Tree-Series-to-Tree-Series Transformations

Andreas Maletti

International Computer Science Institute
Berkeley, CA, USA

maletti@icsi.berkeley.edu

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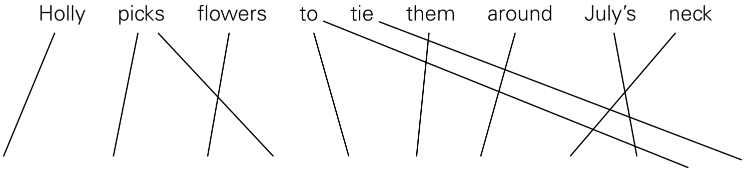
Classic Approach (simplified)



Steps

1. Select scheme
2. Translate words individually
3. Check sequence (bigrams, trigrams, etc.)

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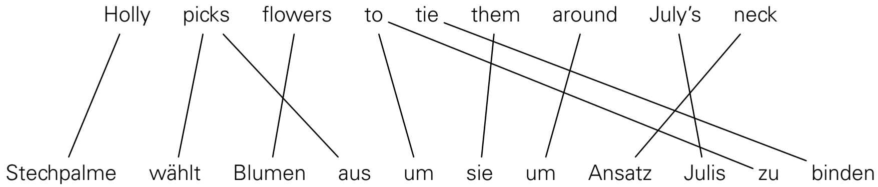

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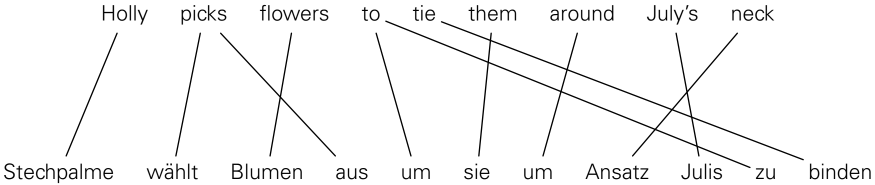

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Syntax-based Approach

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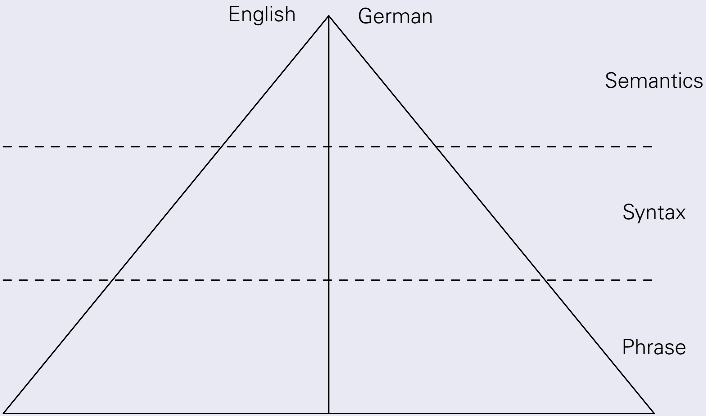

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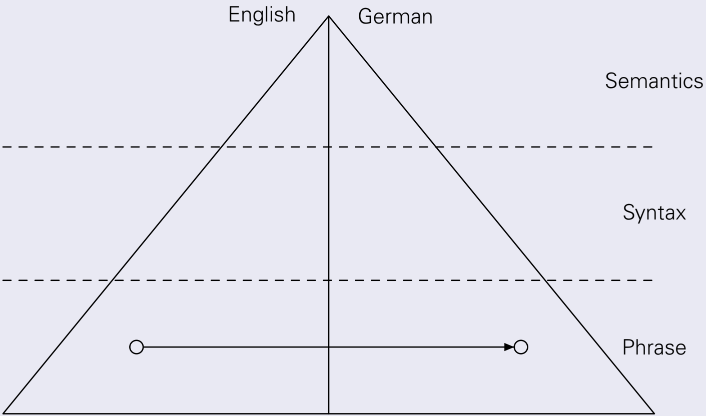

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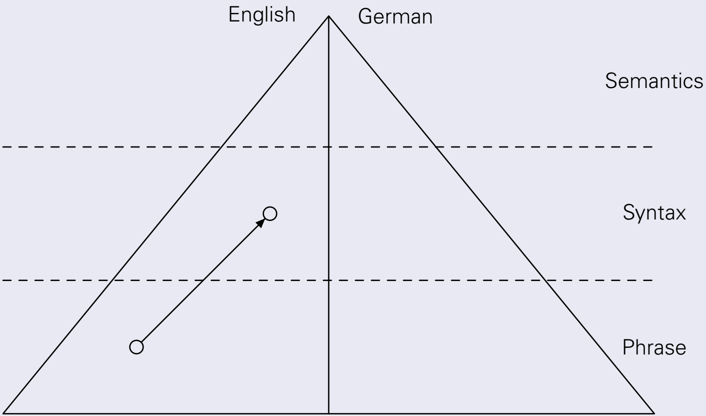

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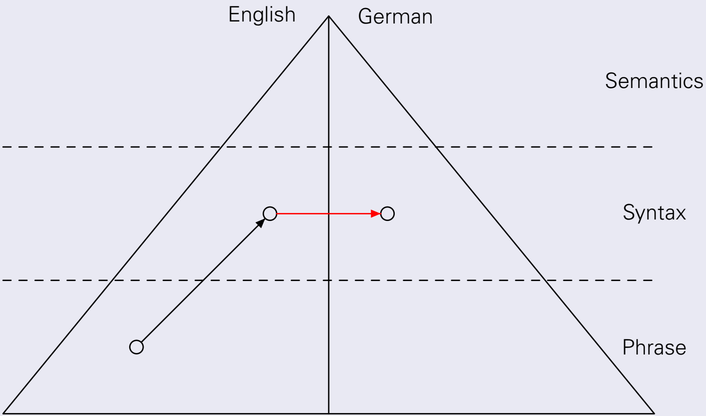

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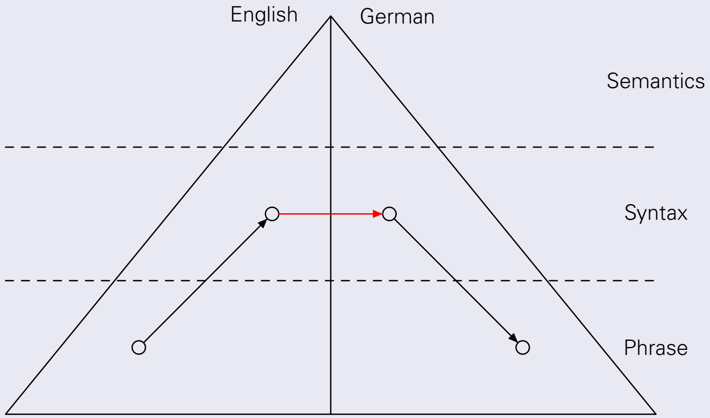
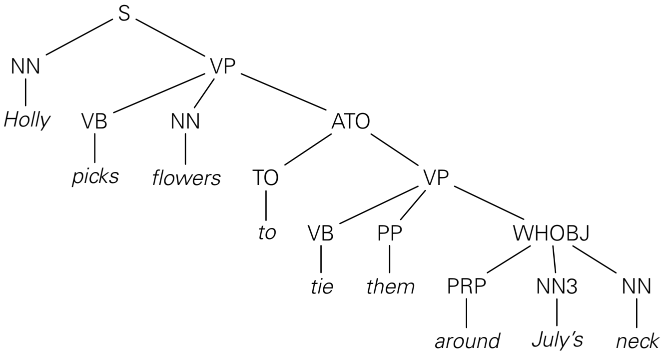


Syntactic Analysis



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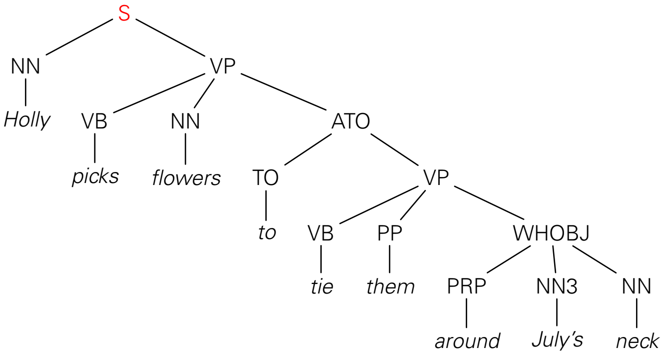

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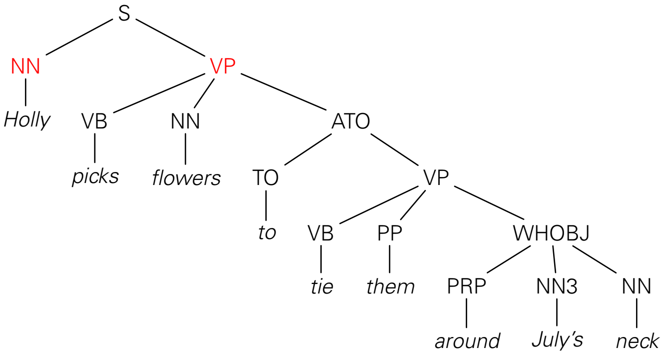

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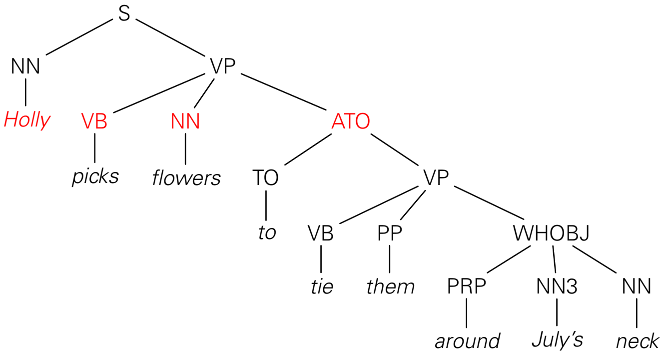

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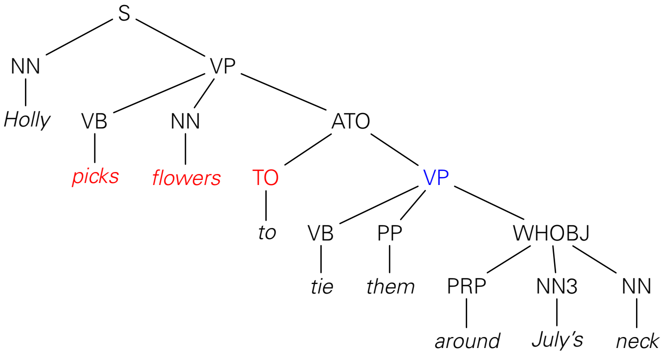

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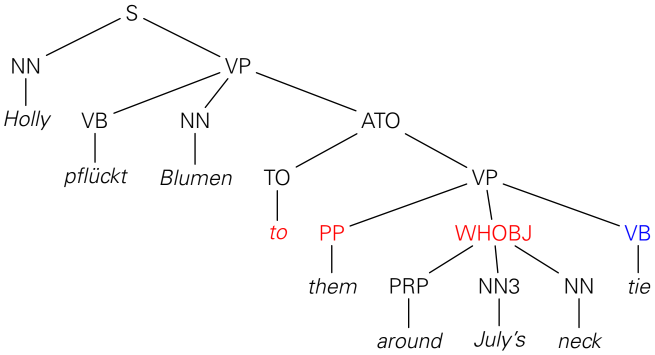

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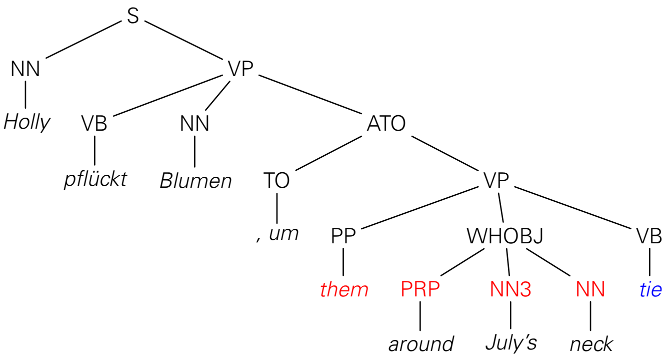

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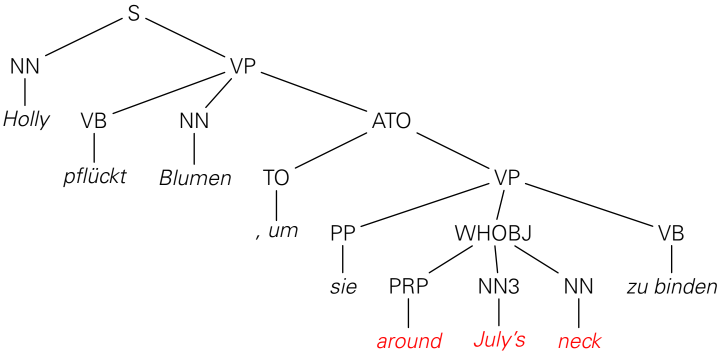

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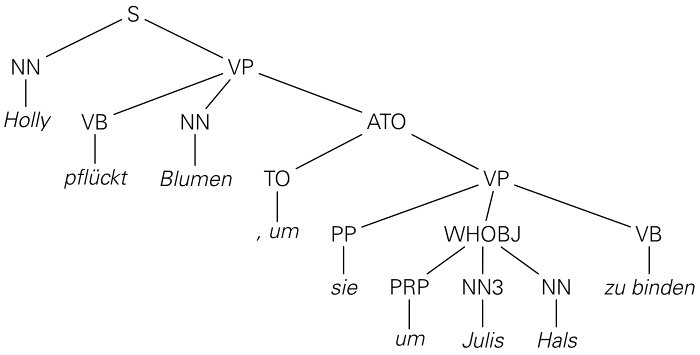

Adding Weights

- Hard decision (yes/no) replaced by soft decision
- Each translation then has a score
- Yields ranking on the alternatives

Problems

- Harder to train (but nowadays there is enough data)
- Destroys nice properties of the tree transducer framework

Addressed here

Well-definedness problem

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Well-definedness problem

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Some Basics

- semiring $\mathcal{A} = (A, +, \cdot, 0, 1)$ is a "ring without subtraction"
- $A\langle\langle T_\Delta(X)\rangle\rangle$: set of all maps $T_\Delta(X) \to A$
- $A\langle T_\Delta(X)\rangle$: 0 almost everywhere maps of $A\langle\langle T_\Delta(X)\rangle\rangle$

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Syntax

Definition

Tree series transducer is tuple $(Q, \Sigma, \Delta, \mathcal{A}, F, \mu)$

- Q finite set of states
- Σ and Δ ranked alphabets of input und output symbols
- $\mathcal{A} = (A, +, \cdot, 0, 1)$ commutative semiring
- $F \subseteq Q$ final states
- $\mu = (\mu_k)_{k \in \mathbb{N}}$ with $\mu_k \colon \Sigma_k \to A\langle\langle T_\Delta(X)\rangle\rangle^{Q \times Q(X_k)^*}$

such that

1. $\mu_k(\sigma)_{q,w} \in A\langle T_\Delta(X_{|w|})\rangle$
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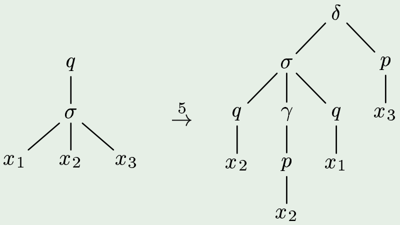
Syntax: Top-down vs. Bottom-up

Definition

- top-down if $\mu_k(\sigma)_{q,w}$ is linear and nondeleting in $X_{|w|}$
- bottom-up if $\mu_k(\sigma)$ is nonzero only at $(q, q_1(x_1) \cdots q_k(x_k))$

Example (Top-down)



$$(\mu_3(\sigma)_{q,q(x_2)p(x_2)q(x_1)p(x_3)}, \delta(\sigma(x_1, \gamma(x_2), x_3), x_4)) = 5$$

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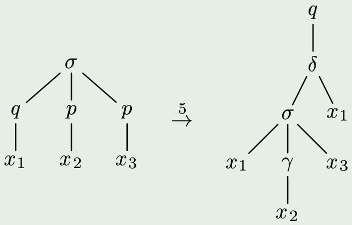
Example (Bottom-up)



$$(\mu_3(\sigma)_{q,q(x_1)p(x_2)p(x_3)}, \delta(\sigma(x_1, \gamma(x_2), x_3), x_1)) = 5$$

Substitution of Tree Series

Definition

$\psi, \psi_1, \ldots, \psi_n \in A\langle\langle T_\Delta(X)\rangle\rangle$

$$\psi \leftarrow (\psi_1, \ldots, \psi_n) = \sum_{t, t_1, \ldots, t_n \in T_\Delta(X)} \left((\psi, t) \cdot \prod_{i=1}^{n} (\psi_i, t_i) \right) t[t_1, \ldots, t_n]$$

Semantics

Definition

Define tree evaluation as:

$$h_\mu(\sigma(t_1, \ldots, t_k))_q = \sum_{w \in Q(X_k)^*} \mu_k(\sigma)_{q,w} \leftarrow (h_\mu(t_{i_1})_{q_1}, \ldots, h_\mu(t_{i_n})_{q_n})$$

where $w = q_1(x_{i_1}) \cdots q_n(x_{i_n})$

Definition

Tree-to-tree-series transformation

$$\tau_M(t) = \sum_{q \in F} h_\mu(t)_q$$

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Tree-series-to-tree-series transformation

$$\tau_M(\psi) = \sum_{t \in T_\Sigma} (\psi, t) \cdot \tau_M(t)$$

Well-definedness

Problem

When is $\sum_{t \in T_\Sigma} (\psi, t) \cdot \tau_M(t)$ well-defined?

Answer

Always in complete semirings.

Rebuke

Most complete semirings are unpractical.

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Our Approach

Answer

It is well-defined if every output tree can be produced only from finitely many input trees

Without weights

It is well-defined if $\tau_M^{-1}(u)$ is finite

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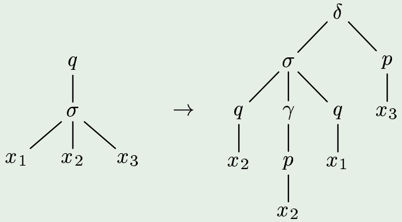
Top-down Case

Example (Nondeleting)

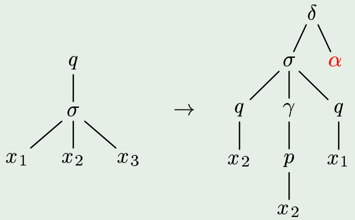


Theorem

If a trim top-down tree transducer is deleting, then $\tau_M^{-1}(u)$ is infinite for some output tree u.

Top-down Case

Example (Deleting)



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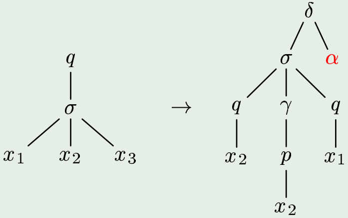

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Top-down Case (cont'd)

Example (Replication)

$$q(\gamma(x_1)) \to q(x_1)$$

Definition

Any state that can be reached from itself without producing output is called replicating.

Top-down Case (cont'd)

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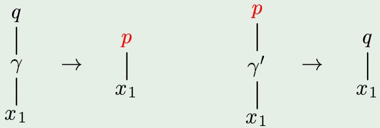

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Top-down Case (cont'd)

Theorem

For a trim top-down tree transducer M, TFAE:

(i) $\tau_M^{-1}(u)$ *is finite for all* $u \in T_\Delta$

(ii) M *is nondeleting and has no replicating states*

Proof.

Using a size argument for the output trees. □

Remarks

- (ii) characterizes well-definedness over positive semirings
- (ii) yields well-definedness of the ts-ts transformation in arbitrary semirings

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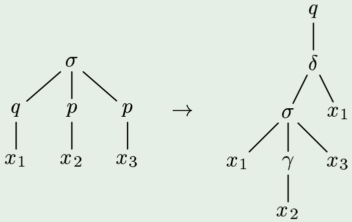

Theorem

If a trim bottom-up tree transducer deletes at a state that can accept infinitely many trees, then $\tau_M^{-1}(u)$ is infinite for some output tree u.

Bottom-up Case

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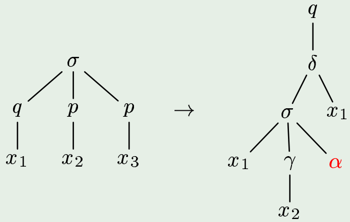

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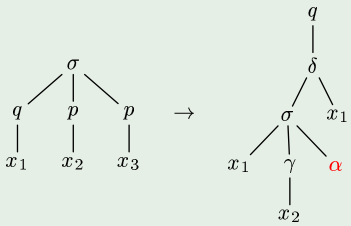

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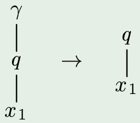

Definition

- Replicating state defined as imagined
- Infinite state accepts infinitely many input trees

Bottom-up Case (con't)

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$$\gamma(q(x_1)) \to p(x_1) \qquad \gamma'(p(x_1)) \to q(x_1)$$

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Bottom-up Case (cont'd)

Theorem

For a trim bottom-up tree transducer M, TFAE:

(i) $\tau_M^{-1}(u)$ *is finite for all* $u \in T_\Delta$

(ii) *M does not delete at an infinite state and has no replicating states*

Proof.

Using again a size argument for the output trees. □

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- Engelfriet, Fülöp, Vogler: Bottom-up and Top-down Tree Series Transformations. *J. Automata, Languages and Combinatorics* 7, 2002
- Gécseg, Steinby: *Tree Automata*. Akadémiai Kiadó, Budapest 1984
- Engelfriet: Bottom-up and top-down tree transformations — a comparison. *Math. Systems Theory* 9(3), 1975

Thank you for your attention!