Introduction to Information Retrieval

http://informationretrieval.org

IIR 15-1: Support Vector Machines

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Overview

1. Recap
2. Vector space classification
3. Linear classifiers
4. Support Vector Machines
5. Discussion

Outline

Feature selection: MI for *poultry*/EXPORT

Goal of feature selection: eleminate noise and useless features for better effectiveness and efficiency

	$e_c = e_{poultry} = 1$	$e_c = e_{poultry} = 0$
$e_t = e_{\text{EXPORT}} = 1$	$N_{11} = 49$	$N_{10} = 27{,}652$
$e_t = e_{\text{EXPORT}} = 0$	$N_{01} = 141$	$N_{00} = 774{,}106$

Plug these values into formula:

$$\begin{aligned}
I(U;C) &= \frac{49}{801{,}948}\log_2\frac{801{,}948\cdot 49}{(49+27{,}652)(49+141)} \\
&+\frac{141}{801{,}948}\log_2\frac{801{,}948\cdot 141}{(141+774{,}106)(49+141)} \\
&+\frac{27{,}652}{801{,}948}\log_2\frac{801{,}948\cdot 27{,}652}{(49+27{,}652)(27{,}652+774{,}106)} \\
&+\frac{774{,}106}{801{,}948}\log_2\frac{801{,}948\cdot 774{,}106}{(141+774{,}106)(27{,}652+774{,}106)} \\
&\approx 0.000105
\end{aligned}$$

Feature selection for Reuters classes coffee and sports

Class: *coffee*

term	MI
COFFEE	0.0111
BAGS	0.0042
GROWERS	0.0025
KG	0.0019
COLOMBIA	0.0018
BRAZIL	0.0016
EXPORT	0.0014
EXPORTERS	0.0013
EXPORTS	0.0013
CROP	0.0012

Class: *sports*

term	MI
SOCCER	0.0681
CUP	0.0515
MATCH	0.0441
MATCHES	0.0408
PLAYED	0.0388
LEAGUE	0.0386
BEAT	0.0301
GAME	0.0299
GAMES	0.0284
TEAM	0.0264

Using language models (LMs) for IR

- LM = language model
- We view the document as a generative model that generates the query.
- What we need to do:
- Define the precise generative model we want to use
- Estimate parameters (different parameters for each document's model)
- Smooth to avoid zeros
- Apply to query and find document most likely to have generated the query
- Present most likely document(s) to user

Jelinek-Mercer smoothing

- $P(t|d) = \lambda P(t|M_d) + (1 - \lambda)P(t|M_c)$
- Mixes the probability from the document with the general collection frequency of the word.
- High value of λ: "conjunctive-like" search – tends to retrieve documents containing all query words.
- Low value of λ: more disjunctive, suitable for long queries
- Correctly setting λ is very important for good performance.

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Take-away today

- Vector space text classification: Classification defined as the problem of finding a separating hyperplane in high-dimensional space
- Linear vector space classifiers: Simple, but often very effective
- Support vector machines: State-of-the-art text classification methods (linear and nonlinear)
- Discussion: Which classifier should I use for my problem?

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Recall vector space representation

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- Normalize vectors (documents) to unit length
- How can we do classification in this space?

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- Premise 1: Documents in the same class form a contiguous region.
- Premise 2: Documents from different classes don't overlap.
- We define lines, surfaces, hypersurfaces to divide regions.

Classes in the vector space

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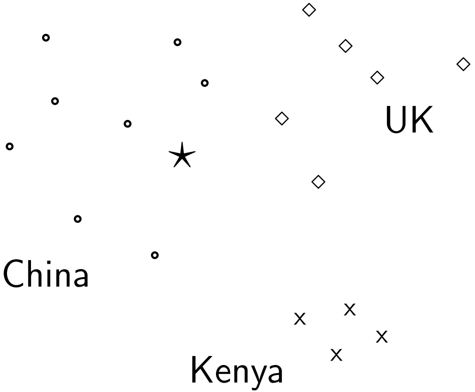
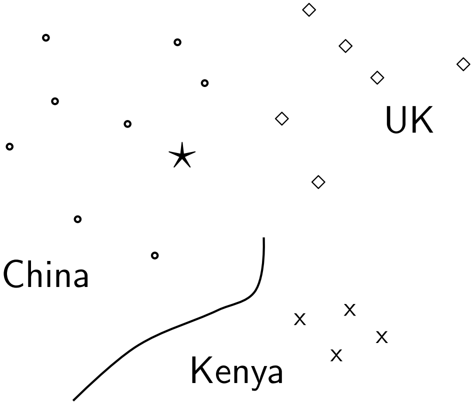


Should the document $\star$ be assigned to *China*, *UK* or *Kenya*?

Classes in the vector space



Find separators between the classes

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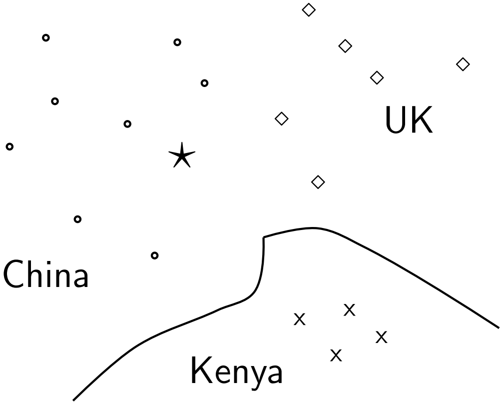

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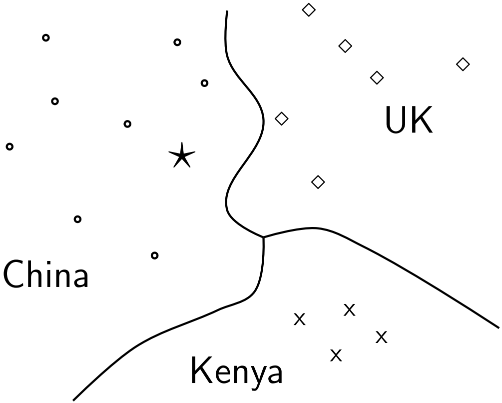

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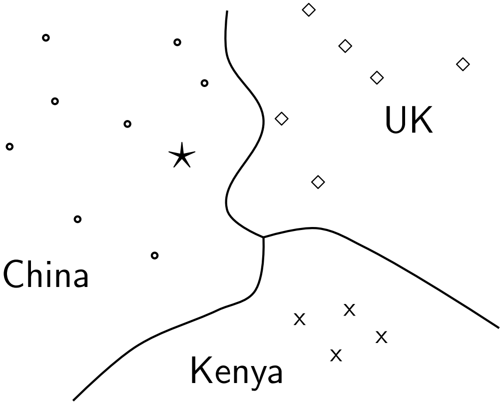
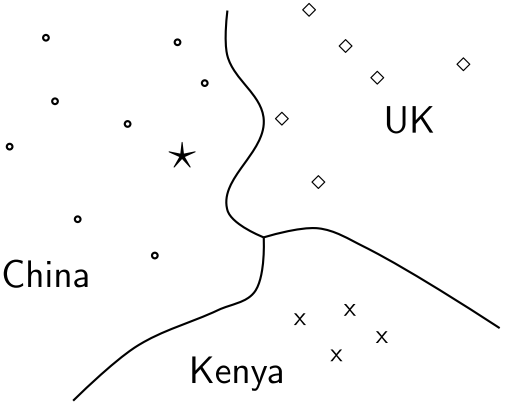


Based on these separators: $\star$ should be assigned to *China*

Classes in the vector space



How do we find separators that do a good job at classifying new documents like $\star$? – Main topic of today

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- Assumption: The classes are linearly separable.

A linear classifier in 1D

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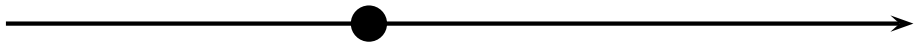
A linear classifier in 1D



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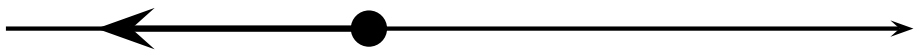

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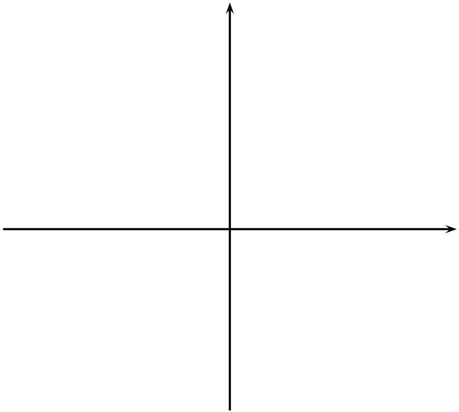

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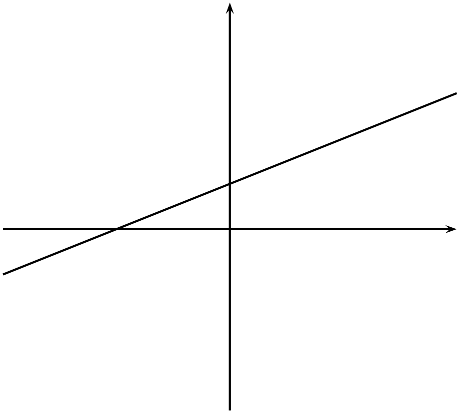

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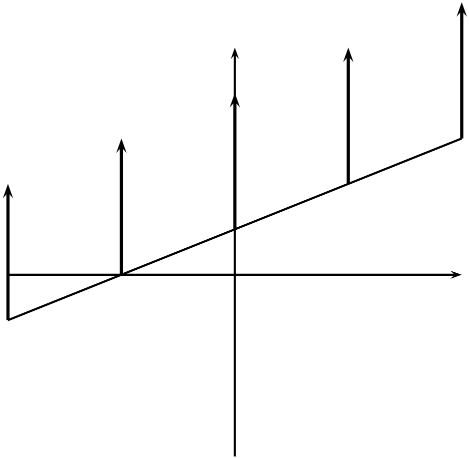

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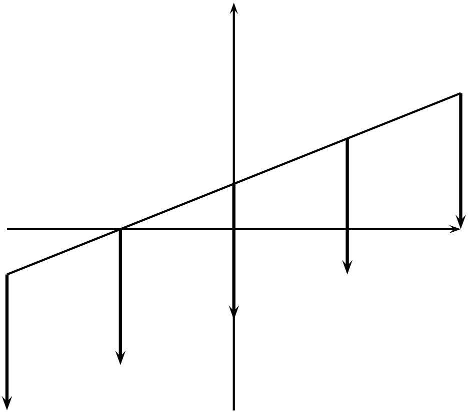

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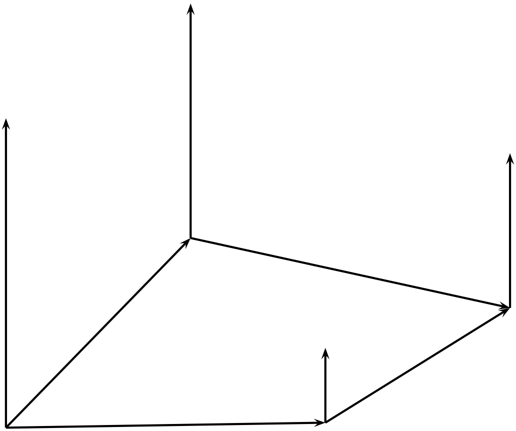

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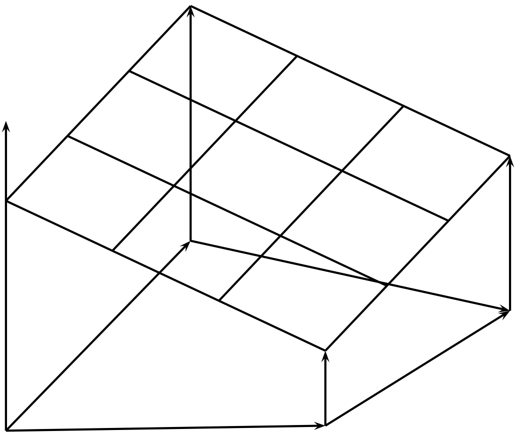

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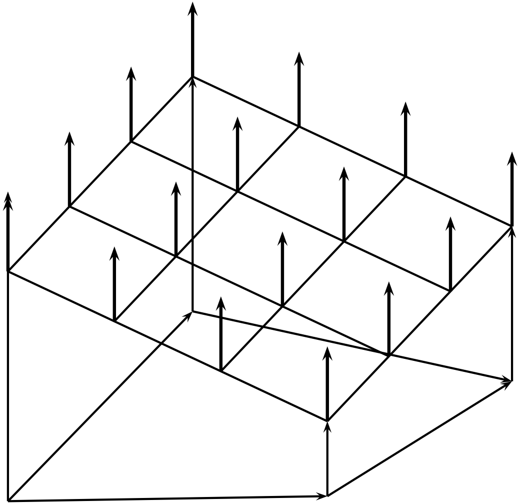

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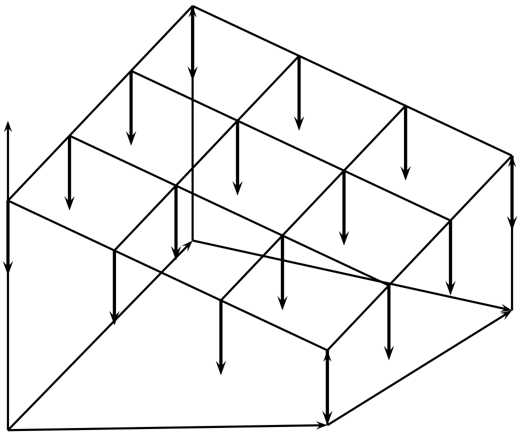

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Naive Bayes as a linear classifier

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Multinomial Naive Bayes is a linear classifier (in log space) defined by:

$$\sum_{i=1}^{M} w_i d_i = \theta$$

where $w_i = \log[\hat{P}(t_i|c)/\hat{P}(t_i|\bar{c})]$, $d_i =$ number of occurrences of t_i in d, and $\theta = -\log[\hat{P}(c)/\hat{P}(\bar{c})]$. Here, the index i, $1 \leq i \leq M$, refers to terms of the vocabulary (not to positions in d as k did in our original definition of Naive Bayes)

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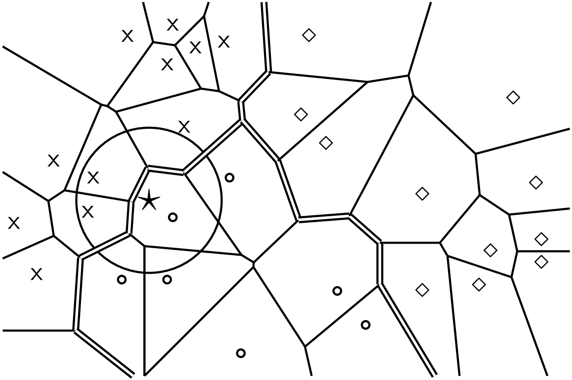

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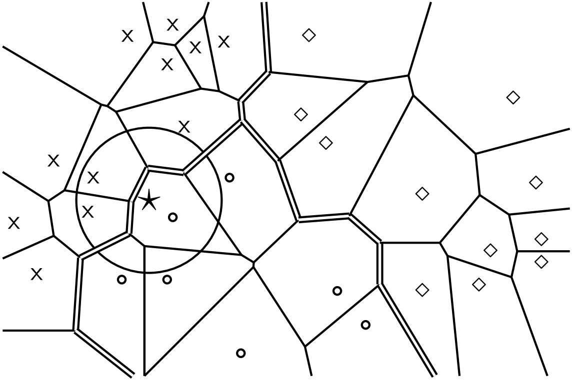

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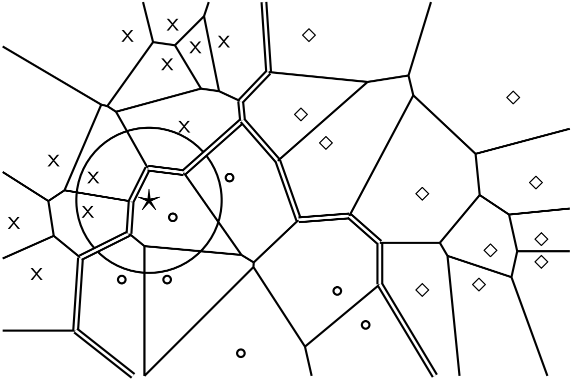

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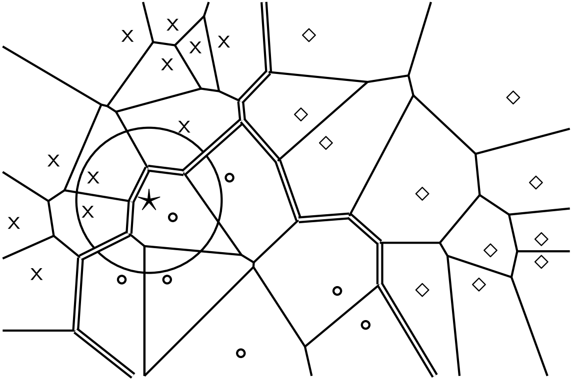

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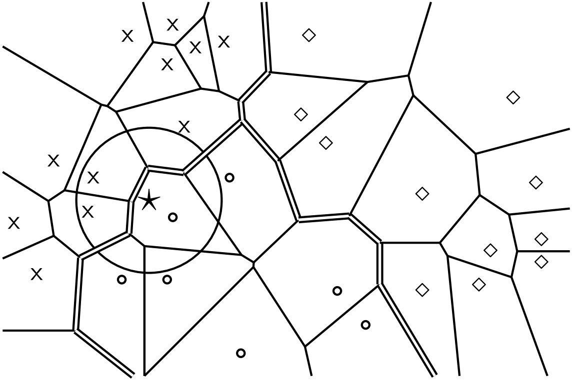

- Classification decision based on majority of k nearest neighbors.
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- ... but they are in general not linear classifiers that can be described as $\sum_{i=1}^{M} w_i d_i = \theta$.

Example of a linear classifier

t_i	w_i	d_{1i}	d_{2i}	t_i	w_i	d_{1i}	d_{2i}
prime	0.70	0	1	dlrs	-0.71	1	1
rate	0.67	1	0	world	-0.35	1	0
interest	0.63	0	0	sees	-0.33	0	0
rates	0.60	0	0	year	-0.25	0	0
discount	0.46	1	0	group	-0.24	0	0
bundesbank	0.43	0	0	dlr	-0.24	0	0

- This is for the class *interest* in Reuters-21578.
- For simplicity: assume a simple 0/1 vector representation
- d_1: “rate discount dlrs world”
- d_2: “prime dlrs”
- $\theta = 0$
- Exercise: Which class is d_1 assigned to? Which class is d_2 assigned to?

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- Exercise: Which class is d_1 assigned to? Which class is d_2 assigned to?
- We assign document $\vec{d}_1$ "rate discount dlrs world" to *interest* since $\vec{w}^T\vec{d}_1 = 0.67 \cdot 1 + 0.46 \cdot 1 + (-0.71) \cdot 1 + (-0.35) \cdot 1 = 0.07 > 0 = \theta$.
- We assign $\vec{d}_2$ "prime dlrs" to the complement class (not in *interest*) since $\vec{w}^T\vec{d}_2 = -0.01 \leq \theta$.

Which hyperplane?

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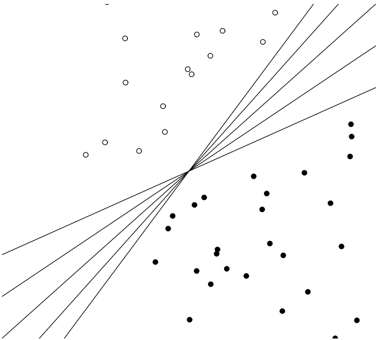

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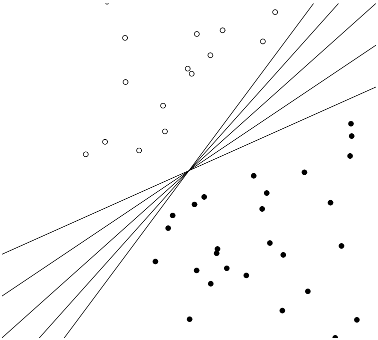
  - Perceptron (example available as PDF on website: http://ifnlp.org/ir/pdfnew/p.pdf)

Learning algorithms for vector space classification

- In terms of actual computation, there are two types of learning algorithms.
- (i) Simple learning algorithms that estimate the parameters of the classifier directly from the training data, often in one linear pass.
  - Naive Bayes, Rocchio, kNN are all examples of this.
- (ii) Iterative algorithms
  - Support vector machines
  - Perceptron (example available as PDF on website: http://ifnlp.org/ir/pdfnew/p.pdf)
- The best performing learning algorithms usually require iterative learning.

Which hyperplane?

Which hyperplane?



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- Error rates on new data are low for some, high for others.
- How do we find a low-error separator?
- Simple perceptron: generally bad; Naive Bayes, Rocchio: ok; linear SVM, "advanced" perceptrons: good

Linear classifiers: Discussion

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Linear classifiers: Discussion

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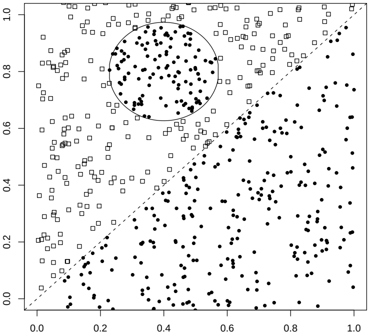
- Each method has a different way of selecting the separating hyperplane
  - Huge differences in performance on test documents
- Can we get better performance with more powerful nonlinear classifiers?
- Not in general: A given amount of training data may suffice for estimating a linear boundary, but not for estimating a more complex nonlinear boundary.

A nonlinear problem

A nonlinear problem



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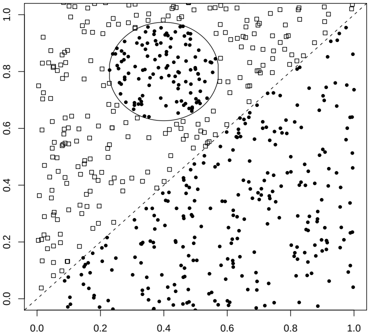

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- kNN will do well (assuming enough training data)

Outline

Support vector machines

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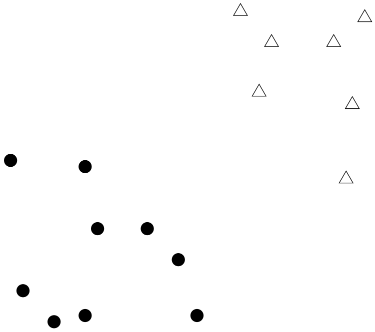
SVMs: A kind of large-margin classifier

Vector space based machine-learning method aiming to find a decision boundary between two classes that is maximally far from any point in the training data (possibly discounting some points as outliers or noise)

Support Vector Machines

- 2-class training data



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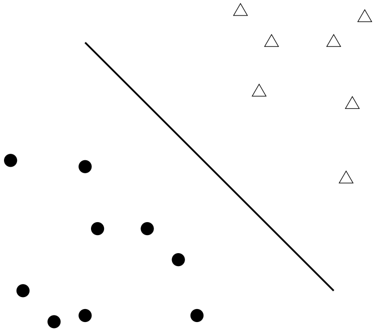

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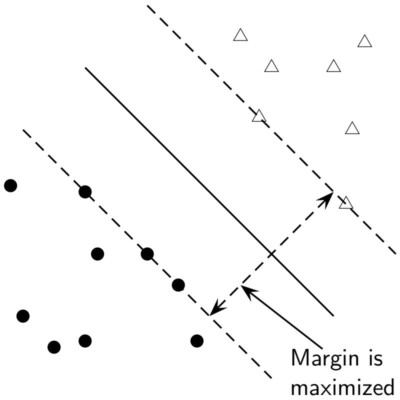

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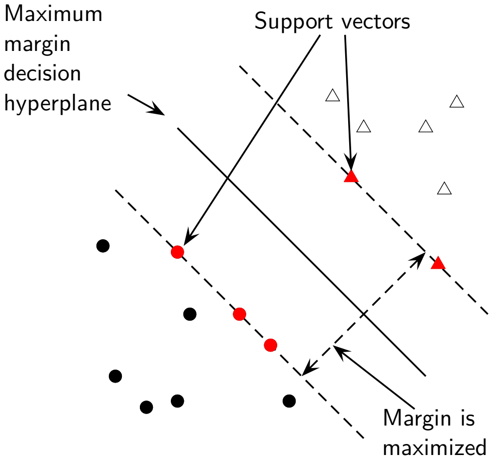

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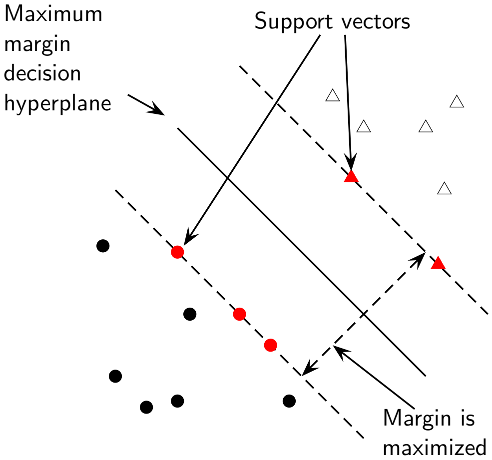

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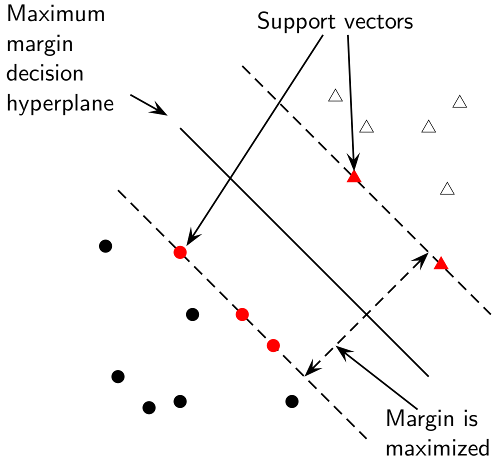

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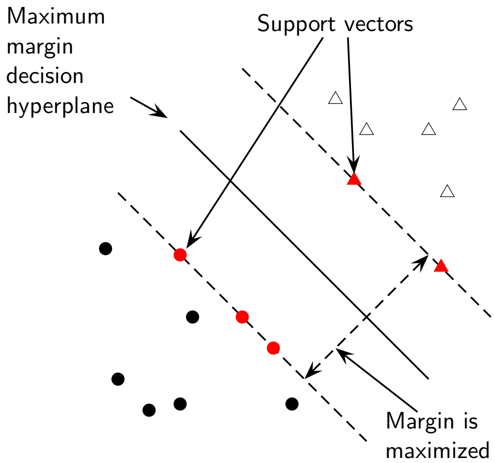
A classifier with a large margin makes no low certainty classification decisions.
Gives classification safety margin w.r.t slight errors in measurement or doc. variation



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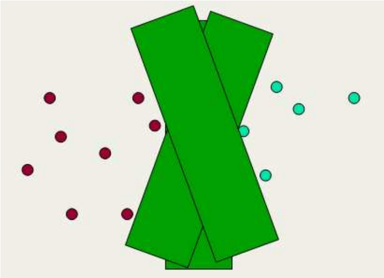
- increased ability to correctly generalize to test data



Separating hyperplane: Recap

Hyperplane

An n-dimensional generalization of a plane (point in 1-D space, line in 2-D space, ordinary plane in 3-D space).

Decision hyperplane

Can be defined by:

- intercept term b
- normal vector $\vec{w}$ (**weight vector**) which is perpendicular to the hyperplane

All points $\vec{x}$ on the hyperplane satisfy:

$$\vec{w}^{\mathsf{T}}\vec{x} = -b$$

Formalization of SVMs

Training set

Consider a binary classification problem:

- $\vec{x}_i$ are the input vectors
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The linear classifier is then:

$$f(\vec{x}) = \text{sign}(\vec{w}^\mathsf{T}\vec{x} + b)$$

A value of -1 indicates one class, and a value of $+1$ the other class.

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But we can increase functional margin by scaling $\vec{w}$ and b.
We need to place some constraint on the size of the $\vec{w}$ vector.

Geometric margin

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$$r = y\frac{\vec{w}^{\mathsf{T}}\vec{x} + b}{|\vec{w}|}$$

The geometric margin is clearly invariant to scaling of parameters: if we replace $\vec{w}$ by $5\vec{w}$ and b by $5b$, then the geometric margin is the same, because it is normalized by the length of $\vec{w}$.

Optimization problem solved by SVMs

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We want to maximize this geometric margin.

That is, we want to find $\vec{w}$ and b such that:

- $\rho = 2/|\vec{w}|$ is maximized
- For all $(\vec{x}_i, y_i) \in \mathbb{D}$, $y_i(\vec{w}^\mathsf{T}\vec{x}_i + b) \geq 1$

Optimization problem solved by SVMs (2)

Maximizing $2/|\vec{w}|$ is the same as minimizing $|\vec{w}|/2$. This gives the final standard formulation of an SVM as a minimization problem:

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Find $\vec{w}$ and b such that:

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We are now optimizing a **quadratic function** subject to linear constraints. Quadratic optimization problems are standard mathematical optimization problems, and many algorithms exist for solving them (e.g. Quadratic Programming libraries).

Recap

- We start with a training set.
- The data set defines the maximum-margin separating hyperplane (if it is separable).
- We use quadratic optimization to find this plane.
- Given a new point $\vec{x}$ to classify, the classification function $f(\vec{x})$ computes the projection of the point onto the hyperplane normal.
- The sign of this function determines the class to assign to the point.
- If the point is within the margin of the classifier, the classifier can return "don't know" rather than one of the two classes.
- The value of $f(\vec{x})$ may also be transformed into a probability of classification

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Soft-margin SVMs minimize training error traded off against margin.

Binary classification → One-of multiclass classification

- Many classification algorithms are binary.
- What do we do for one-of multiclass classification: we have $k > 2$ classes and the k classes are mutually exclusive?
- Common technique: build $|\mathbb{C}|$ one-versus-rest classifiers (commonly referred to as "one-versus-all" or OVA classification), and choose the class which classifies the test data with highest probability (probabilistic classifier) or greatest margin (SVM)
- Another strategy: build a set of one-versus-one classifiers, and choose the class that is selected by the most classifiers. While this involves building $|\mathbb{C}|(|\mathbb{C}| - 1)/2$ classifiers, the time for training classifiers may actually decrease, since the training data set for each classifier is much smaller.

Multiclass support vector machines

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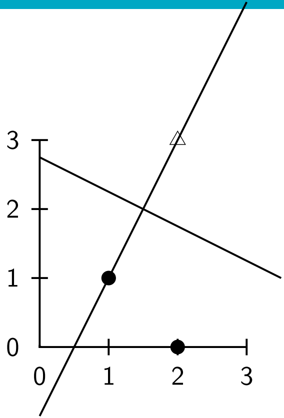
Walkthrough example: building an SVM over the data set shown in the figure

Working geometrically:



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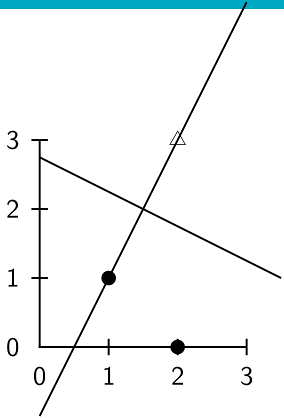

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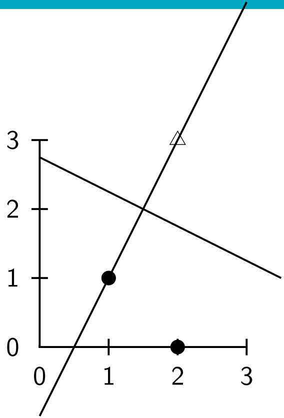

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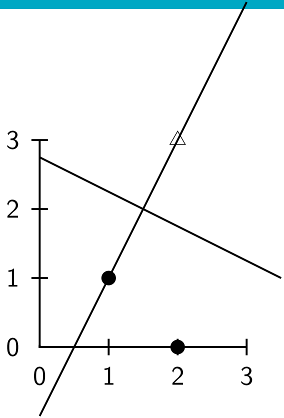
- The SVM decision boundary is:

$$0 = \frac{1}{2}x + y - \frac{11}{4} \Leftrightarrow 0 = \frac{2}{5}x + \frac{4}{5}y - \frac{11}{5}$$



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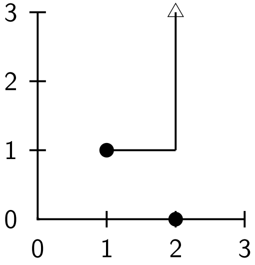

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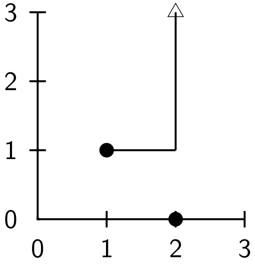

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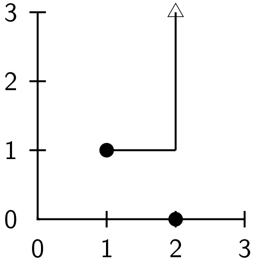

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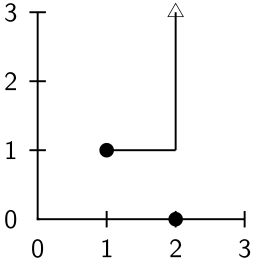

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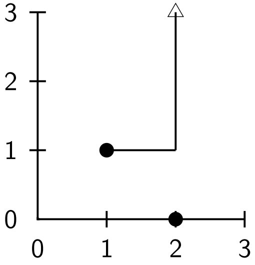
- The margin ρ is $2/|\vec{w}| = 2/\sqrt{4/25 + 16/25} = 2/(2\sqrt{5}/5) = \sqrt{5} = \sqrt{(1-2)^2 + (1-3)^2}$.



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- There are many applications of text classification for corporate Intranets, government departments, and Internet publishers.
- Often greater performance gains from exploiting domain-specific text features than from changing from one machine learning method to another.
- Understanding the data is one of the keys to successful categorization, yet this is an area in which many categorization tool vendors are weak.

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- None?
- Very little?
- Quite a lot?
- A huge amount, growing every day?

If you have no labeled training data

Use hand-written rules

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Example

IF (wheat OR grain) AND NOT (whole OR bread) THEN
$c = \text{grain}$

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In practice, rules get a lot bigger than this, and can be phrased using more sophisticated query languages than just Boolean expressions, including the use of numeric scores. With careful crafting, the accuracy of such rules can become very high (high 90% precision, high 80% recall). Nevertheless the amount of work to create such well-tuned rules is very large. A reasonable estimate is 2 days per class, and extra time has to go into maintenance of rules, as the content of documents in classes drifts over time.

A Verity topic (a complex classification rule)

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comment line	# Beginning of art topic definition
top-level topic	art ACCRUE
topic definition modifiers	/author = "fsmith"
	/date = "30-Dec-01"
	/annotation = "Topic created by fsmith"
subtopictopic	* 0.70 performing-arts ACCRUE
evidencetopic	** 0.50 WORD
topicdefinition modifier	/wordtext = ballet
evidencetopic	** 0.50 STEM
topicdefinition modifier	/wordtext = dance
evidencetopic	** 0.50 WORD
topicdefinition modifier	/wordtext = opera
evidencetopic	** 0.30 WORD
topicdefinition modifier	/wordtext = symphony
subtopic	* 0.70 visual-arts ACCRUE
	** 0.50 WORD
	/wordtext = painting
	** 0.50 WORD
	/wordtext = sculpture

subtopic	* 0.70 film ACCRUE
	** 0.50 STEM
	/wordtext = film
subtopic	** 0.50 motion-picture PHRAS
	*** 1.00 WORD
	/wordtext = motion
	*** 1.00 WORD
	/wordtext = picture
	** 0.50 STEM
	/wordtext = movie
subtopic	* 0.50 video ACCRUE
	** 0.50 STEM
	/wordtext = video
	** 0.50 STEM
	/wordtext = vcr
	# End of art topic

Westlaw: Example queries

Information need: Information on the legal theories involved in preventing the disclosure of trade secrets by employees formerly employed by a competing company

Query: "trade secret" /s disclos! /s prevent /s employe!

Information need: Requirements for disabled people to be able to access a workplace

Query: disab! /p access! /s work-site work-place (employment /3 place)

Information need: Cases about a host's responsibility for drunk guests

Query: host! /p (responsib! liab!) /p (intoxicat! drunk!) /p guest

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Active Learning

A system is built which decides which documents a human should label.
Usually these are the ones on which a classifier is uncertain of the correct classification.

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Good amount of labeled data, but not huge

Use everything that we have presented about text classification.

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Choose classifier based on the scalability of training or runtime efficiency.
Rule of thumb: each doubling of the training data size produces a linear increase in classifier performance, but with very large amounts of data, the improvement becomes sub-linear.

Large and difficult category taxonomies

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Accurate classification over large sets of closely related classes is **inherently difficult**. – No general high-accuracy solution.

Recap

- Is there a learning method that is optimal for all text classification problems?
- No, because there is a tradeoff between bias and variance.
- Factors to take into account:
  - How much training data is available?
  - How simple/complex is the problem? (linear vs. nonlinear decision boundary)
  - How noisy is the problem?
  - How stable is the problem over time?
    - For an unstable problem, it's better to use a simple and robust classifier.

Take-away today

- Vector space text classification: Classification defined as the problem of finding a separating hyperplane in high-dimensional space
- Linear vector space classifiers: Simple, but often very effective
- Support vector machines: State-of-the-art text classification methods (linear and nonlinear)
- Discussion: Which classifier should I use for my problem?

Resources

- Chapter 14 of IIR (basic vector space classification)
- Chapter 15 of IIR (SVMs)
- Resources at `http://ifnlp.org/ir`