
SENT
NP
VP
(Tree)
ADJ
NP
Grammar
Word level
Herzlich (String)
Babel Fish
Cordially (String)

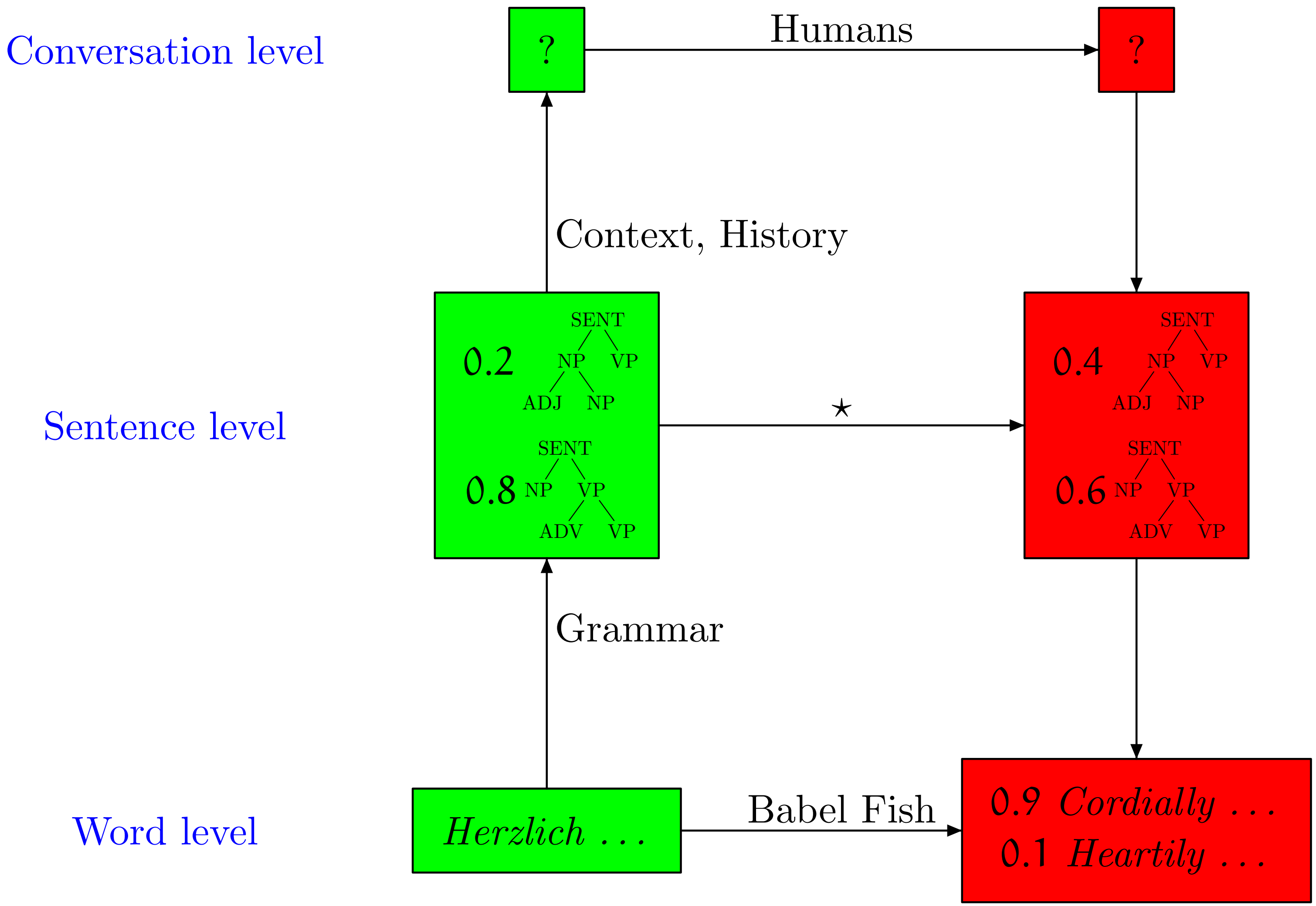
# Motivation

- Automatic translation is widely used (even Microsoft uses it to translate English documentation into German)
- Dictionaries are very powerful word-to-word translators; leave few words untranslated
- Outcome is nevertheless usually unhappy and ungrammatical
- Post-processing necessary

*Major problem:* Ambiguity of natural language

*Common approach:*
- "Soft output" (results equipped with a probability)
- Human choses the correct translation among the more likely ones

# Motivation



Conversation level
Sentence level
Word level
?
Humans
?
Context, History
0.2
SENT
NP VP
ADJ NP
0.8
SENT
NP VP
ADV VP
⋆
0.4
SENT
NP VP
ADJ NP
0.6
SENT
NP VP
ADV VP
Grammar
Herzlich . . .
Babel Fish
0.9 Cordially . . .
0.1 Heartily . . .

# Motivation

Tree series transducers are a straightforward generalization of

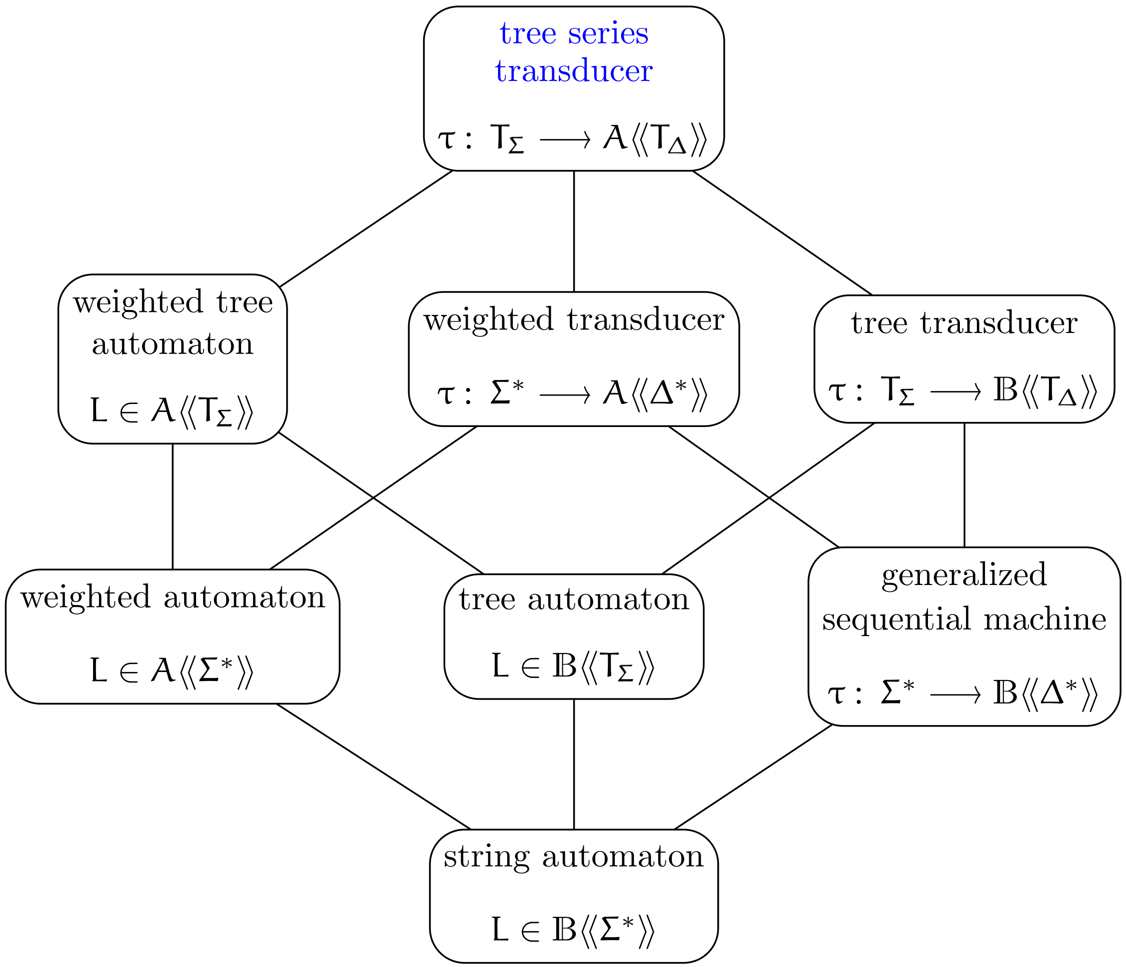
(i) tree transducers, which are applied in

- syntax-directed semantics,
- functional programming, and
- XML querying,

(ii) weighted automata, which are applied in

- (tree) pattern matching,
- image compression and speech-to-text processing.

# Generalization Hierarchy

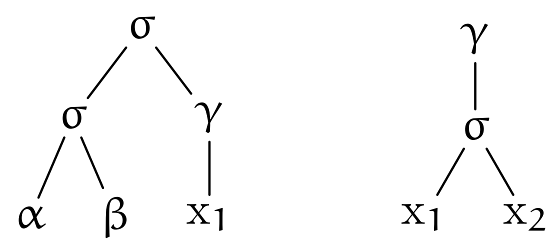


# Trees

$\Sigma$ ranked alphabet, $\Sigma_k \subseteq \Sigma$ symbols of rank $k$, $X = \{ x_i \mid i \in \mathbb{N}_+ \}$

- $T_\Sigma(X)$ set of $\Sigma$-trees indexed by $X$,
- $T_\Sigma = T_\Sigma(\emptyset)$,
- $t \in T_\Sigma(X)$ is *linear* (resp., *nondeleting*) in $Y \subseteq X$, if every $y \in Y$ occurs at most (resp., at least) once in $t$,
- $t[t_1, \ldots, t_k]$ denotes the tree substitution of $t_i$ for $x_i$ in $t$

Examples: $\Sigma = \{\sigma^{(2)}, \gamma^{(1)}, \alpha^{(0)}, \beta^{(0)}\}$ and $Y = \{x_1, x_2\}$




$\sigma\big(\sigma(\alpha, \beta), \gamma(x_1)\big)$ $\gamma\big(\sigma(x_1, x_2)\big)$

# Semirings

A *semiring* is an algebraic structure $\mathcal{A} = (A, \oplus, \odot)$

- $(A, \oplus)$ is a commutative monoid with neutral element $0$,
- $(A, \odot)$ is a monoid with neutral element $1$,
- $0$ is absorbing wrt. $\odot$, and
- $\odot$ distributes over $\oplus$ (from left and right).

Examples:

- semiring of non-negative integers $\mathbb{N}_\infty = (\mathbb{N} \cup \{\infty\}, +, \cdot)$
- Boolean semiring $\mathbb{B} = (\{0, 1\}, \vee, \wedge)$
- tropical semiring $\mathbb{T} = (\mathbb{N} \cup \{\infty\}, \min, +)$
- any ring, field, etc.

# Properties of Semirings

We say that $\mathcal{A}$ is

- *commutative*, if $\odot$ is commutative,
- *idempotent*, if $a \oplus a = a$,
- *complete*, if there is an operation $\bigoplus_I : A^I \longrightarrow A$ such that
  1. $\bigoplus_{i \in \{m,n\}} a_i = a_m \oplus a_n$,
  2. $\bigoplus_{i \in I} a_i = \bigoplus_{j \in J} \big( \bigoplus_{i \in I_j} a_i \big)$, if $I = \bigcup_{j \in J} I_j$ is a (generalized) partition of $I$, and
  3. $\big( \bigoplus_{i \in I} a_i \big) \odot \big( \bigoplus_{j \in J} b_j \big) = \bigoplus_{i \in I, j \in J} (a_i \odot b_j)$.

| Semiring | Commutative | Idempotent | Complete |
|---|---|---|---|
| $\mathbb{N}_\infty$ | YES | no | YES |
| $\mathbb{B}$ | YES | YES | YES |
| $\mathbb{T}$ | YES | YES | YES |

# Tree Series

$\mathcal{A} = (A, \oplus, \odot)$ semiring, $\Sigma$ ranked alphabet

Mappings $\varphi : T_\Sigma(X) \longrightarrow A$ are also called *tree series*

- the set of all tree series is $A\langle\langle T_\Sigma(X)\rangle\rangle$,
- the *coefficient* of $t \in T_\Sigma(X)$ in $\varphi$, i.e., $\varphi(t)$, is denoted by $(\varphi, t)$,
- the *sum* is defined pointwise $(\varphi_1 \oplus \varphi_2, t) = (\varphi_1, t) \oplus (\varphi_2, t)$,
- the *support* of $\varphi$ is $\mathrm{supp}(\varphi) = \{ t \in T_\Sigma(X) \mid (\varphi, t) \neq 0 \}$,
- $\varphi$ is *linear* (resp., *nondeleting* in $Y \subseteq X$), if $\mathrm{supp}(\varphi)$ is a set of trees, which are linear (resp., nondeleting in Y),
- the series $\varphi$ with $\mathrm{supp}(\varphi) = \emptyset$ is denoted by $\widetilde{0}$.

Example: $\varphi = 1\ \alpha + 1\ \beta + 3\ \sigma(\alpha, \alpha) + \ldots + 3\ \sigma(\beta, \beta) + 5\ \sigma(\alpha, \sigma(\alpha, \alpha)) + \ldots$

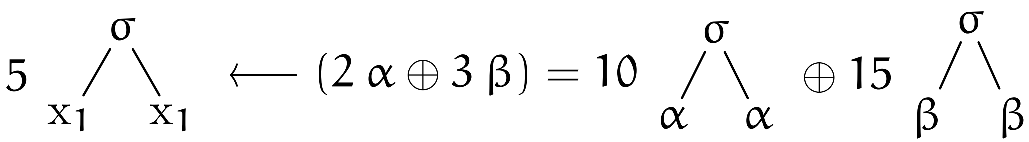
# Tree Series Substitution

$\mathcal{A} = (A, \oplus, \odot)$ complete semiring, $\varphi, \psi_1, \ldots, \psi_k \in A\langle\langle T_\Sigma(X) \rangle\rangle$

*Pure substitution* of $(\psi_1, \ldots, \psi_k)$ into $\varphi$:

$$\varphi \longleftarrow (\psi_1, \ldots, \psi_k) = \bigoplus_{\substack{t \in \mathrm{supp}(\varphi), \\ (\forall i \in [k]):\, t_i \in \mathrm{supp}(\psi_i)}} (\varphi, t) \odot (\psi_1, t_1) \odot \cdots \odot (\psi_k, t_k)\; t[t_1, \ldots, t_k]$$

Example: $5\, \sigma(x_1, x_1) \longleftarrow (2\, \alpha \oplus 3\, \beta) = 10\, \sigma(\alpha, \alpha) \oplus 15\, \sigma(\beta, \beta)$



# Tree Series Transducers

Definition: A *(bottom-up) tree series transducer* (tst) is a system $M = (Q, \Sigma, \Delta, \mathcal{A}, F, \mu)$

- $Q$ is a non-empty set of *states*,
- $\Sigma$ and $\Delta$ are input and output ranked alphabets,
- $\mathcal{A} = (A, \oplus, \odot)$ is a complete semiring,
- $F \in A\langle\langle T_\Delta(X_1)\rangle\rangle^Q$ is a vector of linear and nondeleting tree series, also called *final output*,
- *tree representation* $\mu = (\mu_k)_{k\in\mathbb{N}}$ with $\mu_k : \Sigma_k \longrightarrow A\langle\langle T_\Delta(X_k)\rangle\rangle^{Q\times Q^k}$.

If $Q$ is finite and $\mu_k(\sigma)_{q,\vec{q}}$ is polynomial, then $M$ is called *finite*.

# Semantics of Tree Series Transducers

Mapping $r: \mathrm{pos}(t) \longrightarrow Q$ is a *run* of $M$ on the input tree $t \in T_\Sigma$

$\mathrm{Run}(t)$ set of all runs on $t$

Evaluation mapping: $\mathrm{eval}_r : \mathrm{pos}(t) \longrightarrow A\langle\langle T_\Delta \rangle\rangle$ defined for every $k \in \mathbb{N}$, $\mathrm{lab}_t(p) \in \Sigma_k$ by

$$\mathrm{eval}_r(p) = \mu_k(\mathrm{lab}_t(p))_{r(p),r(p\cdot 1)\dots r(p\cdot k)} \longleftarrow \big(\mathrm{eval}_r(p\cdot 1), \dots, \mathrm{eval}_r(p\cdot k)\big)$$

*Tree-series transformation* induced by $M$ is $\|M\| : A\langle\langle T_\Sigma \rangle\rangle \longrightarrow A\langle\langle T_\Delta \rangle\rangle$ defined

$$\|M\|(\varphi) = \bigoplus_{t \in T_\Sigma} \Big( \bigoplus_{r \in \mathrm{Run}(t)} \mathrm{eval}_r(\varepsilon) \Big)$$

# Semantics — Example

$M = (Q, \Sigma, \Delta, \mathbb{N}_\infty, F, \mu)$ with

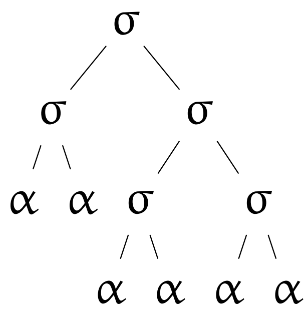
- $Q = \{\perp, \star\}$,
- $\Sigma = \{\sigma^{(2)}, \alpha^{(0)}\}$ and $\Delta = \{\gamma^{(1)}, \alpha^{(0)}\}$,
- $F_\perp = \widetilde{0}$ and $F_\star = 1\ x_1$,
- and tree representation

$$\mu_0(\alpha)_\perp = 1\ \alpha \qquad \mu_0(\alpha)_\star = 1\ \alpha$$

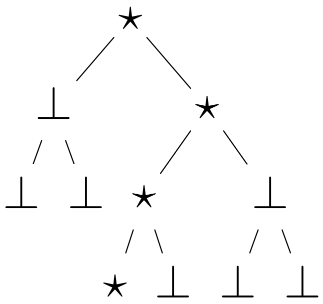
$$\mu_2(\sigma)_{\perp,\perp\perp} = 1\ \alpha \qquad \mu_2(\sigma)_{\star,\star\perp} = 1\ x_1 \qquad \mu_2(\sigma)_{\star,\perp\star} = 1\ x_2$$

# Semantics — Example (cont.)

Input tree $t$



Run $r$ on $t$



$$\|M\|(1\ t) = 2\,\gamma(\alpha) \oplus 4\,\gamma^3(\alpha)$$

# Extension

$(Q, \Sigma, \Delta, \mathcal{A}, F, \mu)$ tree series transducer, $\vec{q} \in Q^k$, $q \in Q$, $\varphi \in A\langle\langle T_\Sigma(X_k)\rangle\rangle$

Definition: We define $h_\mu^{\vec{q}} : T_\Sigma(X_k) \longrightarrow A\langle\langle T_\Delta(X_k)\rangle\rangle^Q$

$$h_\mu^{\vec{q}}(x_i)_q = \begin{cases} 1\, x_i & \text{, if } q = q_i \\ \widetilde{0} & \text{, otherwise} \end{cases}$$

$$h_\mu^{\vec{q}}(\sigma(t_1, \ldots, t_k))_q = \bigoplus_{p_1, \ldots, p_k \in Q} \mu_k(\sigma)_{q, p_1 \ldots p_k} \longleftarrow (h_\mu^{\vec{q}}(t_1)_{p_1}, \ldots, h_\mu^{\vec{q}}(t_k)_{p_k})$$

We define $h_\mu^{\vec{q}} : A\langle\langle T_\Sigma(X_k)\rangle\rangle \longrightarrow A\langle\langle T_\Delta(X_k)\rangle\rangle^Q$ by

$$h_\mu^{\vec{q}}(\varphi)_q = \bigoplus_{t \in T_\Sigma(X_k)} (\varphi, t) \odot h_\mu^{\vec{q}}(t)_q$$

# Composition Construction

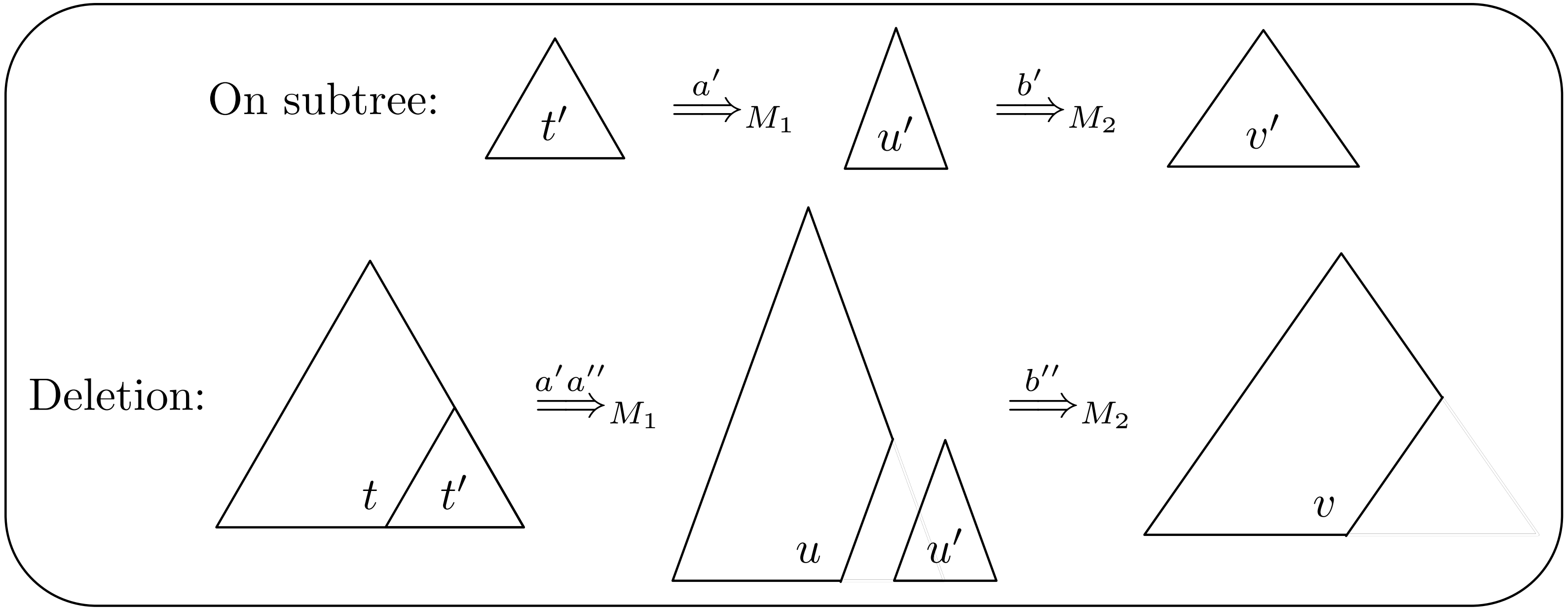
$M_1 = (Q_1, \Sigma, \Delta, \mathcal{A}, F_1, \mu_1)$ and $M_2 = (Q_2, \Delta, \Gamma, \mathcal{A}, F_2, \mu_2)$ tree series transducer

Definition: The *product of* $M_1$ and $M_2$, denoted by $M_1 \cdot M_2$, is the tree series transducer

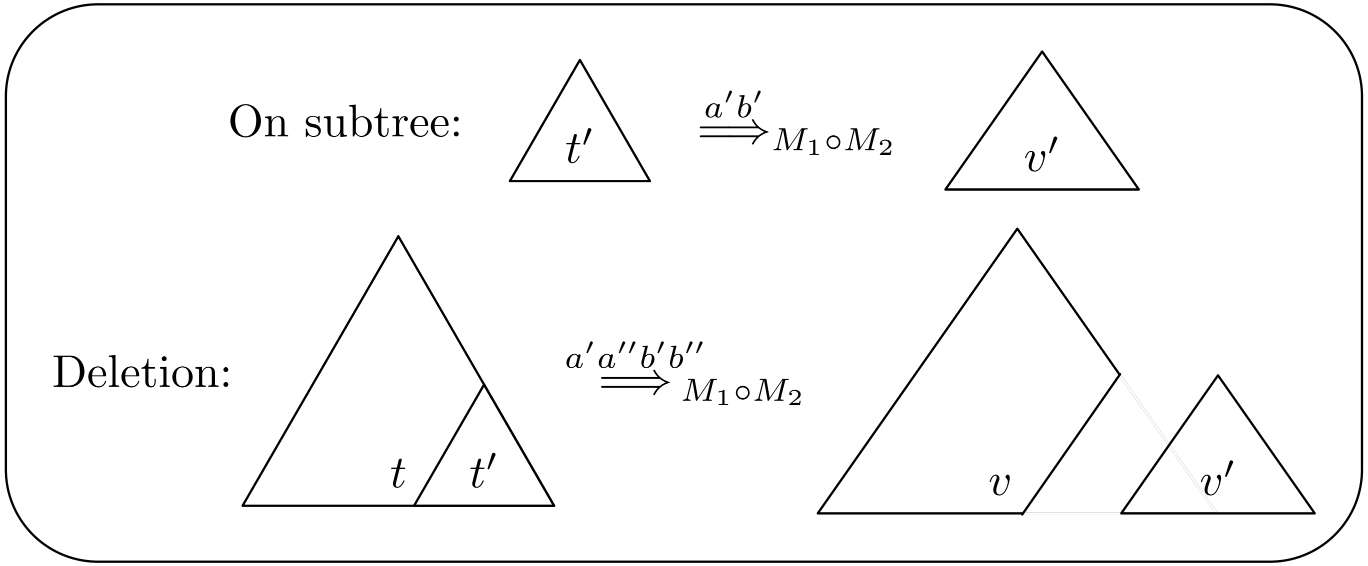
$$M = (Q_1 \times Q_2, \Sigma, \Gamma, \mathcal{A}, F, \mu)$$

- $F_{pq} = \bigoplus_{i \in Q_2} (F_2)_i \longleftarrow h_{\mu_2}^{q}\big((F_1)_p\big)_i$
- $\mu_k(\sigma)_{pq,(p_1 q_1,\ldots,p_k q_k)} = h_{\mu_2}^{q_1 \cdots q_k}\big((\mu_1)_k(\sigma)_{p,p_1 \ldots p_k}\big)_q$.

# Composition



# Composition (cont.)



# Main Theorem

$\mathcal{A}$ commutative and complete semiring

Main Theorem

- $\text{l-BOT}_{\text{ts-ts}}(\mathcal{A}) \circ \text{BOT}_{\text{ts-ts}}(\mathcal{A}) = \text{BOT}_{\text{ts-ts}}(\mathcal{A})$.
- $\text{BOT}_{\text{ts-ts}}(\mathcal{A}) \circ \text{db-BOT}_{\text{ts-ts}}(\mathcal{A}) = \text{BOT}_{\text{ts-ts}}(\mathcal{A})$,
- $\text{BOT}_{\text{ts-ts}}(\mathcal{A}) \circ \text{d-BOT}_{\text{ts-ts}}(\mathcal{A}) = \text{BOT}_{\text{ts-ts}}(\mathcal{A})$, provided that $\mathcal{A}$ is multiplicatively idempotent,

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