1 Outline

- This paper is about the relation between the interpretation of attitude reports and their utterance context.

- I argue that the distinction between readings of attitude reports in terms of scope-based de re and de dicto is too coarse grained and should be given up in favor of an analysis of de-re and de-dicto readings of constituents in terms of mental representations of relations of acquaintance with discourse referents supplied by the utterance context, radicalizing ideas that were brought up in Heim (1992)'s explanation of presupposition projection in attitude reports.

2 Fodor's puzzle

2.1 The de re and de dicto reading of attitude reports

The basic phenomenon to which this paper is devoted are the various options of interpretation for the attitude report in (1).

(1) Adrian wants to buy a jacket like Malte’s.

Quinean structural ambiguity of scope relations (Quine, 1956):

(2) a. De re: \((\exists x)(\text{jacket}(x) \& \text{like-Malte’s-jacket}(x) \& \text{wants}(\text{Adrian}, \text{buy}(\text{Adrian},x)))\)

b. De dicto: \(\text{wants}(\text{Adrian}, (\exists x)(\text{jacket}(x) \& \text{like-Malte’s-jacket}(x) \& \text{buy}(\text{Adrian},x)))\)

2.2 Fodor’s further readings of attitude reports

Fodor (1970) observed that in principle there are two more possibilities besides de dicto and de re to combine the wide resp. narrow scope of the existential quantifier involved in a jacket like Malte’s with the transparency resp. opaqueness induced by the scope of wants. (3c) - (3d) depict the original informal notation which (Fodor, 1970, cf. p. 241) used for the presentation of these additional readings of (1).

(3) a. Wide scope transparent (de re)
    \((\exists x)(\text{jacket}(x) \& \text{like-Malte’s-jacket}(x) \& \text{wants}(\text{Adrian}, \text{buy}(\text{Adrian},x)))\)

b. Narrow scope opaque (de dicto)
    \(\text{wants}(\text{Adrian}, (\exists x)(\text{jacket}(x) \& \text{like-Malte’s-jacket}(x) \& \text{buy}(\text{Adrian},x)))\)

c. Narrow scope transparent (hence third reading)
    x is a jacket like Malte’s and Adrian wants \(\exists x (\text{Adrian buy }x)\)

d. Wide scope opaque (subject to debate, see Gendler-Szabo (2010))
    \(\exists (x \text{ Adrian wants } (x \text{ a coat like Malte’s and Adrian wants to buy }x))\)

Even a superficial look at the informal semantics of the additional reading (3c) – on which I focus in this paper – reveals the problem that Fodor’s proposal of further readings of (1) constitutes. (3c) isn’t a well-formed formula of first order intensional predicate logic. In fact, Fodor (1970) showed that under the transitivity of scope relations, not all of the three conditions (4a) - (4c) imposed by the third reading interpretation of (1) can be satisfied by a formula of first-order intensional predicate logic at once (Fodor, 1970, cf. p. 242)).
(4) a. The first conjunct of (1) must be within the scope of the quantifier $\exists$ if its variable is to be co-referential with the object of Adrian buy . . . 

b. The quantifier $\exists$ must be within the scope of the verb wants if it is to express the narrow scope reading.

c. The first conjunct of (1) must be outside the scope of wants if it is to express the reading which is transparent for descriptive content.

The motivation for Fodor’s third reading is the following context:

(5) Adrian has decided what kind of coat to buy but has no idea that the kind of coat he wants is just like Malte’s coat. (Fodor, 1970, cf. p. 229)

There is a natural interpretation of (1) in context (5) that neither the de re (3a) nor the de dicto (3b) reading render correctly. The de re reading (3a) is wrong for this reading because the quantifier ($\exists x$) entails that there is some particular jacket of which it is true that Adrian wants to buy it. And the de dicto reading (3b) is wrong because it represents the description like Malte’s jacket as part of the content of Adrian’s desire, which in the given scenario it is not.

This is Fodor’s puzzle:

- (1) is true in context (5) while both the de re and the de dicto reading are false
- We can not express the meaning of (1) in context (5) in standard first order intensional predicate logic

2.3 Fodor’s puzzle in the literature

- How does the literature deal with Fodor’s puzzle?

First, consider how Fodor’s puzzle is motivated in the literature: (6a)-(6c) are paradigmatic examples for contexts that have been suggested in the literature to induce a third reading of (1).

(6) a. A store sells some coats that all look like Malte’s and Adrian does not know anything about Malte. Assume further that Adrian wants one of those coats and any of them is an option. (Romoli and Sudo, 2009, cf. p. 427)

b. Adrian’s desire is to buy some jacket or other, and the only important thing is that it be a Burberry jacket. Unbeknownst to him, Malte’s jacket is one of those as well. (von Fintel and Heim, 2011, cf. p. 100)

c. Malte and Adrian do not know each other. Adrian has seen a green Burberry jacket in a catalogue and wants to buy one. Malte happens to own precisely such a green Burberry jacket. (Schwager, 2009, cf. p. 395)

At first glance, it seems as if there is no big difference between the contexts (6a)-(6c).

(7) a. ad. 6a: $\exists ! X : \text{coats} - \text{like} - \text{malte’s}(X)$ and Adrian wants to buy one of $X$

b. ad. 6b: Adrian wants$_{w_0} [\lambda w’[a - \text{jacket} - \text{like} - \text{malte’s}_{w_0}]x_1[\text{PRO to buy}_{w_0}x_1]]$

c. ad. 6c: For the sake of reporting an attitude, a property that is involved in the content of the attitude that is to be reported (the reported property) can be replaced by a different property (the reporting property) as long as the reported property is a subset of the reporting property at all relevant worlds.

(7a) says that the jacket Adrian wants to buy exists and that the interpretation of like Malte’s jacket is established by the fact that Adrian’s choice for one of the jackets from the set of jackets like Malte’s doesn’t matter, they are all like Malte’s and any jacket that is an option for him is de facto a jacket like Malte’s. Things are different for (7b). The jacket that Adrian wants to buy exists only in his desire worlds whereas a jacket like Malte’s exists in the real world $w_0$ in which Adrian makes his buy. Thus, Adrian wants to buy a jacket like a jacket like Malte’s. Consequently, the interpretation of like Malte’s jacket depends on Adrian’s choice in that those jackets Adrian singles out in his desire worlds are drawn upon for the judgement of the report to be true. This
may sound like a subletie but it is not. The truth-conditions in (7a) predict that a report of Adrian’s attitude in context (6b) with (1) is false, because there exists no set of jackets Adrian wants to buy one of which. Next, compare the truth-conditions (7b) with the truth-conditions in (7c). Kaufmann’s point is that a third reading is not about existing jackets at all and thus, unlike in the von Fintel and Heim (2011) context there exists no jacket like Malte’s jacket in the real world \( w_0 \). Consequently, in context (6c), the truth-conditions (7b) predict that a report of Adrian’s attitude with (1) is false. In context (6c) there is no jacket like Malte’s in \( w_0 \) such that Adrian wants to buy a jacket like this jacket in all of his desire worlds.

2.4 Relations of acquaintance

- How can we approach the close relation between context and interpretation of (1)?

(Heim, 1992, p. 210) argued to break down the scope-based de dicto/de re distinction to a more fine grained analysis at the constituent level and proposed that “there is not really just one de re reading (for a given constituent), but there are many - one for each acquaintance relation that the context might supply. . . . In a way, I am blurring the distinction between de re and de dicto readings. But that may not be such a bad thing. (Heim, 1992, footnote 53): It may also make it easier to reconcile the two-way de re/de dicto ambiguity of the standard theory with finer classifications such as the four-way distinctions in (Fodor, 1970, p. 229).”

Applied to Fodor’s puzzle, Heim’s idea amounts to a dependency between context and interpretation of (1) which manifests in the type and amount of acquaintance relations that determine the type and amount of constituents of the semantic representation of (1) supplied by the utterance context. A systematic analysis of Fodor’s puzzle thus has to spell out the various options of how the relations of acquaintance with the constituents of (1) can be realized under the general constraints of Fodor’s third reading.

The first systematic distinction is between relations of acquaintance provided to the reporter of Adrian’s attitude and to Adrian himself, because in third readings, “obviously the speaker must be the source of the description”(Fodor, 1970, p. 227) of the type of jacket that Adrian wants to buy as being like Malte’s jacket. Making precise the role that relations of acquaintance play in the interpretation of (1) is one goal of this paper.

But: once we break down the de dicto/de re/third reading ambiguity of (1) into a context-dependent determination of constituents of semantic representations, no longer is the de dicto/de re/third reading ambiguity a structural ambiguity for which semantic representations can be generated independent of context. The immediate consequence is that context determines the semantic representation of an attitude report insofar it determines its constituents. Any serious account of Fodor’s puzzle thus has to answer the question for how to represent this role of context. Spelling out these consequences that reach beyond Fodor’s puzzle is another goal of this paper.

3 Relations of acquaintance in Discourse Representation Theory

- How should we represent relations of acquaintance?

Let me begin with the first goal of this paper, making explicit the interaction between relations of acquaintance provided by context and the semantic representation of (1). To this end, we need a theory that is able to deal with relations of acquaintance in a principled and formal manner. Right from the start, Discourse Representation Theory (DRT, Kamp et al. (2011) has been used for this purpose, e.g. in Kamp (1984). For the representation of propositional attitudes and relations of acquaintance with discourse referents, a three-place predicate \( Att \) is introduced into the core language of DRT. For the formalization of causal contact with objects and its effect on the status of discourse referents - being directly referential - DRT provides the concept of an external anchor. External anchors represent the acquaintance with an existing object in the real world. In order to distinguish this wide content notion of de re acquaintance with objects from the perceived notion of de re based on quantifier scope relationships (as in (3a)), I call external anchors de-re anchors as opposed to de re scope relationships.

In its basic form, an external anchor for a discourse referent \( x \) in an entity \( b \) fixes the reference of \( x \) to the model-theoretic entity \( b \in Discourse − Universe \) and is represented as in (8). In the course of this paper, I introduce additional more complex types of external anchors, e.g. anchors for sets of individuals and properties and anchors for properties of sets of individuals.
From the viewpoint of semantic representation, an external anchor displays a non-representational relation between a discourse referent and an entity. That is, an external anchor for a discourse referent is not accessible to the agent who entertains a semantic representation in which the so-anchored discourse referent occurs. Consequently, external anchors are not a component of the representation of the mental content which an agent takes as her psychological reality but they are placed outside the scope of the agent’s mental representations. In turn, for an agent to entertain a semantic representation of the relation to the entity $b$ in which she takes herself to stand in a relation of direct reference (via the external anchor of a discourse referent $x$), $x$ must have associated certain conditions with it at the level of semantic representation. These conditions state the relation of acquaintance via which $x$ is non-representationally anchored in $b$ and is called an internal anchor for $x$. An internal anchor takes the form pictured in (9), where $K$ is a Discourse Representation Structure (DRS) of the acquaintance with $x$. In the DRS representations I present in this paper, I leave open the exact specification of the acquaintance representation $K$, as it doesn’t matter to my arguments.

(9)  $\langle \{\text{ANCH}, x\}, K \rangle$

External anchors enter a DRS representing the attitudinal state of an agent as the third argument of the predicate $Att$. The first argument of $Att$ represents the bearer of the attitude that $Att$ is used to describe and the second argument is for descriptions of the attitudinal state that the $Att$-predicate assigns to the bearer. The descriptions occupying the second argument slot of $Att$ consist of pairs $\langle MOD, K \rangle$, where MOD is an attitudinal mode indicator (whether the attitude represented by the pair $\langle MOD, K \rangle$ is e.g. a belief, desire or intention) and $K$ is a representation of the content of the attitude. Unanchored discourse referents occurring in attitude descriptions $K$ are evaluated with respect to a non-specific relation of acquaintance. In parallel to external anchors, I use the term de-dicto for the relation of acquaintance that unanchored discourse referents in attitude descriptions represent (narrow content) and de dicto for a quantifier scope relationship of the type presented in (3b).

4 Representing Adrian’s attitude

- What is the range of possible attitude ascriptions to Adrian that support a third reading?

(1) is based on the ascription of an attitude to Adrian by the reporter compatible with the constraints of Fodor’s puzzle. The ascription of the attitude is determined by the contextually provided relations of acquaintance in which Adrian stands to the object of his desire. In turn, the configuration of Adrian’s desire in a certain situation determines the possibilities for the speaker to report Adrian’s attitude with (1). In the following, I discuss the range of semantic representations of (1) in two steps. First, I discuss the range of possible attitude ascriptions to Adrian and second, I discuss how the attitude ascription to Adrian serves as the starting point of a report with (1).

4.1 Externally anchored object of desire

One end of the spectrum of ascriptions of attitudes to Adrian which the reporter can take as a starting point for a report with a third reading of (1) is based on situations in which Adrian has a de-re relation of acquaintance with the object of his desire. There are at least two variants of Adrian’s attitude under the constraints of the third reading which support such a de-re interpretation of the discourse referent which stands for Adrian’s object of desire.

(10) a. Adrian has seen two jackets. Adrian wants to buy one of them but he has not decided which. This is one version of the situation (6a) that (Romoli and Sudo, 2009) provide. The speaker represents Adrian’s attitude as involving a de-re attitude towards each of the jackets he has seen which together form the set of jackets towards which his desire of buying a jacket is directed.
b. Adrian has seen too many jackets to keep track of them individually. Adrian wants to buy one of them but he has not decided which. This is another version of (Romoli and Sudo, 2009)’s situation (6a). The speaker represents Adrian’s attitude as a de-re attitude towards the externally anchored set of jackets $J$ without requiring Adrian to stand in separate and distinct relations to each of the jackets in $J$. The star $\ast$ turns a predicate of individuals into a predicate of sets of individuals. The DRS in (10b) involves an extension of the concept of external anchoring to anchors in collections of one or more objects. It is not easy to state in general terms what must be the case in order that someone can be said to have such a representation. In many cases the agent must associate some delineating description - such as the jackets on the display in this window, as well as the kind of contact with one or more elements of the set that could also have given rise to anchored representations for those elements on their own. Somewhat simplified, anchors for collections are a compact representation of a conjunction of anchors for each of the members of the collection. Thus, an external anchor of a plural discourse referent (represented in upper case) in a collection codes the expectation that in principle, the plural anchor is reducible to the conjunction of external anchors for each of the members of the set of individuals represented by the plural discourse referent.

4.2 Externally anchored trigger of desire

Another type of attitude ascription to Adrian is induced by situations in which the formation of his desire is based on a causal relation of acquaintance with a jacket like Malte’s but not with the object of his desire. That is, in situations of Adrian of the type proposed by (von Fintel and Heim, 2011) – see (6b) – there are jackets of the type that Adrian wants to buy in the real world, but the jacket that Adrian wants to buy exists only in his desire worlds.

(11) a. Adrian has seen a jacket and wants to buy a jacket which is of the same kind as the jacket he has seen. This is the speaker’s representation of Adrian’s attitude that the context (6b) of (von Fintel and Heim, 2011) suggests.
4.3 Externally anchored properties

Finally, Adrian can be ascribed an attitude in situations which do not provide causal relations of acquaintance with discourse referents at all but where Adrian has only a causal relation of acquaintance with properties (of jackets) that are crucial to his desire. There are several variations of this type of situation, depending on how the causal relation of acquaintance with a property relates to Adrian’s choice of jackets.

(12) a. Adrian is looking through a mail order catalogue and sees a picture of a jacket of a certain brand and make. The speaker represents Adrian’s desire to buy a jacket of the same brand and make (represented as the second order property $DSN$ of being a design property $P$). This is the kind of situation that (Schwager, 2009) suggests for being the cause of Adrian’s attitude.

b. Adrian has formed, in whatever way, the desire to buy a jacket with properties $P_1, \ldots, P_n$. This situation probably comes close to what (Fodor, 1970) originally had in mind.

5 Reporting Adrian’s attitude

- How does the reporter of Adrian’s attitude reach a semantic representation that captures the intuitions behind Fodor’s puzzle?
The range of possible attitude ascription to Adrian which we discussed in the last section are only half the story which we need to tell. In their present form, the DRSs in figures 10a-12b do not support a verbalization with (1). What we have not considered yet is that (1) verbalizes more information than the information which is contained in the ascription of an attitude to Adrian. If Adrian doesn’t know that the jacket he wants to buy is like Malte’s jacket (even if it actually is), someone else needs to do so, as Fodor already remarked: “in this case, obviously the speaker must be the source of the description.” (Fodor, 1970, p. 227). Thus, in order to report Adrian’s attitude with (1), the reporter must relate her representation of the attitude ascribed to Adrian with her representation of how she considers the object of Adrian’s desire to be like Malte’s jacket.

Fodor’s puzzle is so well engineered because the link between Adrian’s attitude and the reporter’s contribution manifests linguistically in the predication of likeness. There is a wide range of situations in which (1) can be used because like can mean any number of things. What is it for one jacket \( j \) to be like some other jacket \( j_1 \)? In what way must they resemble each other – what properties must they share – in order that \( j \) be like \( j_1 \)? The flexibility of the interpretation of like influences the options for the reporter to set up a semantic representation that captures Adrian’s desire as (1). In the following, I will try to give a systematic account of the options of interpretation of like that have been proposed in the literature on Fodor’s puzzle.

5.1 De-facto likeness

If Adrian has a de-re attitude towards certain jackets as in (10a)-(10b), there is a straightforward way of relating Adrian’s desire to Malte’s jacket such that (1) is true. In the cases where Adrian’s object of desire is externally anchored – (13a) and (13b), Adrian’s attitude does not involve any condition of likeness, but the reporter is responsible for the selection of those features relevant to the like-condition with respect to Malte’s jacket and the set of jackets that she perceives as the set of options for Adrian. One intriguing feature of de-facto interpretations of likeness is that the reporter’s justification for her claim of likeness may involve completely different properties of jackets than those properties that Adrian has actually selected as being relevant to his buy. Any jacket that Adrian is going to buy will be like Malte’s and thus the claim ‘like Malte’s jacket’ is a de-facto claim. The common ground in which the difference in the selection of relevant properties converges in the claim of like Malte’s jacket is the set of existing jackets which both Adrian and the reporter perceive and which make up the options for Adrian’s buy.

\[
\begin{align*}
\text{(13) a.} & \quad s_0, s_1, a, w \\
& \quad n \subseteq s_1 \\
& \quad s_0 < s_1 \\
& \quad \text{Adrian(a)} \\
& \quad \text{Reporter(w)} \\
& \quad s_0 : \text{Att} \ a, \\
& \quad \left\{ \begin{array}{c}
\left[ \text{ANCH}, j_1, K \right], \\
\left[ \text{ANCH}, j_2, K \right]
\end{array} \right\} \\
& \quad \left\{ \begin{array}{c}
\text{BEL,} \\
\text{jacket(j1)} \\
\text{jacket(j2)}
\end{array} \right\} \\
& \quad \left\{ \begin{array}{c}
\text{DES,} \\
\text{e : buy(i, j1) \lor e : buy(i, j2)}
\end{array} \right\} \\
& \quad \left\{ \begin{array}{c}
\left[ \text{ANCH}, j_3, K \right], \\
\left[ \text{ANCH}, j_4, K \right]
\end{array} \right\} \\
& \quad \left\{ \begin{array}{c}
\text{BEL,} \\
\text{jacket(j3)} \\
\text{jacket(j4)} \\
\text{maltes \rightarrow jacket(j5)} \\
\text{like(j3, j5)} \\
\text{like(j4, j5)}
\end{array} \right\} \\
& \quad \left\{ \begin{array}{c}
(j_1, u), \\
(j_2, r)
\end{array} \right\}
\end{align*}
\]
5.2 Inferential likeness

The first variant of a context in which the trigger of Adrian’s desire is externally anchored but the object of his desire is only internally anchored is the situation represented by DRS (14). Here, Adrian has seen Malte’s jacket and thus the likeness of the type of jacket Adrian wants to buy can be inferred from the empirical features of Adrian’s de-re attitude towards Malte’s jacket. All that is necessary for the reporter is to acquire an anchored belief that predicates the external anchor of the discourse referent for the jacket that Adrian has seen as Malte’s jacket and to infer that Adrian wants to buy such a jacket. In this case, regardless of what Adrian considers as the relevant features of the jacket he wants to buy, all of them are also properties of Malte’s jacket. Consequently, the condition which introduces like is located inside Adrian’s attitude, the predication of the jacket Adrian has seen as Malte’s jacket inside the reporter’s attitude.

The situation (6b) proposed by von Fintel and Heim (2011) differs from other variants of de-re acquaintances with jackets in that it is not Malte’s jacket which Adrian has seen but some jacket like Malte’s. This makes a difference insofar as the reporter has to relate Adrian’s desired jacket to the source of his desire in a way which involves the review of Adrian’s attitudinal state. On the basis of the existing jacket which triggers Adrian’s desire, the reporter infers that the jacket Adrian wants to buy is like Malte’s jacket because Malte’s jacket is like the jacket which triggered Adrian’s desire. The attitude that Adrian has in this scenario can be paraphrased as in (15).
(15) Adrian wants to buy a jacket like a jacket like Malte’s.

The DRS in (16) for the von Fintel and Heim (2011) scenario contains two like-conditions, one inside Adrian’s attitude and one inside the reporter’s attitude. What this actually amounts to is a more complicated kind of linking the properties of Malte’s jacket to the trigger of Adrian’s desire to the type of jacket Adrian wants to buy than in the de-facto interpretation of \textit{like Malte’s jacket}, while still supporting a report of the situation with (1). Note that Adrian’s attitude representation is the same in both types of inferential likeness but differs in content when judged by an outside observer like the reporter.

(16)

\[ s_0, s_1, a, w \]
\[ n \subseteq s_1 \]
\[ s_0 < s_1 \]
\[ s_{\text{Adrian}}(a) \]
\[ s_{\text{Reporter}}(w) \]

\[ s_0 : \text{Att} \]
\[ a, e \]
\[ \langle \text{ANCH}, j_1 \rangle, K \]
\[ \langle \text{BEL}, j_2 \rangle \]
\[ \text{jacket}(j_1) \]
\[ \text{like}(j_2, j_1) \]
\[ n < e \]
\[ \{ (j_1, u) \} \]
\[ \langle \text{DES}, e : \text{buy}(i, j_2) \rangle \]
\[ n < e \]

\[ s_1 : \text{Att} \]
\[ w, \{ \langle \text{ANCH}, j_3 \rangle, K \} \]
\[ \langle \text{BEL}, j_4 \rangle \]
\[ \text{jacket}(j_3) \]
\[ \text{maltes} – \text{jacket}(j_4) \]
\[ \text{like}(j_3, j_4) \]
\[ \{ (j_3, u) \} \]

5.2.1 Metaphysical likeness

Kaufmann (Schwager, 2009, p. 397, numbering adopted) disagrees with “the assumption that sentences like (1) are about actual jackets”. In her context, the likeness of the jacket Adrian wants to buy and Malte’s jacket can not be judged via some de-re jacket. Consequently, Kaufmann proposes to base the analysis of \textit{like} on the implication of properties according to a principle of property replacement which is given in (17). The attitudinal connection between the reporter and Adrian is then set up via a de-re acquaintance with properties by both Adrian and the reporter.

(17) “For the sake of reporting an attitude, a property that is involved in the content of the attitude that is to be reported (the reported property) can be replaced by a different property (the reporting property) as long as the reported property is a subset of the reporting property at all relevant worlds.” (Schwager, 2009, p. 409)

Kaufmann’s approach to likeness does not pertain to the comparison of existing jackets with existing jackets with features of desired jackets (as in the inferential analysis) but to the relation between properties of jackets Adrian wants to buy and properties that Malte’s jacket has in the actual world, i.e. a relation between certain second-order properties. Instead of a de-re object, it is the relation of de-re properties which link the reporter’s conceptualization of Malte’s jacket with Adrian’s desired jacket through the means of set-theoretic inclusion of the reported property in the reporting property (\textit{like Malte’s}) at all non-empty relevant worlds including the actual world, which de facto leads to an entailment relation between the reported property and the reporting property. The semantics which Kaufmann associates with this idea is given in (18), where \( Q \) is the reporting property (\textit{like Malte’s jacket}) and \( Q' \) is the reported property.
(18) \( \text{Attitude}_w(x, \langle P, Q \rangle) \) (where \( P \) a structured proposition and \( Q \) a property), iff there is a property \( Q' \) s.t. at the \( w \)-closest worlds \( w' \) where \( Q(w') \neq \emptyset \):

- \( Q'(w') \neq \emptyset \)
- \( Q'(w') \subseteq Q(w') \)
- \( \text{Attitude}_w(x, \lambda w' P_w'(Q')) \) is true.

DRSs (19a) and (19b) give the DRSs for two versions of Kaufmann’s context (6c).

(19)

\[
\begin{align*}
\text{s}_0, s_1, w, a \\
\text{n} \subseteq s_1 \\
\text{s}_0 < s_1 \\
\text{Adrian(a)} \\
\text{Reporter(w)}
\end{align*}
\]

\[
\text{s}_0 : \text{Att} \quad \text{a}, \quad \langle \text{ANCH}, P_1 \rangle, K \\
\text{BEL,} \\
\text{j}_1 \\
P_1(j_1) \\
\text{jacket}(j_1) \\
\text{DSN}(P_1) \\
\text{DES,} \\
e \\
n < e \\
e : \text{buy}(i, j_1) \\
\{P_1, Q_1\}
\]

\[
\text{s}_1 : \text{Att} \quad \text{w}, \quad \langle \text{ANCH}, P_2 \rangle, K \\
\text{BEL,} \\
\text{j}_2, P_3 \\
maltes - jacket(j_2) \\
P_3(j_2) \\
P_2(\subseteq P_3) \\
\text{DSN}(P_3) \\
\{P_2, Q_2\}
\]

\[
\text{s}_0, s_1, w, a \\
\text{n} \subseteq s_1 \\
\text{s}_0 < s_1 \\
\text{Adrian(a)} \\
\text{Reporter(w)}
\]

\[
\text{s}_0 : \text{Att} \quad \text{a}, \quad \langle \text{ANCH}, P_1 \rangle, , .., \langle \text{ANCH}, P_n \rangle, \}
\text{BEL,} \\
\text{j}_1 \\
P_1(j_1) \lor .. \lor P_n(j_1) \\
\text{jacket}(j_1) \\
\text{DES,} \\
e \\
n < e \\
e : \text{buy}(i, j_1) \\
\{P_1, Z_1\}, \ldots, \{P_n, Z_n\}
\]

\[
\text{s}_1 : \text{Att} \quad \text{w}, \quad \langle \text{ANCH}, Q_1 \rangle, , .., \langle \text{ANCH}, Q_n \rangle, \}
\text{BEL,} \\
\text{j}_2, P_3 \\
maltes - jacket(j_2) \\
Q_1(j_2) \lor .. \lor Q_n(j_2) \\
\text{DES,} \\
e \\
n < e \\
e : \text{buy}(i, j_1) \\
\{Q_1, Z_1\}, \ldots, \{Q_n, Z_n\}
\]
It may be argued that pictures in a catalogue suggest the existence of the jackets which are pictured - actually this is what a catalogue is about - but the following example (20) by (Schwager, 2009, p. 400) makes explicit the possibility of the denial of the existence of the entity which is the object of Adrian’s desire in the third reading in a report similar to (1).

(20) Mary is looking at the Burj Dubai, which has 191 floors and is currently the highest building in the world. Also, no other building has more floors. Mary doesn’t know this. She also doesn’t know how many floors Burj Dubai has. Mary’s self-reported attitude is “Wow, I want to buy a building that’s even one floor higher!” The following is a faithful report of the situation: “Mary wants to buy a building with (at least) 192 floors.”

It is obvious that in the Burj Dubai case (20), we can not rely on a de-re or hybrid analysis of likeness involving the comparison of objects but have to take into account properties in the way that Kaufmann proposes. Informally, the attitude that Adrian has in Kaufmann’s context is the one in (21)

(21) Adrian wants to buy a jacket which has properties that are properties of Malte’s jacket in a relevant manner, too.

5.3 Intentional likeness

Up to now, we focused on the constitution of constituents of the semantic representation of (1) via the spectrum of possible relations of acquaintance provided by perception. But we were not concerned with attitudes and their specific properties themselves. Consider scenario (22).

(22) Adrian has seen a jacket which has three stripes on its sleeves and wants to buy such a jacket. However, he read that Adidas supports child labour in the production of its jackets, so the additional condition for his buy is that the jacket is not from Adidas. If Adrian does not know that Adidas is the brand with the three stripes, he has a desire that he would paraphrase as “I want to buy a jacket from the brand with the three stripes but not from Adidas.” Fritz hears Adrian’s utterance and as he has seen Malte’s jacket which has three stripes and as he also knows about the problem with child labour and Adidas he believes that Malte would never buy a jacket which is made by children. Fritz also doesn’t know that Adidas is the brand with the three stripes. He reports Adrian’s desire as “Adrian wants to buy a jacket like Malte’s.”

Because Adidas is the brand with three stripes in the actual world, in the formal semantics we can not make use of Kaufmann’s replacement principle. The property of being a jacket from the brand with the three stripes but not from Adidas does not exist (in the extensional expression 'there exists a property of being a jacket from the brand with the three stripes but not from Adidas', if we substitute 'brand with the three stripes’ for 'Adidas’ we run into a contradiction, which in turn would entail any reported property) but only in Adrian’s desire worlds and (probably coincidently) in Fritz’ belief worlds. But Fritz can truthfully report Adrian’s attitude with (1), where all of the approaches to the third reading discussed in this paper would predict that the report is false. That is, in (22) there is no de-re property $Q'$ under which Adrian buys his jacket involved in the third reading.

The problem can be nicely illustrated with the parallel that Kaufmann draws between her semantics of the third reading and counterfactuals. Consider Kaufmann’s counterfactual example for the Burj-Dubai context.

(23) If there was a building with 192 floors, that building would be one floor higher than the Burj Dubai currently is.

But while (23) is true in the Burj-Dubai context, a similar paraphrase does not work for the Adidas case.

(24) If there was a jacket like Malte’s, that jacket would be from the brand with the three stripes but not from Adidas as Malte’s jacket is in the real world.

If we can’t apply Kaufmann’s principle of property substitution to (22), how can we set up a connection between Fritz’ belief that Malte’s jacket is like the type of jacket Adrian wants to buy and the type of jacket that Adrian wants to buy? (22) obviously involves the sharing of mental contents across agents and this sharing rests
upon a connection which isn’t mediated through externally anchored objects or properties, but it concerns the intentionality of attitudes themselves. The situation seems to be somehow similar to the problem of intentional identity which is usually illustrated with (Geach, 1967, p. 627)’s Hob-Nob example (25).

(25) Hob thinks a witch has blighted Rob’s mare and Nob wonders whether she (the same witch) killed Cob’s sow.

As different as the Hob-Nob situation may be, Geach’s characterization of intentional identity is what we find in the Adidas case (22).

“[W]e have intentional identity when a number of people, or one person on different occasions, have attitudes with a common focus, whether or not there actually is something at that focus” (Geach, 1967, p. 627).

An informal semantics of the third reading in context (22) takes the form of (26).

(26) Adrian wants to buy a jacket of which the reporter believes that it (the same jacket) will be like Malte’s.

Intuitively, what we want to say about the way in which the discourse referent that the reporter uses to predicate it as being like Malte’s jacket and the discourse referent which represents the object of Adrian’s desire is the following: whatever the value is that Adrian’s attitude assigns to his discourse referent for the jacket he wants to buy, it will be picked up by the reporter. Such cases of shared reference are reminiscent of what is called “vicarious” anchoring in DRT (see e.g. (Kamp and Benders-Farkas, 2006)). The version of vicarious anchoring I adopt in this paper assigns intentional anchors the following form, where v is a discourse referent and yz a discourse referent stemming from an attitude which is entertained by an agent z.

(27) ⟨v, yz⟩

With the concept of intentional anchoring, (28) gives a representation of (22).

(28)

6 Context and Logical Form

- If context determines the relation of acquaintance with discourse referents and if in turn the relations of acquaintance with discourse referents determine the constituents of the semantic representation of an attitude report, in which sense does the semantics of (1) relate to the principle of compositionality that the semantic representation of a sentence is determined by its constituents?
This question of course generalizes to de dicto/de re, as was the main observation of Heim (1992). The straightforward de re (2a) and de dicto (2b) readings of attitude reports behave similar to the third reading with respect to the role of contextually induced variation of semantic representations that can not be captured by scope relationships. If Adrian knows Malte’s jacket, all situations we discussed support a de dicto reading of (1), as long as Adrian has not decided to buy a specific jacket. Similar considerations hold for the de re case. If Adrian has decided which jacket he is going to buy, (but doesn’t know that it is like Malte’s), there is also a range of possibilities by which he can be acquainted with the jacket he wants to buy and how the reporter of his attitude relates Adrian’s desire to Malte’s jacket.

- Is there an interpretation of attitude reports out of the blue, i.e. without any context?

For Hob-Nob Sentences, (van Rooy and Zimmermann, 1996, p. 134) suggest that “the literal reading of any cross-attitude anaphor is de re, and all other readings only become available if there is reason to rule out this literal reading. Inspection of [...] examples [...] shows that they only seem to work fine when accompanied by a longer text setting up the background that eliminates all unwelcome reading.” The out-of-the-blue interpretation of attitude reports thus essentially relies on the idea that interpreters automatically assume a default, “natural” or “normal” (Heim (1992)) situation based on which they determine the out-of-the-blue interpretation of the attitude report. Thus, we can’t escape context in determining the truth-conditions of attitude reports.

- How is it that context constrains the interpretation possibilities of attitude reports?

Obviously, context decreases the set of possibilities of representing the semantics of the attitude report to be interpreted. This function of context as decreasing options of interpretation very much reminds me of the function that context plays in dynamic semantics, where discourse context does a similar job. But while we have a clear understanding of what it e.g. means for an anaphora to depend on the constraints expressed by its antecedent, what does it mean for the semantic representation of an attitude report to depend on constraints expressed by its context?

Two types of constraints are relevant to this question. The first type of constraints pertains to the construction of semantic representations of attitude reports from a pair of sentence and context: how does previous discourse (the determination of relations of acquaintance) constrain the options for constructing a semantic representation of an attitude report?

The second type of constraints pertains to the interpretation of semantic representations of attitude reports. While cross-agent attitudinal dependencies can be captured in a straightforward manner on the level of semantic representation (e.g. via intentional or vicarious anchors), the model-theoretic interpretation of such representations poses a problem: what does it ‘mean’ for the attitude of an agent x to depend on another attitude entertained by agent y?

To me it seems as if both types of constraints make up the essential ingredients of a dynamic theory of attitude reports which is required to deal with the semantics of attitude reports such as (1) in a satisfactory manner. It is when advancing attitudinal semantics beyond the static cases of Quinean ambiguity to a dynamic theory of the semantics of attitudes and their reports that we should seriously think about how much of the structure and semantics of attitudes we want to import into the structure and semantics of their reports. Our considerations have actually carried us beyond the horizons of the third reading. In the light of what has been said, it seems doubtful to me whether talk about the third reading, the de dicto or the de re interpretation of attitude reports is really all that helpful. The discussion suggests that it is not so much a third reading that is involved in the examples that Fodor first brought up, but rather that these examples show that we need a different semantics for attitude reports generally. This new semantics applies not only to the cases that Fodor recognized as problematic for the semantic methods and frameworks that were available at the time when she wrote – and that appears to be a situation that seems to have changed but little since that time – but also to those cases that we knew, or thought we knew, what to do with then, including most saliently the classical de re cases such as that where Adrian has seen a particular jacket and decides that that is the jacket he wants to buy. If we decide to take into account the structure of attitude reports below the coarse-grained tools of scope relationships, then semantic representationalism is the appropriate instrument to carve out the subelements involved in attitude ascriptions and their reports. This is much in line with the conclusion of Kora (2011) (even if I do not agree with the way in which he reached this conclusion): “[T]he correct theory of descriptions requires a representationalist theory of interpretation. On such a theory, particular interpretations of sentences correspond to components of
(mental) representations of the discourse in which they occur. The intuitive truth conditions of interpretations of utterances in discourse correspond to the truth conditions of the (mental) representations that the interpreter builds of the discourse.” If my thoughts are on the right track, then the implications of Fodor’s observations may in the end be even more dramatic than seems implied by recent treatments of the cases she has brought to our attention.

7 Syntax and semantics of the extended DRS language

This section presents the formal syntax and semantics of the DRT-language which was employed for the representation of Fodor’s third reading in this paper. Although the semantics for attitude DRSs is already available in print (Kamp et al., 2011), I decided to repeat the specifications given there in order to make this paper self-contained and because the problems involved in the definition of vicarious anchors can only be made precise against the background of Kamp’s proposal. However, the reader interested in a detailed discussion of attitude semantics in the framework of DRT and DRT in general is referred to (Kamp et al., 2011).

7.1 A compromise between intensionality and intentionality

The model-theoretic semantics that (Kamp, 2003) and (Kamp et al., 2011) offer for the extension of DRT with attitudes and anchors assigns intensions to the DRSs \( K \) that occur in the expressions filling the second argument slot of \( \text{Att} \). But such an intensional semantics is not optimal, in that it does away with some of the potential of this approach towards the structure of mental states and the meaning of mental state descriptions in natural language to escape the problems of logical omniscience. Descriptions of attitudinal states that are not just formally different, but are also meant to be different in a cognitively relevant sense – an agent with an attitudinal state answering to the one description can be expected to reason and behave differently from an agent with a state answering to the other description – will collapse under this kind of intensional interpretation because cognitively distinct content representations \( K_1 \) and \( K_2 \) are intensionally equivalent and thus their semantic values coincide.

Because of this an intensional model-theory for the \( \text{Att} \)-extension of DRT is a compromise: it captures some of the important inferential properties of complex attitudinal states, and therefore also of some aspects of the cognitive dynamics of such states, but at the same time the coarseness of its granularity conceals many of the finer points of such a dynamics. This pertains in particular to the modelling of referential dependency among attitudinal states of two or more agents. What I am going to present in section (7.4.7) is no more than a work-around solution to this problem in the hope that future research will come up with proposals that do better.

7.2 The DRS Language \( \mathcal{L}_{\text{prop}} \)

7.2.1 Vocabulary

**Definition 1** Categories of symbols included in the vocabulary

- a set \( \text{Ref} \) of discourse referents
- a set \( \text{Rel} \) of predicates
- a set \( \text{Name} \) : of proper names

**Definition 2** The vocabulary for the DRS Language \( \mathcal{L}_{\text{prop}} \)

- Sorts of Discourse Referents: The set \( \text{Ref} \) is the union of the following three mutually disjoint sets of discourse referents
  - \( \text{Ind} = \{x_1, \ldots, x_n, \ldots\} \), a set of referents for individuals
  - \( \text{Plu} = \{Q_1, \ldots, Q_n, \ldots\} \), a set of referents for sets of individuals
  - \( \text{Prop} = \{X_1, \ldots, X_n, \ldots\} \), a set of referents for predicates (i.e. properties)
  - \( \text{Event} = \{e_1, \ldots, e_n, \ldots\} \), a set of referents for events
  - \( \text{State} = \{s_1, \ldots, s_n, \ldots\} \), a set of referents for states

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Relation Symbols: The set Rel is the union of the following sets of relation symbols:

- Pred₁: a set of n-place predicates of individuals
- Pred₂: a set of 1-place predicates of predicates
- Event: a set of (n + 1)-place predicates (with n ≥ 0) where the first argument is of type event and the remaining arguments are of type individual
- State: a set of (n + 1)-place predicates (with n ≥ 0) where the first argument is of type state and the remaining arguments are of type individual
- 2-place predicate symbols denoting temporal relations between events and states: <, ⊆ and 2-place predicate symbols denoting set-theoretic relations: ⊆, ∈

- The set of logical symbols Sym: {=, ¬, ∧, ⇒}
- The indexical discourse referents i and n
- The predicate Att
- A set Name: of 1-place relation constants

7.2.2 Syntax of DRSs and DRS conditions

Definition 3 Syntax of DRSs and DRS conditions of \( L_{\text{prop}} \)

- If \( U \subseteq \text{Ref} \) and Con a (possibly empty) set of conditions, then \( \langle U, \text{Con} \rangle \) is a DRS.
- If \( x_i, x_j \in \text{Ref} \) then \( x_i = x_j \) is a condition.
- If \( N \in \text{Name} \) and \( x \in \text{Ind} \) then \( N(x) \) is a condition.
- If \( P \) is a n-place predicate constant in \( \text{Pred}_1 \) and \( x_1, \ldots, x_n \in \text{Ind} \), then \( P(x_1, \ldots, x_n) \) is a condition.
- If \( P \) is a 1-place predicate constant in \( \text{Pred}_1 \) and \( Q \in \text{Plu} \), then \( P*(X) \) is a condition.
- If \( X \in \text{Pred}_2 \) and \( Q \in \text{Prop} \), then \( X(Q) \) is a condition.
- If \( e \in \text{Event}, x_1, \ldots, x_n \in \text{Ind} \) and \( R \in \text{Pred}_1 \) an (n + 1)-place event predicate, then \( e : R(x_1, \ldots, x_n) \) is a condition.
- If \( s \in \text{State}, x_1, \ldots, x_n \in \text{Ind} \) and \( R \in \text{Pred}_1 \) an (n + 1)-place state predicate, then \( s : R(x_1, \ldots, x_n) \) is a condition.
- If \( \tau, \delta \in \{\text{Event} \cup \text{State}\} \), \( R \) one of the predicates \( \subseteq, < \) then \( \tau R \delta \) is a condition.
- If \( K \) is a DRS then \( \neg K \) is a condition.
- If \( x_i \in \text{Ind} \) and \( X_j \in \text{Plu} \) then \( x_i \in X_j \) is a condition.
- If \( K_1 \) and \( K_2 \) are DRSs, then \( K_1 \lor K_2 \) is a condition.
- If \( K_1 \) and \( K_2 \) are DRSs, then \( K_1 \Rightarrow K_2 \) is a condition.
- If \( P_1, P_2 \in \text{Prop} \) then \( P_1 \subseteq P_2 \) is a condition.
- If \( Q \in \text{Plu} \), \( x \in \text{Ind} \) then \( x \in Q \) is a condition.
- If \( x_i, x_j, z \in \text{Ref} \) then \( \langle x_i, x_j, z \rangle \) is a vicarious anchor.
- An Attitude Description Set (ADS) of \( L_{\text{prop}} \) is a set of pairs each of which has one of the following two forms:

\[1\] I do not elaborate on the semantic interpretation of \( i \) and \( n \) in the following, but refer the interested reader to (Kamp et al., 2011).
\[ \langle \text{MOD}, K \rangle \text{ where MOD} \in \{\text{BEL}, \text{DES}\} \text{ and } K \text{ is a DRS of } \mathcal{L}_{\text{prop}}. \]

\[ \langle [\text{ANCH}, \Upsilon], K \rangle \text{ where } \Upsilon \text{ is a discourse referent and } K \text{ is a DRS of } \mathcal{L}_{\text{prop}} \text{ sth. } \Upsilon \in U_K. \]

- If \( K \) is an ADS, then \( IA(K) \) is the set of internal anchors of \( K \), i.e. those members of \( K \) whose first component is of the form \([\text{ANCH}, \Upsilon]\).

- If \( K \) is an ADS, then an external anchor for \( K \) is a function \( f \) sth. \( \text{Dom}(f) \subseteq IA(K) \), i.e.
  
  \[ \{x : \text{for some DRS } K, ([\text{ANCH}, x], K) \in K\} \text{ or } \{X : \text{for some DRS } K, ([\text{ANCH}, X], K) \in K\} \text{ resp.} \]

- If \( s \) is a state discourse referent, \( x \) a discourse referent for individuals, \( K \) an ADS and \( EA \) a set of external anchors for \( K \), then \( s : \text{Att}(x, K, EA) \) is a DRS condition.

### 7.2.3 Free discourse referents and properness

Next, we define the set of free discourse referents of a DRS \( K \), \( FV(K) \) and the notion of a proper a DRS \( K \).

**Definition 4** \( FV(K) \), the set of free discourse referents of \( K \) is defined by:

- \( FV([U_K, Con_K]) := (\bigcup_{\gamma \in Con_K} FV(\gamma)) - U_K \)
- \( FV(x_i = x_j) := [x_i, x_j] \)
- \( FV(P(x_1, \ldots, x_n)) := [x_1, \ldots, x_n] \)
- \( FV(P(X)) := [X] \)
- \( FV(\neg K) := FV(K) \)
- \( FV((K_1 \lor K_2)) := FV(K_1) \cup FV(K_2) \)
- \( FV(K_1 \Rightarrow K_2) := FV(K_1) \cup (FV(K_2) - U_{K_1}) \)

A DRS \( K \) is proper iff \( FV(K) = \emptyset \).

### 7.2.4 Accessibility

**Definition 5** \( K_1 \) is an immediate sub-DRS of \( K \), \( K_1 < K \), if any of the following conditions holds:

- \( \neg K_1 \in Con_K \)
- There is a DRS \( K_2 \) sth. \( K_1 \Rightarrow K_2 \in Con_K \) or \( K_2 \Rightarrow K_1 \in Con_K \)
- There is a DRS \( K_2 \) sth. \( K_1 \lor K_2 \in Con_K \) or \( K_2 \lor K_1 \in Con_K \)

**Definition 6** Given DRSs \( K \) and \( K_1 \), \( K \) is accessible from \( K_1 \), in symbols \( K \text{ acc } K_1 \), iff

- \( K_1 \leq K; \) or
- there exist DRSs \( K_2 \) and \( K_3 \) sth. \( K_2 \Rightarrow K_3 \) and \( K \text{ acc } K_2 \) and \( K_3 \text{ acc } K_1 \).

Given DRSs \( K, K_1 \) and discourse referents \( x \) and \( y \), \( x \) is accessible from \( y \), in symbols \( x \text{ acc } y \) iff \( x \in U_K, y \in U_{K_1} \) and \( K_3 \text{ acc } K_1 \).

### 7.3 Semantics for DRSs

The semantic scaffolding for the interpretation of \( \mathcal{L}_{\text{prop}} \) is an intensional model theory. The central definition of this section is that of a context change potential CCP of a DRS \( K \) relative to a model \( \mathcal{M} \). This section discusses only the standard part of DRT’s model theoretic semantics, the evaluation of ADSs is considered in the next section.
7.3.1 Intensional Models

**Definition 7** An intensional model $\mathcal{M}$ for the DRS language specified in definition 3 is a tuple $(W, U, \mathcal{I}, EV, P)$, where

- $W, U$ is a set of possible worlds
- $U$ is a non-empty set of individuals
- $P$ is a non-empty set of properties
- $\mathcal{I}$ is an interpretation function
  - for names, $\mathcal{I}_M: Name \mapsto \{\{d\} | d \in U_M\}$
  - for n-ary relations, $\mathcal{I}_M: Rel^n \mapsto (W_M \rightarrow \mathcal{P}(U^n))$
  - for predicates of first order $\mathcal{I}: Pred_1^n \mapsto \mathcal{P}(U^n M)$
  - for predicates of second order $\mathcal{I}: Pred_2 \rightarrow \mathcal{P}(P_M)$
- $EV$ is an eventuality structure (see (Kamp and Reyle, 1993, chapter 4))

**Definition 8** An embedding $g : (Ind \cup Plu \cup Prop) \rightarrow (U \cup \mathcal{P}(U) \cup P)$ is defined as an overriding union $g \oplus g' \oplus g''$ that relates any element of the domain of $g''$ to its image under $g''$, any element of the domain of $g'$ to its image under $g'$ and any other element of the domain of $g$ to its image under $g$, where

- $g : Ind \mapsto U$
- $g' : Plu \mapsto \mathcal{P}(U)^2$
- $g'' : Prop \mapsto P$

7.3.2 DRS verification

The core of the dynamic notion of truth involved in the semantics of DRSs is that of a verifying embedding.

**Definition 9** Verifying embeddings for DRSs and DRS conditions of $\mathcal{L}$:

- $(g, h) \models_{\mathcal{M}_w} (U, Con)$ iff $g \subseteq U$ and for all $\gamma \in Con : h \models_{\mathcal{M}_w} \gamma$
- $g \models_{\mathcal{M}_w} x_i = x_j$ iff $g(x_i) = g(x_j)$
- $g \models_{\mathcal{M}_w} N(x)$ iff $\mathcal{I}(N) = g(x)$
- $g \models_{\mathcal{M}_w} P(x_1, \ldots, x_n)$ iff $(g(x_1), \ldots, g(x_n)) \in \mathcal{I}(P)$
- $g \models_{\mathcal{M}_w} X(Q) \iff g(Q) \in \mathcal{I}(X)$
- $g \models_{\mathcal{M}_w} K$ iff there does not exist an $h$ s.t. $(g, h) \models_{\mathcal{M}_w} K$
- $g \models_{\mathcal{M}_w} K_1 \lor K_2$ iff there is some $h$ s.t. $(g, h) \models_{\mathcal{M}_w} K_1$ or there is some $h$ s.t. $(g, h) \models_{\mathcal{M}_w} K_2$
- $g \models_{\mathcal{M}_w} K_1 \Rightarrow K_2$ iff for all $m$ such that $(g, m) \models_{\mathcal{M}_w} K_1$ there exists $k$ s.t. $(m, k) \models_{\mathcal{M}_w} K_2$
- $g \models_{\mathcal{M}_w} e : R(x_1, \ldots, x_n)$ iff $(g(e), g(x_1), \ldots, g(x_n)) \in \mathcal{I}(R(w))$
- $g \models_{\mathcal{M}_w} s : R(x_1, \ldots, x_n)$ iff $(g(s), g(x_1), \ldots, g(x_n)) \in \mathcal{I}(R(w))$
- $g \models_{\mathcal{M}_w} P(x)$ iff for all $u \in g(X)$, $g(x/u) \models_{\mathcal{M}_w} P(x)$
- $g \models_{\mathcal{M}_w} x \in X$ iff $g(x) \subseteq g(X)$
- $g \models_{\mathcal{M}_w} P_1 \subseteq P_2$ iff $g(P_1) \subseteq g(P_2)$

\footnote{For a more adequate treatment of the plural, see (Kamp and Reyle, 1993, chapter 4)}
7.3.3 Propositions and Information States

**Definition 10** Given a proper DRS $K$, the proposition $[K]_M$ expressed by $K$ relative to $M$ is defined as:

- $[K]_M := \{(w, f) | (\Delta, f) \models_{M, w} K\}$

**Definition 11** Given an intensional model $M$, a DRS $K$ and a set of discourse referents $X$ we define

- $I$ is an information state relative to $M$ and $X$ iff $I \subseteq \{(w, f) | \text{Dom}(f) = X \land \text{Ran}(f) \subseteq U_M, w \in W_M\}$
- $I$ is an information state relative to $M$ and $X$ when there is an $X$ such that $I$ is an information state relative to $M$ and $X$
- the empty information state $\Delta_I := \{(w, \emptyset) | w \in W_M\}$
- the proposition $\text{Proposition}(I)$ determined by $I$: $\text{Proposition}(I) := \{w | \exists f (\langle w, f \rangle \in I)\}$

7.3.4 Context Change Potentials

**Definition 12** The context change potential $[K]_M^d$ of a DRS $K$ relative to a model $M$ is defined as a partial function from information states to information states sth.

- $[K]_M^d$ is defined for those information states $I$ relative to $M$ sth. $\text{FV}(K) \subseteq X_I$
- if $I_i \in \text{Dom}([K]_M^d)$, then $[K]_M^d(I_i) = \{(w, g) | \exists f (\langle w, f \rangle \in I_i \land \langle f, g \rangle \models_{M, w} K)\}$

**Definition 13** Let $M$ be an intensional model and $S$ a set of information states relative to $M$. The consistent merge of the $I \in S$, denoted $\cup S$, is the information state defined by:

- $\cup S := \{(w, h) | \text{there exists a function } F \text{ sth. } \text{Dom}(F) = S, \text{ for all } I \in S, \langle w, F(I) \rangle \in I \text{ and } h = \cup F(I | I \in S)\}$ is a function.

7.4 Semantics of anchored attitude DRSs

We now turn to the main point of the semantics of $\mathcal{L}_{\text{prop}}$, the definition of a semantics for anchored attitude DRSs. The challenge for such a semantics is to assign ADSs the right type of intensional constructs which can be used for their evaluation. Those intensional constructs are what we call “Information-State-Based-Attitudinal-State-Descriptions” (ISBAS). They are designed to resolve the problem that not all DRSs which are part of an ADS are proper, but may referentially depend on other DRSs which are part of the same ADS. ISBASs deal with this problem by defining information states for improper DRSs on the basis of a merge of the information states defined by the proper components of an ADS. Two assumptions are necessary as a basis, the well-foundedness of the recursion through referentially dependent DRSs and that the merge of DRSs of an ADS contains no free variables.

7.4.1 Well-foundedness of ADSs

A basic assumption that underlies the commerce with referential dependencies of some components of a mental state on others is that we deal only with ADSs which satisfy the following well-foundedness constraint.

**Definition 14** Well-foundedness

The transitive closure of $\prec$ of the relation $\prec$ between the DRS components $K_1$ and $K_2$ of a DRS $K$ is well-founded: $K_1 \prec K_2$ iff there is a discourse referent $x$ which occurs free in $K_2$ and belongs to the universe of $K_1$. 

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7.4.2 Proper-over-all ADSs

In addition, we restrict attention to ADSs $K$ which are “proper over all”.

**Definition 15** Given two ADSs $K_1$ and $K_2$, $K$ is a ‘proper over all’ ADS iff

- $FV(K) \subseteq \cup \{U_{K'} | (\exists MOD')(MOD', K') \in K \land K' \prec_K K\}$

7.4.3 Relating Attitudes and Information States

The definition of the intensional constructs for the evaluation of ADSs proceeds in two steps. First, we define the notion of a “Potential Information State Based Attitudinal State Description” (PISBAS) and then narrow this concept down to that of an ISBAS. ISBAS are those objects that we use for the definition of the semantics of ADSs.

**Definition 16** Let $M$ be a model and let $J, J_1, J_2, J'$ be CCPs:

- A Potential Information State Based Attitudinal State Description (PISBAS) relative to $M$ is any set of pairs $(MOD, J)$ with MOD a mode indicator an $J$ a regular CCP relative to $M$.

- Let $J$ be a PISBAS relative to $M$. Let $\prec_J$ be the transitive closure of the relation $\prec$ between the members of $J$. $\prec_J$ is defined as

  - $J_1 \prec_J J_2$ iff there is a discourse referent $x$ which belongs to $FV(J_2)$ and to a base of $J_1$.

- We say that a PISBAS relative to $M$ is an Information State Based Attitudinal State Description (ISBAS) relative to $M$ iff

  - $\prec_J$ is well-founded and

  - it is possible to assign, by induction along $\prec_J$, to each CCP $J$ occurring in $J$ an information state $I(J)$ as follows:

    * Suppose that $J$ has no predecessors according to $\prec_J$ Then $J$ is a total CCP and the associated information state $I(J)$ is defined as $I(\Delta)$.
    * Suppose that for all $J'$ occurring in $J$ s.th. $J' \prec_J J$, $I(J')$ has been defined. Then $J$ is defined on $\cup \{I(J') | J' \prec_J J\}$ and $I(J) = J(\cup \{I(J') | J' \prec_J J\})$.

7.4.4 Models for ADSs

**Definition 17** We extend the intensional model $M$ with

- A set of cognitive Agents $CA_w$; the set of cognitive agents of $M$ in each possible world $w$ of $M$.

- A function $AS_M(a, w, t)$ such that $AS_M$ assigns in each possible world $w$ of $M$ to each member $a$ of $CA_w$ of the universe of the model at each moment of time $t$ belonging to a certain interval or set of intervals an ISBAS which identifies a’s mental state at the time in question.

7.4.5 Truth of an ADS

The discourse referents of an ADS $K$ need not be the same as those occurring in the bases of the CCPs of the ISBAS. Consequently, we rename$^3$ the discourse referents occurring in the ADS under the additional constraint that the discourse referents occurring in ISBAS are entirely disjoint from those which belong to the language $L_{prop}$. $r(K)$ is the DRS obtained by replacing each discourse referent $x$ occurring in $K$ throughout $K$ by $r(x)$.

**Definition 18** Truth conditions for ADSs:

- $f \models_{M, w} s : Att(a, K)$ iff there exists

\[\text{Suppose that } r \text{ is a 1-1 map from the set of discourse referents occurring in an ADS } K \text{ onto some other set of discourse referents. Then the alphabetic variant of } K \text{ determined by } r \text{ is the set of all pairs } \langle MOD, r(K) \rangle \text{ such that } \langle MOD, K \rangle \text{ belongs to } K \text{ together with the pairs } \langle [ANCH, x] \rangle \text{ such that } (\langle ANCH, x \rangle) \text{ belongs to } K.\]
– a renaming function r sth. Dom(r) consists of the discourse referents occurring in K and
– a function H with Dom(H) = r(K) sth.

* H(⟨MOD, K⟩) is of the form ⟨MOD, J⟩
* for all timepoints in the duration of s and each ⟨MOD, K⟩ ∈ r(K), H(⟨MOD, K⟩) belongs to AS_M(f(a), w, t) and
* for each ⟨MOD, K⟩ ∈ r(K), [K]_{M,w,t}^{*} ≤ I(J), where I(J) is the information state determined within AS_M(f(a), w, t) by the CCP J of H(⟨MOD, K⟩).

7.4.6 Truth of an anchored ADS

Two requirements should be captured by a semantics of ADSs that takes into account the role of external anchors. First, the verification condition for s : Att(a, K, EA) should be undefined when K contains discourse referents which are internally but not externally anchored. The idea which is adopted here is to remove all internal anchors of such discourse referents in K, via a reduction of K with respect to EA.

**Definition 19 Reduction of K with respect to EA, Red(K, EA)**

- Red(K, EA) := K\{([ANCH, x], K)|([ANCH, x], K) ∈ K ∧ ∃x′(x, x′) ∈ EA\}

Second, a DRS K in which an external anchor for x occurs should be considered to express a proposition that is singular with respect to the value x of the external anchor for x. This is achieved by evaluating the proposition expressed by K with respect to embeddings f ∪ (EA ∪ f), which has each of the externally anchored discourse referents x in its domain and assigns to x the value that f assigns to x'.

**Definition 20 Truth conditions for anchored ADSs**

- f ⪯ M,w,s : Att(a, K, EA) iff
  - for all t ∈ dur(f(s)) there exists a function H from Red(K, EA) into AS_M(f(a), w, t) sth.
  - for each ⟨MOD, K⟩ ∈ Red(K, EA),
    [K]_{M,w,t}^{*} ≤ I(J)
  - where I(J) is the information state determined within AS_M(f(a), w, t) by the CCP J of H(⟨MOD, K⟩)

The truth conditions in definition 20 considered the ‘wide’ content interpretation of an ADS. For the ‘narrow’ content interpretation, i.e. for an attitude which does not depend on the environment of the agent who entertains the attitude, we can ignore the external anchor set EA and treat internally anchored discourse referents of K existentially. That is, the idea which is adopted here is to replace each internal anchor ⟨[ANCH, x], K⟩ in K by ⟨[BEL, K]⟩.

**Definition 21 Existentialization of internal anchors**

- NC(K) = (K\{([ANCH, x], K)|([ANCH, x], K) ∈ K\})
  ∪{⟨[BEL, K]⟩ : ⟨[ANCH, x], K⟩ ∈ K}

**Definition 22 Narrow content verification of an ADS**

- The narrow content verification of an ADS s : (Att(a, K) is the verification of the condition s : (Att(a, NC(K)) according to definition 20.
7.4.7 Semantics of vicarious anchors

The definitions in the preceding paragraphs considered only referential dependencies between DRSs which are part of the same ADS. In order to deal with vicarious anchors, we now need to extend the evaluation of ADSs against the ISBASs assigned to the an $x$ whose mental state is represented by an ADS $K_1$ to cases where the interpretation of discourse referents in $K_1$ depends on the interpretations that are assigned to discourse referents from an ADS $K_2$ of an agent $y$. The problem with which we are faced in capturing this intuition about dependencies between attitudes of different agents is obvious: the semantics which we defined for the mapping from ADSs to information states is not a dynamic one in that it does not consider the mapping from ADSs to information states to depend on “previous” mappings from ADSs to information states. But the interpretation of a vicarious anchor occurring in an ADS $K_1$ of an agent $x$ should not be determined with respect to the function $\mathcal{AS}_{M,x,w,t}$ but with respect to the value that $\mathcal{AS}_{M,y,w,t}$ assigns to the ADS $K_2$ of agent $y$ in which the second argument of the vicarious anchor occurs. That is, the objects which were singled out by Adrian’s desire as jackets he wants to buy should constitute the set of objects with respect to which conditions involving a vicariously anchored discourse referent in the attitude of the reporter should be evaluated. But while the evaluation of ADSs takes into account the difference between agents $\alpha$ in the function $\mathcal{AS}_{M,\alpha,w,t}$, this difference is lost at the level of information states, which are defined only with respect to embeddings and possible worlds but not with respect to agents. The question how a notion of an agent-relative information state could be defined is a matter which I won’t deal with in this paper.

What we propose in the following is a work-around to this problem. For the evaluation of vicarious anchors of the form $\langle x, y_z \rangle$, we assume that the dependency between attitudes of different agents can be captured as a temporal dependence. Before the reporter forms a representation referring to Adrian’s attitude, she needs to have a representation of Adrian’s attitude supporting one of the variants of the third reading that we discussed. In addition, the representation of Adrian’s attitude must also have an interpretation at the time that the full attitude report is evaluated. That is, we propose to break up the interpretation of DRSs containing several attitudes into separate representations and to valuate sets of conditions of the form $s : \text{Att}(\alpha, K)$ incrementally, according to their temporal order. The incrementality of evaluation could be modelled on the assumption that there are stop-points in the interpretation algorithm for DRSs and that conditions of the form $s : \text{Att}(\alpha, K)$ are such stop-points which enforce an interpretation of the ADS in which they occur. Then, we store for each ADS $K$ its verifying embeddings $g$ together with the agent who is the first argument of the ADS $K$. Given that we rename ADSs in order to assign them CCPs, we have to ensure that we are able to link vicarious anchors to their interpretations in the right manner. In order to do so, we also have to store the translation function $r_z$ associated with a certain ADS $K_1$ of which the first argument is an agent $z$. Then, the proposition expressed by an ADS $K$ should not be evaluated with respect to embeddings $f \cup (EA \circ f)$ but with respect to the extension $f \cup (EA \circ f) \cup (VA)$, where $VA$ has each of the vicariously anchored discourse referents $\langle x, y_z \rangle \in K$ in its domain and assigns to $x$ the value that $g_z$ assigned to $r(y_z)$. Thus, $VA$ is a function from vicariously anchored discourse referents $\langle y, x_z \rangle$ to $g_z(r_z(x_z))$. Definition 23 makes use of this function $VA$ in the evaluation of ADSs with vicarious anchors.

**Definition 23** Wide content verification of an anchored ADSs with vicarious anchors.

- $f \models_{M, w, s} : \text{Att}(a, K, EA)$ iff
  - for all $t \in \text{dur}(f(s))$ there exists a function $H$ from $\text{Red}(K, EA)$ into $\mathcal{AS}_{M}(f(a), w, t)$ s.th.
  - for each $(MOD, K) \in \text{Red}(K, EA)$,
    - $[K]^{\text{\textbf{w}}, f \cup (EA \circ f) \cup VA} \in K^{I(J)}$
    - where $I(J)$ is the information state determined within $\mathcal{AS}_{M}(f(a), w, t)$ by the CCP $J$ of $H((MOD, K))$

**Definition 24** Narrow content verification of an anchored ADSs with vicarious anchors.

- The narrow content verification of an ADS $s : \text{Att}(a, K)$ is the verification of the condition $s : \text{Att}(a, NC(K))$ according to definition 23.

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4 First steps into this direction have been undertaken in [Pross, 2010].
References


