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# DRT: AN UPDATED SURVEY

## Lecture III: INTERPRETING DRS-ES

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# Contents

# 1 What Sentences Mean

Semantic objects for sentences/formulae/DRS-es:

- Extensional (static) logic: truth values; different but equivalent df: sets of verifying assignments; e.g.

  (1) $[\![\exists x.\varphi]\!]^{s,g} = \{g' | g[x]g' \ s.t. \ [\![\varphi]\!]^{s,g'} \ true\}$

- Intensional (static) logic: functions from possible worlds into truth values; equivalently: sets of possible worlds where the sentence/formula is true.

  (2) a. $[\![\varphi]\!]^f \in \left(2^W\right)$
  b. $[\![\varphi]\!] = \{w \in W | [\![\varphi]\!]^f(w) = 1\}$

- Stalnaker: the meaning of a sentence is its contribution to context. Putting it slightly differently, the meaning of a sentence is the way it changes a given context. More recent terminology: *context change potential*.

'Static' and 'dynamic' sentence meaning. Roughly, truth conditions vs context change potential.

Syntactic correlate: Groenendijk and Stokhof' s $\uparrow \varphi$ and $\downarrow \Phi$ (Groenendijk and Stokhof (1991)). ($\uparrow$ lifts a 'static' formula to the dynamic level, $\downarrow$ lowers a 'dynamic' formula.)

Why sentences?

In the classic, static case, the meaning of a subsentential consituent is its contribution to the truth-conditions of the sentence it appears in. In the dynamic case:

- truth-conditions, as always;
- contribution to context change potential;
- links to preceding context (or: imposing constraints on what kind of contexts the sentence has to connect to).

# 2 Truth and Other 'Static' Notions in DRT

## 2.1 Syntax of DRS-es and DRS-Conditions

**Definition 1** • Given $U \subseteq Ref$, a set $Con$ of conditions, $\langle U, Con \rangle$ is a DRS;

- given drefs $x_{i,j}$, $x_i = x_j$ is a condition;
- given a name $N \in \mathcal{N}$ and $x \in Ref$, $N(x)$ is a condition;
- given $P \in Rel^n$ and $x_1, \ldots, x_n \in Ref$, $P(x_1, \ldots, x_n)$ is a condition;
- if $K$ is a DRS, $\neg K$ is a condition;
- if $K_{1,2}$ are DRS-es, $K_1 \vee K_2$ and $K_1 \Rightarrow K_2$ are conditions;
- *duplex* conditions (needed to represent generalised quantifiers): not discussed here;
- conditions of the form $\Sigma x.K$ ('abstracted' conditions): not discussed, either.

## 2.2 Some Terminology

Calculating the set of *free variables* in a DRS $K$ ($FV(K)$):

- $FV(\langle U, Con \rangle) =_{df} (\bigcup_{\gamma \in Con_K} FV(\gamma)) \setminus U_K$;
- $FV(x_i = x_j) =_{df} \{x_i, x_j\}$;
- $FV(P(x_1, \ldots, x_n)) =_{df}$ ????;
- $FV(\neg K) =_{df} FV(K)$;
- $FV(K_1 \vee K_2) =_{df} FV(K_1) \cup FV(K_2)$;
- $FV(K_1 \Rightarrow K_2) =_{df} FV(K_1) \cup (FV(K_2) \setminus U_{K_1})$.

*Bound variables*: $BV(K) =_{df} V(K) \setminus FV(K)$, where $V(K)$ is the set of drefs occurring *somewhere* in $K$.

**Definition 2** A DRS $K$ is *proper* iff $FV(K) = \emptyset$.

A *subordination* relation between DRS-es: the reflexive transitive closure of the relation $\leq$ defined below.

$K_2 \leq K_1$ iff any of the following conditions holds:

1. $\neg K_2 \in Con(K_1)$, or
2. $\exists K_3$ s.t. $K_3 \Rightarrow K_2$ or $K_2 \Rightarrow K_3 \in Con(K_1)$, or
3. $\exists K_3$ s.t. $K_3 \vee K_2$ or $K_2 \vee K_3 \in Con(K_1)$.

**Definition 3** Let $\leq_{cl}$ be the reflexive transitive closure of $\leq$. If $K_j \leq_{cl} K_i$, we say that $K_j$ is a *sub-DRS* of $K_i$.

**Definition 4** DRS $K$ is *pure* iff for any two distinct $K_{1,2}$ s.t. $K_2 \leq_{cl} K_1$ and $K_1 \leq_{cl} K$, the following holds: $U_{K_2} \cap (U_{K_1} \cup FV(K)) = \emptyset$. That is, iff $U_{K_2} \cap U_{K_1} = \emptyset$ and $U_{K_2} \cap FV(K)) = \emptyset$.

*Accessibility of DRS*-es:

**Definition 5** $K$ is accessible from $K_1$, $K acc K_1$, iff

1. $K_1 \leq_{cl} K$, or
2. $\exists K_{2,3}$: $K_2 \Rightarrow K_3$ & $K acc K_2$ & $K_3 acc K_1$.

*Accessibility of drefs*:

**Definition 6** Given $K_{1,2}, x_{1,2}$: $x_1$ is *accessible* from $x_2$ iff $x_1 \in K_1, x_2 \in K_2$, and $K_1 acc K_2$.

## 2.3 Extensional, Not-so Dynamic Interpretation

(3) $M =_{df} \langle U, W, I \rangle$

(4) $I$: $\mathcal{N} \rightarrow U$
$I$: $Rel^n \rightarrow \mathcal{P}(U^n)$

*Verifying embeddings*:

1. $\langle g, h \rangle \models_M \langle U, Con \rangle$ iff $g \subset_U h$ and $\forall \gamma \in Con$: $h \models_M \gamma$;
2. $g \models_M x_i = x_j$ iff ...;
3. $g \models_M N(x)$ iff $I(N) = g(x)$;
4. $g \models_M P(x_1, \ldots x_n)$ iff ...;
5. $g \models_M \neg K$ iff $\nexists h$ s.t. $\langle g, h \rangle \models_M K$;
6. $g \models_M K_1 \vee K_2$ iff ...;
7. $g \models_M K_1 \Rightarrow K_2$ iff $\forall k$: $\langle g, k \rangle \models_M K_1$: $\exists l$: $\langle k, l \rangle \models_M K_2$.

When $g \models_M \gamma$: *$g$ verifies $\gamma$* in $M$.
When $\langle g, h \rangle \models_M K$: *$h$ verifies $K$* in $M$.

*Truth* of a proper DRS $K$ in $M$:

**Definition 7** A proper DRS $K$ is true in $M$ iff there exists a verifying embedding $h$ for $K$ w.r.t. the empty assignment $\Lambda$:
If $\langle \Lambda, h \rangle \models_M K$, then $\models_M K$.

**NB** Two 'uses' of *truth* (Berg (1996)):

- $g, h$ stand in the relation 'appropriate' for $K$; i.e. it is true that the relation $R_K$ holds; Van den Berg calls this the 'dynamic condition';
- $K$ expresses a true statement: truth conditions.

## 2.4 Intensional, Still Not-so-dynamic Interpretation

Let $L$ be a DRS language with a family $Rel^n$ of atomic predicates and a set $\mathcal{N}$ of names. We say that an *intensional model $M$ for $L$* is a triple $\langle W_M, U_M, I_M \rangle$, where

- $W_M$ is a set of possible worlds;
- $U_M$ is a non-empty set;
- $I\colon \mathcal{N} \to \{\{d\} | d \in U_M\}$, and $I\colon Rel^n \to (W_M \to \mathcal{P}(U_M))$.

Verifying embeddings are given *globally*, i.e. $g\colon X \to U_M$. Intensional semantics for DRS-es and the DRS-conditions we have handled so far: **Exercise**. Assuming a total accessibility relation on $W_M$, we have in addition:

(5) a. $g \models_{M,w} \Box K$ iff $\forall w' \in W_M\colon g \models_{M,w'} K$;
b. $g \models_{M,w} \Diamond K$ iff ...

**NB1** for those familiar with work on modality in Dynamic Semantics: the modalities in (5) are not *epistemic* modalities. *Might* in DS: a consistency check on context, as in (6). The clauses in (5) serve different purposes, cf. Hans Kamp's lecture tomorrow.

(6) It *might* be raining. It isn't raining.

**NB2** Times in DRT:
Now assume: $L$ contains in addition

- discourse referents for times, events and states;
- constants $\prec, \circ, \subseteq, :$ (complete precedence, overlap, temporal inclusion, 'is of type') and perhaps others.

An *intensional* model $M$ for $L$ is now of the form $\langle W_M, \mathcal{T}_M, E_M, S_M, U_M, I_M \rangle$; $E_{M,w}, S_{M,w}, Per(\mathcal{T}_M) \subseteq U_{M,w}$ for $w \in W$;

**Definition 8** A DRS $K$ is *true in $M$ in $w$ at $t$* iff there is a verifying embedding $f$ of $U_K$ into $U_{M,w}$ such that

*1.* $f$ verifies all conditions of $K$ in $M$ in $w$, and

*2.* $f(n) \subseteq t$.

# 3 Changing the Context

## 3.1 Contexts as Sets of Possible Worlds

Stalnaker: sentence meaning $\approx$ context change potential.

(7) $[\![\varphi_i]\!]^d : C \to C$

Still Stalnakerian, but see also Veltman (1996):
Suppose that $[\![\varphi_i]\!]^d : c \mapsto c'$, where $c = \bigcap_{1 \leq h \leq i_i} [\![\varphi_h]\!]^s$, i.e. $c = \{w \in W | \varphi_h \textit{ true in } w, 1 \leq h \leq i-1\}$.
Then $c' = \{w \in W | \varphi_h \textit{ true in } w, 1 \leq h \leq i\}$.

**NB0** A dilemma that is not a dilemma: where to locate context change: in the contribution of the (new) sentence or operate with a standard, static sentence meaning and define an operation that adds the (semantic object contributed by the) sentence to context: n

- The simplest implementation of Stalnaker's core idea: let contexts and sentences *alike* denote sets of possible worlds, and define an operation + from $\mathcal{P}(W) \times \mathcal{P}(W)$ into $\mathcal{P}(W)$. (Better said, + is intersection, and that is that.)
- Define sentences as functions from contexts to contexts, s.t. $[\![\varphi]\!]^f(c) =$ as above.

**NB1**: $c' \subseteq c$. Groenendijk and Stokhof (1990b): *eliminativity.*
**NB2**: sentences (their translations/representations) play two roles:

- As context change potentials, when $\varphi_i$ is added to context $c$;
- as forming (part of a ) context that undergoes change.

DRT: a DRS $K$ has two semantic values, a context change potential (CCP/$\pi$), i.e. a function, and an information state, that can be the argument of such a function.

$$
\begin{array}{lllll}
S_1, \ldots S_n & , & S_{n+1} & & \\[1em]
K_{1,\ldots,n} & ; & K_{n+1} & \to & K_{1,\ldots,n+1} \\[1em]
[\![K_{1,\ldots,n}]\!]^s_M & & [\![K_{n+1}]\!]^p_M & & [\![K_{1,\ldots,n+1}]\!]^s_M
\end{array}
$$

### 3.2 Adding Information about (Discourse) Entities

Insufficient to define $C$ as a set (of sets of) possible worlds:

(8)

a. Nine of the ten marbles are in the bag. #It is under the sofa.
b. Exactly one of the ten marbles is not in the bag. It is under the sofa.

'Context': *information state.*

| | | | |
|---|---|---|---|
| (9) | a. | $I_{ext} = \{g \mid g \in \left(U^{Var}\right)\}$ | DPL |
| | b. | $I_1 = \{\langle w, g\rangle \mid w \in W, g \in \left(U^{Var}\right)\}$ | Heim |
| | c. | $I_2 = \{\langle n, w, g\rangle \mid n \in \mathbf{N}, w \in W, g \in \left(U^{Var}\right)\}$ | Gr&St&V'94 |
| | d. | $I_3 = \{\langle w, g\rangle \mid w \in W, \exists X \subseteq U : g \in \left(X^{Var}\right)\}$ | DRT |

## 4 'Dynamic' Meanings for DRS-s

### 4.1 DRS-es as Part of Context

**Definition 9** Information states: given $M$, DRS $K$ and a set $X$ of dref-s,

1. $s$ is an information state (i.s., henceforth) relative to $M$ and $X$ iff $s \subseteq \{\langle w, f\rangle \mid Dom(f) = X \ \&\ Ran(f) \subseteq U_M \ \&\ w \in W_M\}$.
2. $s$ is an i.s. relative to $M$ iff there is a set $X$ of drefs s.t. ...;
3. $X$ is the *base* of $s$; notation: $X_s$;
4. the empty information state: $\Lambda^s_M =_{df} \{\langle w, \Lambda\rangle \mid w \in W_M\}$;
5. The proposition determined by $s$: $Prop(s) =_{df} \{w \mid \exists f\colon \langle w, f\rangle \in s\}$.

Given a proper DRS, the *information state* $[\![K]\!]^s_M$ *expressed by* $K$ *relative to* $M$*:*

(10) $[\![K]\!]^s_M =_{df} \{\langle w, f\rangle \mid \langle \Lambda, f\rangle \models_{M,w} K\}$.

**Definition 10** Given an intensional model $M$, a possibly improper DRS $K$ and an assignment $g$ of $FV(K)$ in $M$. Then

1. *the proposition expressed by $K$ relative to $M, g$,*
   $[\![K]\!]^p_{M,g} =_{df} \{w \in W | \exists h : g \subseteq_{U_K} h \ \& \ \langle g, h\rangle \models_{M,g} K\}$;

2. the information state expressed by $K$ relative to $M, g$,
   $[\![K]\!]^s_{M,g} =_{df} \{\langle w, h\rangle | g \subseteq_{U_K} h \ \& \ \langle g, h\rangle \models_{M,w} K\}$.

## 4.2 DRS-es as Context Change Potentials

**Definition 11** The *Contex Change Potential* $[\![K]\!]^d_M (= \pi_K)$ of a DRS $K$ relative to model $M$ is a partial function from i.s-s to i.s.-s s.t.:
$[\![K]\!]^d_M =_{df} \in S^{S'}$, where $S' \subseteq S$, s.t.

1. $s \in Dom([\![K]\!]^d_M)$ iff $FV(K) \subseteq X_s$;

2. if $s \in Dom([\![K]\!]^d_M)$, then
   $[\![K]\!]^d_M(s) = \{\langle w, g\rangle | \exists f : (\langle w, f\rangle \in s \ \& \ \langle f, g\rangle \models_{M,w} K)\}$.

If $K$ is proper, $[\![K]\!]^d_M$ is a total function. That is, it will be defined even for the empty i.s. $\Lambda^s_M$.

## 4.3 Useful Notions

*Informativity* of information states:

**Definition 12** Given two i.s-s $s, s'$, $s'$ is at least informative as $s$, $s \leq s'$, iff
$\forall w, g : \langle w, g\rangle \in s' \rightarrow \exists \langle w, f\rangle \in s \ \& \ f \subseteq g$.

*Survival:*

**Definition 13** An i.s. $s$ *survives* in i.s. $s'$ iff
$\forall \langle w, f\rangle \in s : \exists g : f \subseteq g \ \& \ \langle w, g\rangle \in s'$.

(Dekker (1993))

*Merging* information states:

**Definition 14** Given an intensional model $M$ and a set $S$ of is-s relative to $M$, the *consistent merge* of the members of $S$, denoted $\underline{\bigcup} S$, is the information state defined by:

$\underline{\bigcup} S =_{df} \{\langle w, h\rangle |$there is a function $F$ s.t. $Dom(F) = S,$ and for $\forall s \in S, \langle w, F(s)\rangle \in s,$ and $h = \bigcup\{F(s)|s \in S\}$ is a function$\}$.

Conditions satisfied by context change potentials determined by DRS-es:

1. Given an i.s. $s$ with base $X_s$, it depends exclusively on $X_s$ whether $s$ is in the domain of some ccp $\pi_K$.

   More precisely: there exists a set $X_\pi$ of drefs s.t. $s \in Dom(\pi_K)$ iff $X_{\pi_K} \subseteq X_s$. ($X_\pi$ will be called the *base* of $\pi_K$.)

2. Ccps of the form $[\![K]\!]^d_M$ have the property that the information states in their range are always at least as informative as the information states in their domain: $\forall s \in Dom(\pi_K)$: $s \leq \pi(s)$.

**Definition 15** *Regular ccps* are ccp-s that satisfy the two conditions above.

The set $X_\pi$ mentioned in condition 1 is unique (unless $\pi_K$ is empty).

A rigourous definition of a *total* ccp:

**Definition 16** A context change potential $\pi$ is *total* iff $\pi(\Lambda^s_M)$ is defined.

Relation(s) between the i.s. and ccp expressed by the same DRS $K$ (assuming an extensional DRS-language):

**Theorem 1** Given $s \in Dom([\![K]\!]^d_M)$ and $\langle w, g\rangle \in s$. Then for any assignment function $h$ s.t. $g \subseteq_{U_K} h$:

$\langle w, h\rangle \in [\![K]\!]^d_M(s)$ iff $\langle w, h\rangle \in [\![K]\!]^s_{M,g}$.

### 4.4 Other Useful Notions

**Definition 17** An information state $s$ *supports* a DRS $K$ iff $s \in Dom(\pi_K)$, $s \leq \pi_K(s)$, and $\forall \langle w, f\rangle \in s\colon \exists \langle w, g\rangle \in \pi_K(s)$ s.t. $f \subseteq g$.

This definition is from Groenendijk, Stokhof and Veltman (1994), and as an **exercise** you can check that this notion of support corresponds to DRT's truth definition.

Building on the notion of support, one can define *coherence*:

**Definition 18** Given a DRS $K$ and a set $S$ of information states, $K$ is *coherent* iff there is $s \in S$ such that $s \neq \emptyset$ and $s$ supports $K$.

*Dynamic* entailment:

**Definition 19** Given a set $S$ of i.s-s $K_1, \ldots, K_n \models K_{n+1}$ *iff* $\forall s \in S$ :

if the update of $s$ with $K_1, \ldots, K_{n+1}$ exists, then the information state obtained from updating $s$ with the premise-DRS-es supports the conclusion-DRS $K_{n+1}$.

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